# INTERNATIONAL PHYSICS OLYMPIADS

## PROBLEMS AND SOLUTIONS FROM 1967-1995

*C. Manilerd*

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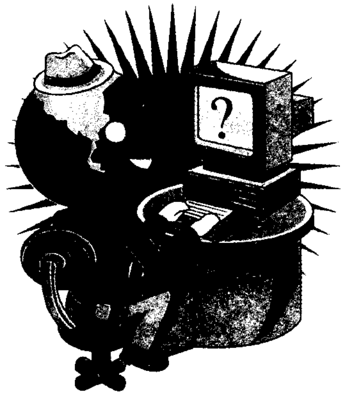


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**Problems and Solutions**

**From 1967 to 1995**



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# International Physics Olympiads Problems and Solutions

# From 1967 to 1995

By

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# ABOUT THE AUTHOR

**CHALEO MANILERD** was born in Tak on 9 January, 1933. He attended Norwich City College, Norwich, UK before going on to the Royal School of Sciences, Imperial College where he recieved a B.Sc. in Physics in 1958. After having taught Physics for three years at what is now known as Chandrakasem Rajbhat Institute, he went to further study in Physics in USA, where he took an M.S. and a PhD. in Physics at University of Detroit and Wayne State University in 1964 and 1968 respectively. He was a lecturer in the Graudate Schools of Chulalonghorn University and Sri Nakarinvirote University. He has been deeply involved in the training of science teachers, particu-larly physics teachers. Dr. Manilerd is currently Senior Lectrurer in the Faculty of Sciences of Rangsit University.

# Acknowledgements

This book is designed to provide a guide for Physics teachers and tutors who are in charge of preparing or wish to prepare their students for physics competitions at the local levels leading on to the International Physics Olympiads. This volume is also available in a Thai version, so as to give a choice to the reader who may prefer to read physics in one language rather than the other.

The first half of the questions i.e. from 1966 to 1986 is taken from the material presented to the author during his participation in a workshop for physics teachers, organized in 1990, Jassberrenyi, Hungary -as part of the activity related to International Physics Olympiads. The rest of the problems has been collected by the author during his terms as Director of the Institute for the Promotion of Teaching Sciences and Technology - with the help of his colleagues who chaperoned Thai students to various International Physics Olympiad competitions during subsequent years.

In writing this book, the author has deliberately avoided excessive quotations on the grounds that most of the physics principles used in this writing are, relatively speaking, are of basic kind ; another reason is that the author is divided between referring and involving fellow physicists in matters which in the final anyalysis must be the responsibility of the author. In spite of this, the author wishes to express his many sincere thanks to Prof. Waldemar Gorzkowski of the Institute of Physics, Polish Academy of Sciences for his encouragement and advice concerning the way in which the teaching and learning of Physics could be improved locally, and also Prof. George Marx and Babara Tosh, both of whom are physicists well known not only in their native country , Hungary, but also around the world, for much of the information on the International Physics Olympiads.

I am also grteatly indebted to Dr. Supat Poopaka, President of Rangsit University - whose encouragement has been the source of inspiration since the inception of this book. Without his help and support, this book would never have been brought to the public in its present ofrm.

Last but not least I wish to express my appreciation and thanks to my dear wife, Ann and my two daughters, Julie and Rosalyn for their sympathy and understanding throughout the time of the preparation of the manuscript of this book.

To all of them I owe my gratitude.

Chaleo Manilerd

Rangsit University

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# Introduction

International Physics Olympiad is a competition on theoretical and practical physics for secondary school students of various countries. It is one of the five competitive events held annually under the aegis of International Mathematical and Sciences Olympiads Committee - with the other four being International Mathematical Olympiad, International Chemistry Olympiad , International Olympiad in Informatics and International Biological Olympiad. The number of entrants from each country varies from four to six depending on Olympiads. For International Physics Olympiad, the number of contestants is limited to no more than five per country.

The first International Physics Olympiad was organized in Poland in 1967 through the personal efforts of a group of physicists under the leadership of Prof. CZesiaw Scistowski. The running of the first competition followed the International Mathematical Olympiad as a model with questions on practical work thrown in. In the first three competitions the number of participating countries was fewer than ten and the future of the comption itself was beset by uncertainty. However due to the dedication and cooperation of Physicists in Hungary, Poland and a few other eastern European countries, the International Physics Olympiads have not only survived,but also evolved into one of the most popular international events and one of the most successful international cooperation programmes in science and technology.

## The Running of International Physics Olympiads

Countries participating in the Olympiad events take turn to host the competition. Besides being responsible for board and lodging of the contest ants and tutors - two from each country, the host country is in charge of preparing questions for the competition based on the approved guidelines as well as the general management and running of the competition.

From the academic point of view , the most important role of the host country is to set problems for the International Committee to consider before giving approval for use in the competition. The International Committee consists of tutors from all participating countries.

The actual competition lasts only two days, with a few intervening days for rest. The contestants and the tutors are of course required to report at the venue two or three days prior to the beginning of the competition. On the day preceding the day of each contest, the International Committee holds the meeting to anylyze the problems in detail . The problems may be altered , modified or added to as deemed necessary before they are approved for the contest. The approved problems are issued in English and Russian versions. In some cases, French or German versions may be added .

After the problems in English and Russian are distributed to all tutors as members of the International Committee, the problems are translated by the tutors or tutor into the languages which are normally used by their respective students. For instance in the case of Thailand, Thai tutors will translate the problems from English into Thai. As a rule, the whole procedure from the scrutiniztion of the problems until the translation of the approved problems into all languages of the participating countries begins from 9.00 o'clock in the morning and lasts throughout the night until the morning of the examination day. The contestants are allowed to answer the problems in their respective languages. At the end of each contest, the tutors are required to translate the answers of their students into English or Russian so that the answers can be marked by the referees appointed for the contest.

Full marks for each problem and also for various steps in each problem are set by the host country with the condition that the ratio of full marks for theoretical and practical parts must be 3:2. Normally, each theoretical problem carries a full mark of 10; while a practical problem, 20, in case there is only one practical problem and 10 each, when there are two practical problems, giving a total of 100 as a maximum mark.

The marks of all contestants are arranged in order of merit. In order to decide the winners of gold , silver and bronze medals including honourable mentions, the highest mark is normally set aside for a special award and the next three highest marks are combined to find the average which is converted to 100%.

Entrants who score:
-more than 90% ; gold medals
-more than 78% but less than 90%; silver medals
-more than 65% but less than 78%; bronze medals.
-more than 50% but less than 65%; honourable mention.

All remaining entrants are given certificates of honour of representing their countries in the contest.

Other awards may be set up and given at the recommendation of the International Committee for Physics Olympiads.

The criteria for all awards given at each Interantional Physic Olympiad are based entirely on performance and not on a quota basis. That is to say if all entrants score more than 90%, all will receive gold medals. By the same token, if all entrants score less than 50%, the only award that each may be given is certificate of honour of representing his or her country in the contest.

The translation of the answers of the contestants into either English or Russian as well as preliminary marking is the responsibility of their respective tutors and must be completed within one day following the end of the contest. While the tutors are busy translating and marking papers, it is time for their charges to go sightseeing and visiting places of historical and cultural importance of the host country, resting and preparing for the remaining contest.

## Characteristics of International Physics Olympiad Problems

International Physics Olympiads are not merely contests between students but also a competition between host countries in designing problems for each year. The quality of the problems of International Physics Olympiad in any year naturally depends on the effort and dedication of physicists of the host country. If one goes through the problems of the International Physics Olympiads of the past twenty four events, we will find that the problems have evolved in a very positive way. Some questions are of course better or more interesting than others.

To decide on the appropriateness or the quality of the problems of International Physics Olympiads in different years, Prof. Waldemar Gorzkowski Secretariate of International Physics Olympiadshas suggested 12 criteria as follows:

1. Beauty - requiring the use of concept or concepts to explain phenomena which are not clearly understood in a concise manner.
2. Originality
3. Evoking interest in the latest application of physics to modern technology.
4. Encouraging the students to learn more on the subjects being asked.
5. Optimum length of the question.
6. The topic is within the syllabus for gifted students.
7. The conciseness and clarity of the question.
8. Allowing the students to express creativeness.
9. Possibility of being solved in more than one way.
10. Providing hints in a subtle manner.
11. Capability of separating students of varying and different levels of abilities.
12. Covering several branches of physics in one problem.

A typical International Physics Olympiad problem considered a good problem may not meet all twelve criteria but should have at least two or three parameters. Undoubtedly the reader will have his or her own idea about what make a good International Physics Olympiad problem based on the criteria suggested above or any other additional cri teria.

# International Physics Olympiad I

# 1967

## Warsaw, Poland

## Theory

### Problem 1

A ball with mass M = 0.2 kg rests on a vertical post of height h = 5 m. A bullet of mass m = 0.01 kg travelling with veloci ty 500 m/s in a horizontal direction before passing through the centre of the ball. The ball hits the ground at a distance of 20 m from the foot of the post. Determine the distance of the spot where the bullet strikes the ground. What part of the kinetic energy of the bullet is transferred to the bullet as heat?

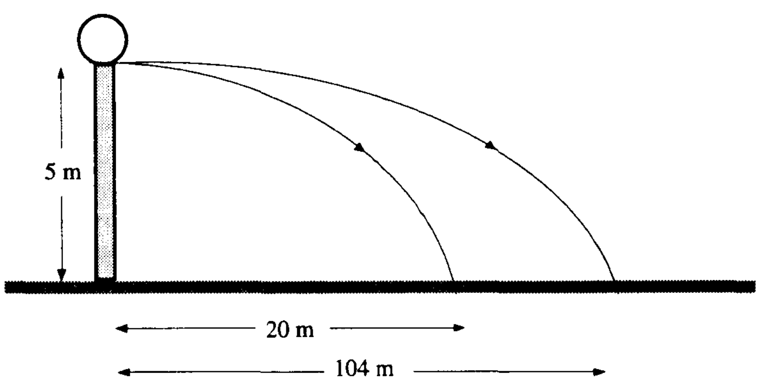



Fig. 1.1 (Not drawn to scale)

$M = 0.2$ kg

$m = 0.01$ kg

$v = 500$ m/s

## Solution

Analysis of the vertical motion of the ball.

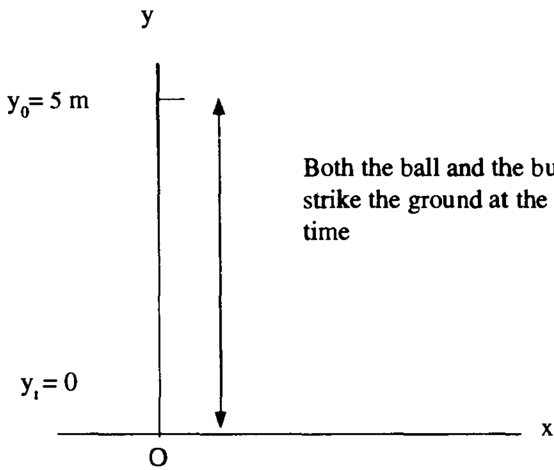



Fig.1.2 Vertical motion of the ball

Apply the formula $\quad y_t = y_0 + u\,t + \frac{1}{2} g\,t^2$

where $y_0$ is the position of the ball measured from the ground at t = 0 s.
$y_t$ is the position of the ball measured from the ground at t s.
u is the initial vertical velocity of the ball = 0 m/s.
g is the gravitational acceleration = - 9.8 m/s$^2$.

Hence

$$0 = 5 - \frac{1}{2} \times 9.8 \times t^2$$

$$t^2 = \frac{10}{9.8} \ s^2$$

$$t = 1.01 \quad s$$

The ball as well as the bullet reaches the ground after 1.01 s.

Analysis of the hoizontal motion of the bullet and the ball.

Let $\mathbf{v}_i$ be the velocity of the bullet before hitting the ball.(before collision)
$\mathbf{v}_f$ be the velocity of the bullet after emerging from the ball. (after collision)
**V** be the velocity of the ball just after the collision.
$\mathbf{v}_i$ $\mathbf{v}_f$ and **V** are directed along the positive x-axis.

From the principle of the conservation of momentum:

$$m\,v_i = m\,v_f + M\,V \qquad (1)$$

$$|V| = \frac{20}{1.01}\ \text{m/s} = 19.8\ \text{m/s}$$

Substitute $V = 19.8$ m/s  $v_i = 500$ m/s
$M = 0.2$ kg  $m = 0.01$ kg

$$0.01 \times 500 = 0.01 \times v_f + 0.02 \times 19.8$$

$$v_f = \frac{1.04}{0.01}$$

$$= 104 \quad \text{m/s}$$

The magnitude of the velocity of the bullet just after the collision $v_f$ is 104 m/s
The bullet also reaches the ground after 1.01 s.
During the time interval of 1.01 s the bullet covers the horizontal distance given by,

$$x = 1.01 \times 104 \quad \text{m}$$

$$= 105 \quad \text{m}$$

The bullet strikes the ground at the distance of 105 m **Ans**

Kinetic energy of the bullet before the collision $= \frac{1}{2}mv^2_i$

$$= \frac{1}{2} \times 0.01 \times 500^2 \qquad \text{J}$$

$$= 1250 \qquad \text{J}$$

Kinetic energy of the bullet just after the collision $= \frac{1}{2}mv^2_i$

$$= \frac{1}{2} \times 0.01 \times 104^2 \qquad \text{J}$$

$$= 54 \qquad \text{J}$$

Kinetic energy of the ball just after the collision $= \frac{1}{2}MV^2$

$$= \frac{1}{2} \times 0.2 \times 19.8^2 \qquad \text{J}$$

$$= 39.2 \qquad \text{J}$$

The kinenic energy of the ball and the bullet just before the collision = 1250 J
The kinetic energy of the ball and the bullet just after the collsion = 54 + 39.2 = 93.2 J

The part of the energy of the bullet transferred into heat = 1250 - 93.2 = 1156.8 J **Ans**

## Problem 2.

Fig. 1.2 below represents an infinite network of resistors each of which is equal to r Ω. Determine the netr esistance across points A and B.

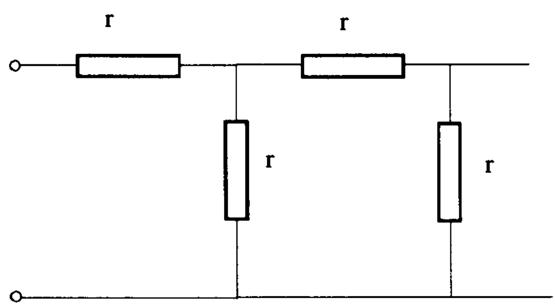



Fig. 1.3 An infinte network of resistors.

## Solution

a. If the network is cut at C and D, let the effective resistance of the separated network to the right of C and D be $R_0$.

b. Now connect $R_0$ across C and D, we have the circuit as shown in Fig. 1.3 below

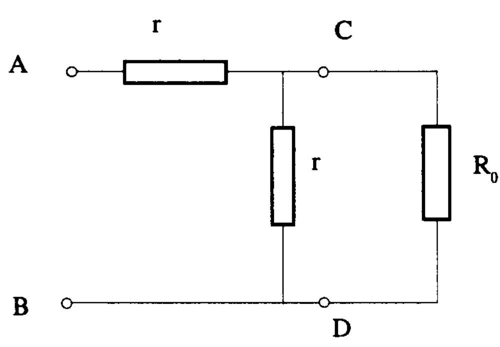



Fig. 1.4 An equivalent circuit to the circuit in Fig.1.2

Let R be the resultant resistance of r and $R_0$ in parallel.

Thus $$\frac{1}{R} = \frac{1}{r} + \frac{1}{R_0}$$

$$R = \frac{rR_0}{(r + R_0)}$$

The resultant resistance of R and r in series is $r + \frac{rR_0}{(r+R_0)}$ .

As the network consists of an infinite number of unit cells, one more unit added to or taken out of the network does not change the net resistance of the network. That is to say the resultant resistance across AB is the same as that across CD, i.e. the resistance of the network to the right of CD.

Hence $$r + \frac{rR_0}{(r+R_0)} = R_0$$

$$r^2 + rR_0 + rR_0 = rR_0 + R_0^2$$

$$R_0^2 - rR_0 - r^2 = 0$$

$$R_0 = \frac{r \pm \sqrt{r^2+4r^2}}{2}$$

$$= \frac{r+\sqrt{5r^2}}{2}$$

$$= 3.2r/2$$

The resistance of the network across AB is 1.6 r Ω **Ans**

# Problem 3.

Given are two steel balls of the same size. One ball is at rest on the horizontal plane, while the other is suspended from a fine chord made from a good insulator. The centres of gravity of the two balls are at the same horizontal level at the beginning of the experiment. The same quantity of heat is applied to both balls. Are the temperatures of the two balls the same? Explain your reasons. ( All kinds of heat losses are negligible. )

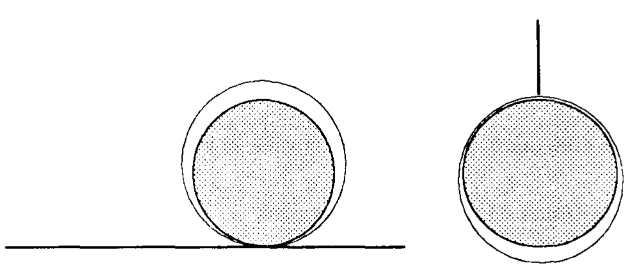


Fig. 1.5

## Solution

Upon being heated, both balls expand in size. For theball resting on the horizontal plane, the centre of gravity is raised upwards from the plane, while for the second ball the centre of gravity is lowered. This implies that part of the heat energy supplied is converted to the potential energy of the first ball, causing the temperature of the first ball to be less than that of the second ball.

# Experiment

## Problem 1.

Determine specific heat of petroleum, using balance, calorimeter, thermometer, electric heater, stop-watch, petroleum liquid, water and accessories provided.

## Solution

The specific heat of petroleum provided can be found by mixing petroleum and water of known masses and temperatures in the calorimeter. The temperature of the mixed solution can be easily measured. Knowing the specific heat and mass of the calaorimeter and the same for water,we can readily calculate the specific heat of the petroleum.

Other alternative is to heat water in the calorimeter using the electric heater provided, and take temperature readings at convennient time intervals. Conduct a similar experiment using the petroleum in the same quantity as that of water used in the preceding experiment. Plot a graph of temperature against time for water and the petroleum.

Let $m_1$ be mass of the water
$c_1$ the specific heat of water
$m_c$ mass of the caloarimeter
$c_c$ specific heat of the calorimeter
$m_p$ mass of the petroleum used in the experiment
$c_p$ specific heat of the petroleum
$t$ heating time
$T$ temperature of the mixed solution at time t
$T_0$ temperautre of the mixed solution at $t = 0$
$H$ heat absorbed by calorimeter per second

Thus $(m_1 c_1 + m_c c_c)(T - T_0) = Ht$

$$T = \frac{Ht}{(m_1 c_1 + m_c c_c)} + T_0$$

Let a be the slope of the graph for water and
b be the slope of the graph for the petroleum liquid.

$$a = \frac{Ht}{(m_1 c_1 + m_c c_c)}$$

and

$$b = \frac{Ht}{(m_p c_p + m_c c_c)}$$

where $m_p$ is mass of petroleum liquid, and $m_p = m_l$

$$\frac{a}{b} = \frac{m_l c_p + m_c c_c}{m_l c_l + m_c c_c}$$

With known values of the specific heat of water and the calorimeter( $c_l$ and $c_c$ ) the specific heat of petroleum can be calculated.

## Problem 2. (An option for those who are not familiar with the use of heater)

A dry air having the volume of 10 $dm^3$ at normal atmospheric pressure and temperature $0^\circ$ C is in a closed container. A quantity of 3 g of water is sent into the container through a specially fitted valve. The container is heated to $100^\circ$ C.Determine the pressure of the container.

## Solution

At $100^\circ$ C the water in the container is in the vapour form.

3 g of water is converted into 3/18 g.mol or 1/6 g.mol
1 g.mol of vapour at 273 K and 1 atm occupies volume of 22.4 $dm^3$
1/6 g.mol of vapour at 273 K and 1 atm correspond to thevolume of 22.4/6 = 3.7 $dm^3$.

Calculation of the vapour pressure at 373 K having the same volume i.e. 10 $dm^3$ .

From the equation of state

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

in which $P_1$ = 1 atm $P_2$ the value of the pressure to be calculated
$V_1$ = 3.7 $dm^3$ $V_2$ = 10 $dm^3$
$T_1$ = 273 K $T_2$ = 373 K

Substitution gives

$$\frac{22.4}{6 \times 273} = \frac{P_2 \times 10}{373}$$

$$P_2 = 0.51 \text{ atm}$$

$P_1$ is the air pressure at 273 K and 1 atm $T_1 = 273$ K
$P_2$ is the air pressure at 373 K and to be determined $T_2 = 373$ K

$$\frac{1}{P_2} = \frac{273}{373}$$

$$P_2 = 1.37 \quad \text{atm}$$

The combined presssure of air and vapour at $100^\circ C$ is $0.51 + 1.37 = 1.88$ atm **Ans**

# International Physic Olympiad II

# 1968

## Budapest, Hungary

## Theory

### Problem 1.

On an inclined plane making an angle $\theta = 30^o$ with the horizontal plane, a solid cylinder of mass $m_1 = 8$ kg and radius $r = 5$ cm is connected to a rectangular block with mass $m_2 = 4$ kg by means of a light chord. (See Fig. 2.1)

1.1 The system is held at rest and then released, calculate the acceleration. The kinetic as well as static coefficient of friction between the block and the inclined plane is $\mu = 0.2$. Friction at the bearing is negligible.

1.2 Find the condition under which the block and the cylinder move with the same acceleration **a** and $a > 0$. Also find the angle which will result in the block and the cylinder moving with the same acceleration.

1.3 Find the angle $\theta$ if the block and the cylinder do not move together with the same velocity.

1.4 Find the condition under which the cylinder will go into pure sliding.

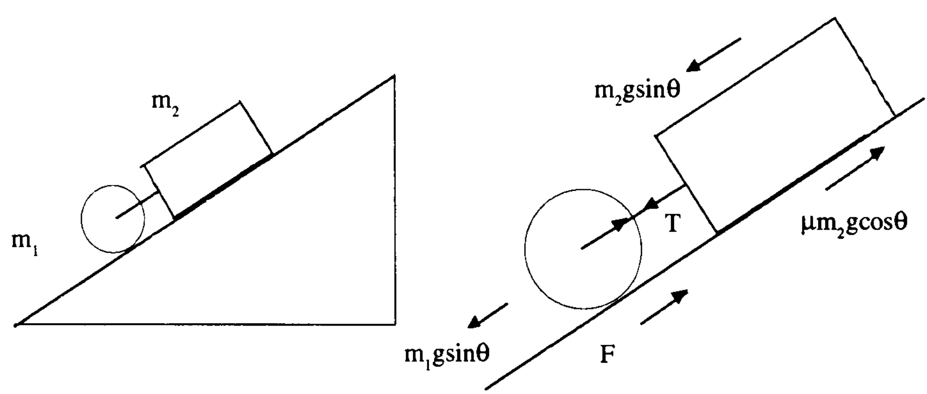



Fig. 2.1

## Solution

1.1 When the cylinder and the block move with the same acceleration **a**, the chord is taut. Let tension in the chord be **T** ($|\mathbf{T}| = T$)
and the friction which the plane exerts on the cylinder be **F** ($|\mathbf{F}| = F$)
The equation of motion of the block of mass $m_2$ is

$$m_2 a \quad = \quad m_2 g \sin\theta - \mu\, m_2 g \cos\theta + T \qquad (1)$$

and that of the cylinder of mass $m_1$ is

$$m_1 a \quad = \quad m_1 g \sin - F - T \qquad (2)$$

( Note that $F \leq \mu N$. In the case of the rectangular block about to go into motion, the frictional force is maximum and equal to $\mu N$, while for the cylinder friction is less than $\mu N$ -sufficient enough to keep the ball rolling without sliding. )
F can be determined from the equation of rotational motion,

Torque $|\tau| = I\ddot{\alpha}$

$$\mathbf{t} = \mathbf{r} \times \mathbf{F}$$

$$F = \frac{\tau}{r} = \frac{I\ddot{\alpha}}{r}$$

$$I = \frac{1}{2} m_1 r^2$$

$$\dot{\alpha} = \frac{|\mathbf{v}|}{r}$$

$$\ddot{\alpha} = \frac{1}{r}\left|\frac{d\mathbf{v}}{dr}\right| = \frac{|\mathbf{a}|}{r} = \frac{a}{r}$$

Substituting $F = Ia / r^2$ in (2) gives

$$m_1 a \quad = \quad m_1 g \sin\theta - I \frac{a}{r^2} \quad - T \qquad (3)$$

(3)+(4) $$(m_1 + m_2)\, a \quad = \quad (m_1 + m_2)\, g \sin\theta \; - \mu g\, m_2 \cos\theta - I \frac{a}{r^2} \qquad (4)$$

$$\left(m_1 + m_2 + \frac{1}{r^2}\right) a \quad = \quad (m_1 + m_2)\, g \sin\theta - \mu\, g\, m_2 \cos\theta$$

$$a \quad = \quad g.\, \frac{(m_1 + m_2)\sin\theta - \mu m_2 \cos\theta}{(m_1 + m_2 + \frac{1}{r^2})} \qquad (5)$$

$$F \quad = \quad I \frac{a}{r^2}$$

$$F = \frac{Ig}{r^2} \cdot \frac{(m_1 + m_2)\sin\theta - \mu m_2 \cos\theta}{(m_1 + m_2 + \frac{1}{r^2})} \tag{6}$$

Substitute F and a in (2) to obtain

$$T = \text{mg}\sin\theta - -\frac{m_1 g[(m_1 + m_2)\sin\theta - \mu m_2 \cos\theta]}{(m_1 + m_2 + \frac{I}{r^2})} - \frac{\frac{Ig}{r^2}(m_1 + m_2)\sin\theta - \mu m_2 \cos\theta}{(m_1 + m_2 + \frac{I}{r^2})}$$

$$= \frac{m_1 g \sin\theta (m_1 + \frac{I}{r^2}) + m_1 m_2 g \sin\theta - (m_1 + \frac{I}{r^2}) g[(m_1 + m_2)\sin\theta - \mu m_2 \cos\theta]}{(m_1 + m_2 + \frac{I}{r^2})}$$

$$= \frac{g(m_1 + \frac{I}{r^2})[m_1 \sin\theta - (m_1 + m_2)\sin\theta \mu m_2 \cos\theta] + m_1 m_2 g \sin\theta}{(m_1 + m_2 + \frac{I}{r^2})}$$

$$= \frac{g(m_1 + \frac{I}{r^2})(\mu m_2 \cos\theta - m_2 \sin\theta) + m_1 m_2 g \sin\theta}{(m_1 + m_2 + \frac{I}{r^2})}$$

$$= \frac{g(m_1 + \frac{I}{r^2})\mu m_2 \cos\theta - g m_1 m_2 g \sin\theta - g m_2 \sin\theta \frac{I}{r^2} + g m_1 m_2 \sin\theta}{(m_1 + m_2 + \frac{I}{r^2})}$$

$$= g m_2 [\frac{\mu(m_1 + \frac{I}{r^2})\cos\theta - \sin\theta \frac{I}{r^2}}{(m_1 + m_2 + \frac{I}{r^2})}]$$

For the cylinder $\frac{I}{r^2} = \frac{1}{2} \times 8 = 4$

Substituting $\frac{I}{r^2} = 4 \quad \mu = 0.2$ in (5) gives

$$a = 9.81 \times \frac{(8 + 4) \times \frac{1}{2} - 0.2 \times 4 \times \sqrt{\frac{3}{2}}}{12 + 4}$$

$$= 9.81 \times \frac{6 - 0.69}{16} = 3.25 \text{ m/s}^2$$

The acceleration of both the block and the cylinder in 1.1 is 3.25 m/s² **Ans**

$$F = a\frac{I}{r^2}$$
$$= 3.25 \times 4 = 13\ \text{N}$$

The friction which the plane exerts on thecylinder is 13 N **Ans**

The friction which the plane exerts on the block is $0.24 \times \cos 30^\circ = .71$ N **Ans**

From (7)
$$T = 9.81 \times 4$$
$$= 39.24 \times = 0.17\ \text{N}$$

Tension in the chord is 0.17 N **Ans**

1.2 The problem here is to find the minimum value of $\theta$ which causes the block and the cylinder to move with the same acceleration $a > 0$

Substituting $a = 0$ in (5) gives

$$0 = g\frac{(m_1 + m_2)\sin\theta - \mu m_2 \cos\theta}{(m_1 + m_2 + \frac{I}{r^2})}$$

$$\tan\theta = \frac{\mu m_1}{(m_1 + m_2)}$$
$$= \frac{0.2}{3} = 0.667$$
$$\theta_2 = 3^\circ 49'$$

The minimum value of the angle of inclination which results in the block and the cylinder moving with the same acceleration $a > 0$ is $3^\circ\ 49'$ **Ans**

1.3 When the block and the cylinder do not move together with the same velocity, tension T in the chord is 0.

Substituting $T = 0$ in (7) gives

$$0 = \frac{gm_2[\mu(m_1 + \frac{I}{r^2})\cos\theta - \frac{I}{r^2}\sin\theta]}{(m_1 + m_2 + \frac{I}{r^2})}$$

$$\tan\theta = \mu(1 + \frac{m_1 r^2}{I}) \qquad \tan\theta = 3\mu = 0.6$$

$$\theta_3 = 30^\circ\ 58'$$

The minimum value of the angle of inclination which results in the block and the cylinder moving with different velocities is 30° 58′ **Ans**

1.4 The cylinder begins to slide when friction F reaches a maximum value i.e. $\mu\ m_1 g \cos\theta$
Substituting $F = \mu\ m_1 g \cos\theta$ in (2) and solve the equation for a from (1) + (2) to obtain

$$a = g(\sin\theta - \mu \cos\theta)$$

and from

$$F = \mu\ m_1 g \cos\theta$$

$$g(\sin\theta - \mu\cos\theta) = 2mg\cos\theta$$

$$\tan\theta = 3\mu$$

$$\theta_4 = \theta_3 = 30^\circ 58'$$

The cylinder begins to slide when the angle of inclination of the inclined plane is 30° 58′ **Ans**

The graph below illustrates change of the tension in the chord T, the friction F exerted by the plane on the cylinder, and the acceleration a of the system consisting of the cylinder and the block for the range of the angle of inclination from θ = 3°49′ to 30° 58′ and also the friction exerted by the inclined plane on thecylinder, the friction exerted by the plane on the block, the accelertion of the cylinder, and the acceleration of the block for the range of the angle of inclination from θ = 30° 58′ to 90°.

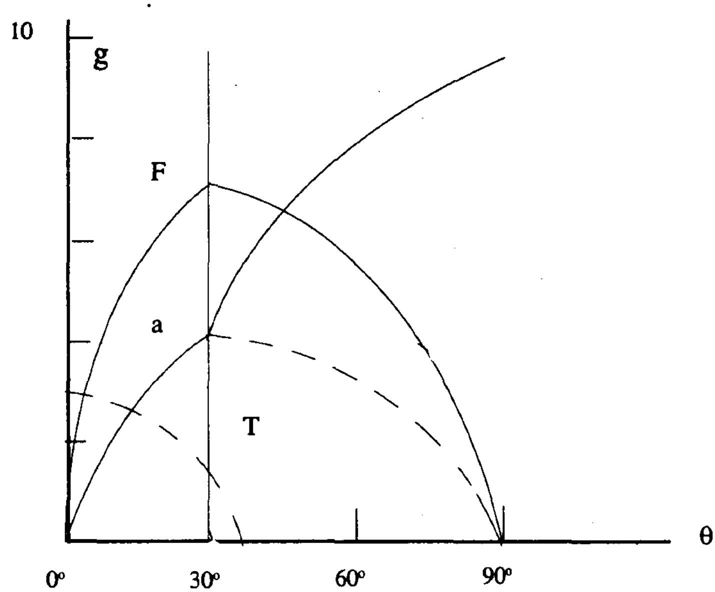



Fig.2.2 Graph of T, F and a pertaining to the bolck and the cylinder as a function of θ

## Problem 2

Toluene liquid of volume 300 $cm^3$ at $0^\circ$ C is contained in a beaker and another quantiy of toluene of volume 110 $cm^3$ at $100^\circ$C is in another beaker. (The combined volume is 410 $cm^3$.) Determine the total volume of the mixture of the toluene liquids when they are mixed together. Given the coefficient of volume expansion $\alpha = 0.001\ C^{-1}$, and all forms of heat losses can be ignored.

## Solution

Let $V_{10}$ be the volume of the toluene liquid in the first beaker at $0^\circ$C
$V_{20}$ be the volume of the toluene liquid in the second beaker at $0^\circ$ C
$\rho_0$ be the density of toluene at $0^0$ C
$m_1$ be mass of toluene in the first beaker
$m_2$ be mass of toluene in the second beaker

Thus

$$m_1 = V_{10}\,\rho_0$$
$$m_2 = V_{20}\,\rho_0$$

After the toluene liquids in the two beakers are mixed together, let the temeperature of the mixture be t.

Heat gained by toluene in the first beaker $= m_1(t - t_1)$
Heat loss suffered by toluene in the second beaker $= m_2(t_2 - t)$
in which $t_1$ and $t_2$ are temperatures of toluene in the first and second beakers respectively prior to mixing.

From the principle of the conservation of heat energy and the experimental conditions

$$m_1(t - t_1) = m_2(t_2 - t)$$

$$(m_1 + m_2)t = m_2t_2 + m_1t_1$$

$$t = \frac{(m_1t_1 + m_2t_2)}{(m_1 + m_2)}$$

The combined volume of the toluene mixture at $t^\circ$ C $= V_{10}(1+\alpha t) + V_{20}(1+\alpha t)$

$$= (V_{10}+V_{20}) + \alpha t(V_{10}+V_{20})$$

$$= V_{10}+V_{20} + \alpha\frac{(m_1t_1 + m_2t_2)}{(m_1 + m_2)} \times \frac{(m_1 + m_2)}{\rho_0}$$

$$= V_{10} + V_{20} + \alpha \frac{m_1 t_1}{\rho_0} + \alpha \frac{m_2 t_2}{\rho_0}$$

$$= V_{10}(1 + t_1 \alpha \frac{m_1 t_1}{\rho_0}) + V_{20}(1 + t_2 \alpha \frac{m_2 t_2}{\rho_0})$$

$$= V_{10} + V_{20} + \alpha V_{10} t_1 + \alpha V_{02} t_2$$

$$= V_{10}(1 + \alpha t_1) + V_{20}(1 + \alpha t_2)$$

$$= V_1 + V_2$$

in which $V_1$ the volume of toluene in the first beaker attemperature $t_1$ ($300\ cm^3$)
$V_2$ the volume of toluene in the second beaker at temperature $t_2$ ($100\ cm^3$)

The outcome of the analysis shows that the volume of toluene mixture is the same as the volumes of toluene in individual beakers combined before mixing. **Ans**

# Problem 3

A parallel beam of light falls on a plane surfaceof a semi-circle cylindrical prism making 45° incident angle. The beam undergoes refraction at the plane surface and strikes the curved surface. ( See Fig. 2.2 ) If the refractive index of the prism is $\sqrt{2}$ , analyze all the beams that emerge from the curved surface.

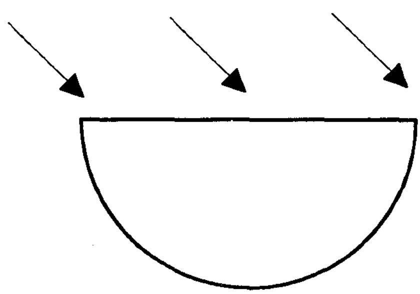


Fig. 2.2

# Solution

In the diagram of Fig 2.3, a straight line drawn from the centre of the prism to the position where a refracted beam meets the curved surface of the prism makes an angle $\phi$ with the horizontal surface. (See Fig. 2.3)

The analysis of the problem will be using angle $\phi$ as the parameter of reference.

Consider the beam which falls on the extreme left end of the prism at A.

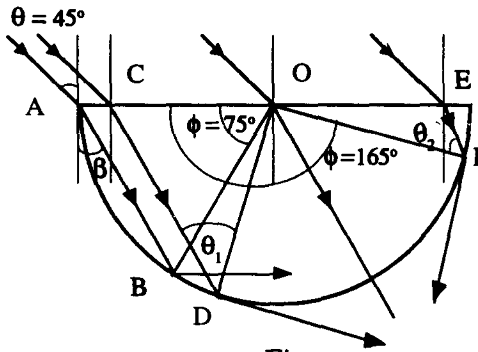



Fig. 2.3

$$\frac{\sin 45^o}{\sin \beta} = {}_a\mu_g = \sqrt{2}$$

$$\sin\beta = \frac{1}{2} \qquad \beta = 30^o$$

From the diagram $\angle OAB = \angle OBA = 60^o$

It is obvious from the diagram that $\phi$ may not be smaller than $60^o$.
Also from the diagram the critical angle is effected at D.
Let $\angle$CDO be represented by $\theta_1$. The law of refraction gives

$$\frac{\sin \theta_1}{\sin 90^o} = {}_g\mu_a = \frac{1}{{}_a\mu_g} = \frac{1}{\sqrt{2}}$$

$$\theta_1 = 45^o$$

$$\phi = 180^o - 60^o - 45^o = 75^o$$

This impies that the beams to the right of A will be refracted at the curved surface until a point E where another critical angle is encountered.

Let $\angle EFO = \theta_2$ likewise

$$\frac{\sin \theta_2}{\sin 90^o} = {}_g\mu_a = \frac{1}{{}_a\mu_g} = \frac{1}{\sqrt{2}}$$

$$\theta_2 = 45^o$$

$$\angle EOF = 180^o - 90^o - 30^o - 45^o = 15^o$$

$$\phi = 180^o - 15^o$$

**Conclusion**. From $\phi = 75^o$ to $165^o$ all the refracted beams will undergo another refraction out of the prism. If $\phi$ is outside this range, the beam will suffer relection at the curved surface. **Ans**

# Experiment

## Problem 1

Each contestant is provided with 3 "Black Boxes" each of which contains one kind of electronic piece of equipment and two terminals on the outside. The contestant is to identify the electronic piece in each black box without opening th box. and then deter mine the characteristic value of the associated piece identified. The following set of equipment is at thedisposal of the contestant, i.e.

1. AC and DC voltmeter and ohmmeter with specified internal resistance and accuracy.
2. AC DC signal generator 50 Hz and DC source.

## Solution

1. With AC and DC voltmeters, measure voltage across the terminals of each black box. No signal is detected, hencethere is no power source such as DC battery.

2. With AC and DC ohmmeters, measure resistance of each box.One box gives the same value of AC and DC resistance, an indication of a simple rsistor. Recordresistance reading R and estimate associated error.

3. For the second box, DC resistance as measured by DC ohmmeter is very high, while AC resistance as measured by AC ohmmeter is rather low - an indiation of a capacitor. Send AC signal with constant voltage into the circuit consisting of known resistance R from 1.1 and capacitor C in box 2 in series.

Let $V_R$ and $V_C$ be AC votages across R and C respectively.

$$V_R = IR$$
$$V_C = \frac{I}{\omega C} \qquad (1)$$

With the known values of R $V_R$ and $V_C$ from the experiment and $\omega = 2\pi\nu = 100\pi$, C can be calculated.

4. For the third box, DC resistance as measured from DC ohmmeter is near zero value, while AC resistance is rather high -an indication of an inductor. Pass AC signal with constant voltage into a circuit consisting of known resistance R from 1.1 and inductor L

in series. (2)

Let $V_R$ and $V_L$ be voltages across R and L respectively.

$$V_R = I R$$

Since the DC resistance of L is very low , voltage across L can be written as

$$V_L = I \omega L$$

$$\frac{V_R}{V_L} = \frac{R}{\omega L}$$

From which L can be calculated.

# International Physics Olympiad III

# 1969

# Brno, Czechoslovakia

## Theory

### Problem 1

A mechanical system depicted in Fig. 3.1 consists of 3 cars having mass $m_A = 0.3$ kg, $m_B = 0.2$ kg, $m_C = 1.5$ kg respectively.

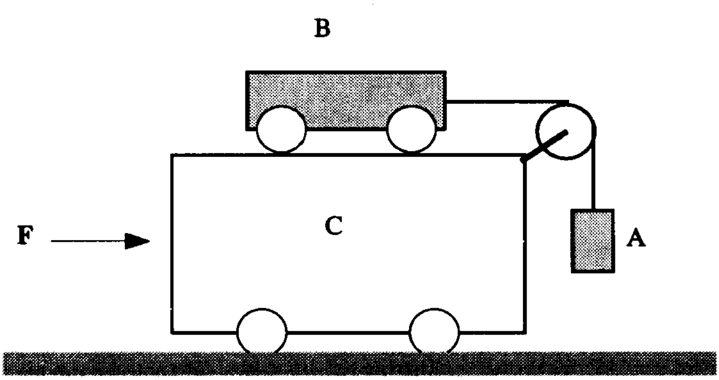



Fig.3.1

1.1. Force **F** acting on car C is large enough to cause cars A and B to remain at rest relative to car C, i.e. all three cars have the same acceleration. Determine tension in the string, acceeration of of A, B and C and the magnitude of force **F**.

1.2 If car C is at rest while force **F** is removed, find the acceleration of A and B and tension in the string.

(N.B. Friction and the moment of inertia of the pulleys are negligible.)

## Solution

1.1 Since car A has no acceleration in the vertical direction ascars A and B are at rest, we may consider cars A, B and C constitute one single system.

Tension in the string $T = m_A g = 0.3 \times 9.81 = 2.94$ N **Ans**

This tension provides the force which causes B to have acceleration a.

Hence $m_A g = m_B a$

$$a = \frac{m_A}{m_B}$$

$$= \frac{0.3}{0.2} \times 9.81 = 14.7 \quad m/s^2$$

The acceleration of the three cars is 14.7 $m/s^2$ **Ans**

Equation of motion in the horizontal direction gives

$$F = (m_A + m_B + m_C)\, a$$

$$= (0.3 + 0.2 + 1.5) \times 14.7$$

$$= 29.4 \quad N$$ **Ans**

1.2 Car C is at rest (no force F), the system consisting ofcars A and B is acted upon by mAg producing acceleration a.

Equation of motion gives

$$m_A g = (m_A + m_B)\, a$$

$$a = \frac{m_A}{(m_A + m_B)}\, g$$

$$= 0.6 \times 9.81$$

$$= 5.9 \quad m/s^2$$ **Ans**

Tension in the string $T = m_A g$

$$= 0.3 \times 5.9$$

$$= 1.77 \quad N$$ **Ans**

## Problem 2

Water of mass $m_2$ is contained in a copper calorimeter of mass $m_1$ . The temperature of the water and the calorimeter is $t_{12}$. A lump of ice of mass $m_3$ and temperature $t_3$ is gently dropped into the calorimeter.(See Fig. 3.2)
Determine all possible temperatures of the mixture if $t_3$ is negative..

Given are:

| | | |
|---|---|---|
| specific heat of copper be | $c_1$ | kcal /kg.C |
| specific heat of water | $c_2$ | kcal/kg.C |
| specific heat of ice | $c_3$ | kcal/kgC |
| latent heat of ice | L | kcal/kg |

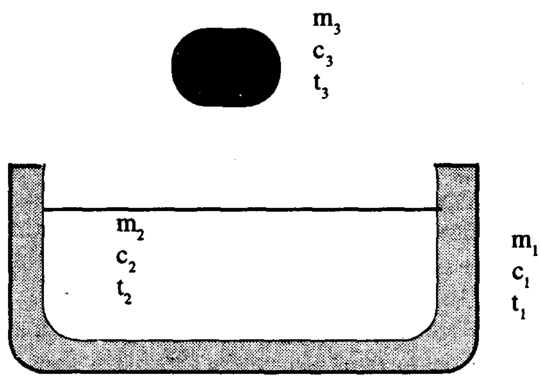



Fig.3.2

## Solution

There are three possible situations after the lump of ice is dropped into the calorimeter i.e.

a. The whole mixture in the calorimeter becomes ice.

b. The mixture in the calorimeter becomes partly ice and partly water at 0° C

c. The mixture in the calaorimeter becomes water with temperature greater than 0° C

**Case a** The lump of ice which begins with temperature lower than 0°C has the temper ature raised to a higher value. Let this temperature be $t_a$

Heat lost = Heat gained

$$c_3m_3(t_a-t_3) = (c_1m_1+c_2m_2)(t_{12}-t_a)+m_2L$$
$$c_3m_3t_a- c_3m_3t_3 = c_1m_1t_{12}-c_1m_1t_a+c_2m_2t_{12}-c_2m_2t_a+m_2L$$

$$(c_1m_1 + c_2m_2 + c_3m_3)t_a = (c_1m_1 + c_2m_2)t_{12} + c_3m_3t_3 + m_2L$$

$$t_a = \frac{(c_1m_1 + c_2m_2)t_{12} + c_3m_3t_3 + m_2L}{c_1m_1 + c_2m_2 + c_3m_3}$$

As the temperature of the mixture lower than 0° C s **Ans**

$$t_a < 0$$

$$(c_1m_1 + c_2m_2)t_{12} + c_3m_3t_3 + m_2L < 0$$

$$(c_1m_1 + c_2m_2)t_{12} < -c_3m_3t_3 - m_2L$$

As the condition for case a to take place. Ans

Consider case C when the mixture becomes water having temperature greater than 0° Cs
Let the temperature of the mixture be $t_c$.

Heat gained = Heat lost

$$c_3m_3(0 - t_3) + m_2L + c_3m_3t_3 = (c_1m_1 + c_2m_2)(t_{12} - t_c)$$

$$-c_3m_3t_3 + m_3L + c_3m_3t_c = c_1m_1t_{12} - c_1m_1t_c + c_2m_2t_{12} - c_3m_3t_3 - m_3L$$

$$(c_1m_1 + c_2m_2 + c_3m_3)t_c = (c_1m_1 + c_2m_2)t_{12} + c_3m_3t_3 - m_3L$$

$$t_c = \frac{(c_1m_1 + c_2m_2)t_{12} + c_3m_3t_3 - m_3L}{c_1m_1 + c_2m_2 + c_3m_3}$$

As temperature of the mixture. **Ans**

$$\text{As } t_c > 0$$

$$(c_1m_1 + c_2m_2)t_{12} + c_3m_3t_3 - m_3L > 0$$

$$-c_3m_3t_3 + m_3L < (c_1m_1 + c_2m_2)t_{12}$$

As a condition for case C to take place. **Ans**

Consider case B when the mixture becomes partly ice and partly water at 0° C.
The condition for case B to take place is the combination of cases A and C together.

$$-c_3m_3t_3 + m_3L < (c_1m_1 + c_2m_2)t_{12} < -c_3m_3t_3 - m_3L$$

If only x g of ice is changed to water, then

$$- c_3m_3t3 + xL = (c_1m_1 + c_2m_2)t_{12}$$

$$x = \frac{(c_1m_1 + c_2m_2)t_{12} + c_3m_3t_3}{L}$$

The quantity of ice remains in the calorimete is $m_3 - \dfrac{(c_1m_1 + c_2m_2)t_{12} + c_3m_3t_3}{L}$

The mass of water in the calorimeter is $m_2 + \dfrac{(c_1m_1 + c_2m_2)t_{12} + c_3m_3t_3}{L}$ **Ans**

## Problem 3

A circular piece of wire with radius R = 5 cm (See Fig. 3.3) is held in a vertical plane. A light chord made from a perfect insulator of length l is suspended from the highest point of the ring. The other end of the chord is attached to a small conducting sphere of mass m = 1 g . Electic charge of the same sign and the same quantity $q = 9 \times 10^{-7}$ C is given to the ring as well as the small conducting sphere. As a result the sphere finds itself in equilibrium at a point on the axis of symmetry perpendicular to the plane of the ring, with the light chord making an angle $\alpha$ with the axis of symmetry. Determine length l.

( Given $k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N.m²/C² )

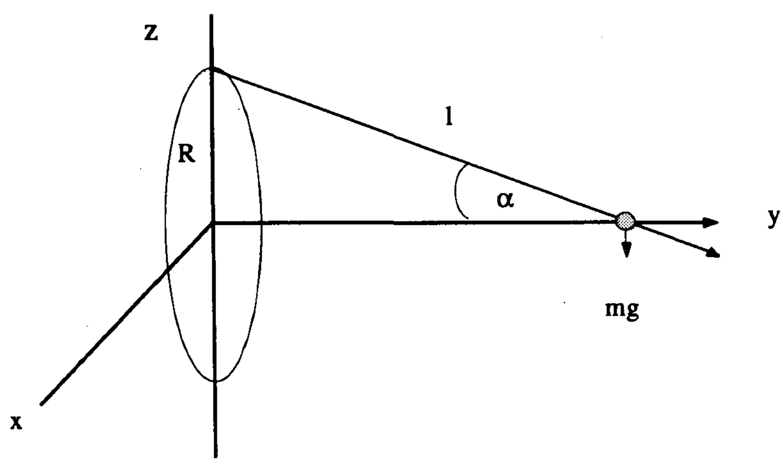



Fig.3.3

## Solution

Linear charge density per unit length $= \dfrac{q}{2\pi R}$

Each section designated by $\Delta s$ along the ring exerts force on the sphere $= \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2\Delta s}{l^2}$

with the direction of force along the line of the chord as indicated in the diagram.

However only the components along the axis of the symmetryonly that contribute to the net force.

$$\text{Net force} \qquad F = \frac{q^2}{4\pi\varepsilon_0}\int_0^{2\pi R}\frac{\cos\alpha}{l^2}ds$$

$$= \frac{kq^2\cos\alpha}{l^2}$$

$$\frac{mg}{F} = \tan\alpha$$

$$\frac{mgl^2}{kq^2\cos\alpha} = \tan\alpha$$

$$\frac{mgl^2}{kq^2} = \sin\alpha$$

Substituting $\sin\alpha = \dfrac{R}{l}$ and $l^3 = \dfrac{kq^2}{mg}$

and k $= 9\times10^9$ $Nm^2/C^2$

q $= 9\times10^{-7}$ C

m $= 10^{-3}$ kg

g $= 9.81$ $m/s^2$

gives $$l^3 = \frac{5\times10^{-2}\times9\times10^9\times9\times10^{-4}}{10^{-3}\times9.81}$$

$$l = 7.2\times10^{-2} \text{ m} \quad \textbf{Ans}$$

## Problem 4

A thin glass slab is held over a glass cube having each side in the order of 2 cm in length, creating an air gap of uniform thickness between the slab and the cube. (See Fig. 3.4) Electromagnetic waves having wavelengths from 0.4 $\mu$m to1.15 $\mu$m travel along the direction perpendicular to the slab are reflected at the boundaries of the air gap and interfere. It is found that only two wavelengths in the range described above give constuctive interference. One of these two wavelengths is $\lambda_1$ = 0.4 $\mu$m, determine the second wave length and the thickness of the air gap.

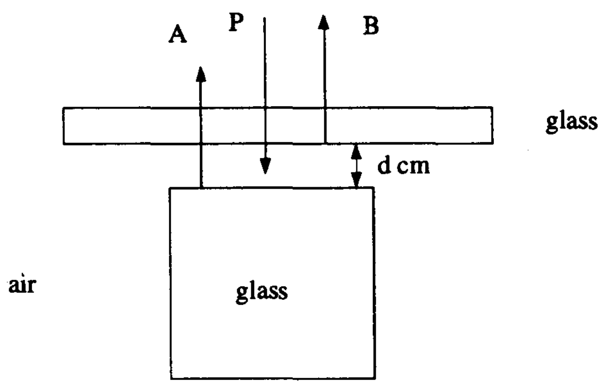



Fig. 3.4

## Solution

Consider a plane wave travelling in the direction represented by P. Part of the wave is transmitted through the upper glass slab and reflected at the top surface of the cube, while the other part is reflected at the lower surface of the glass slab.

From the principle of the reflection, we observe that when light beam is reflected at the boundary separating two media of lower density and higher density, the light beam reflected back into the medium of lower density undergoes a phase change of $180^\circ$ corresponding to the distance of $\lambda/2$ . This is what happens to light A, while light B that is reflected back into the slab suffers no phase change.

Let the thickness of the air gap be d $\mu$m.

The condition that light A reflected at the upper surface of the cube and light B reflected at the lower surface of the glass slab will interfere constructively is, both beams must be in the same phase.
From the diagram, light A travesres a distance to and back of 2d inside the air gap and upon emerging from lower surface of the glass slab must be in the same phase with light B

From the diagram, light A travesres a distance to and back of 2d inside the air gap and upon emerging from lower surface of the glass slab must be in the same phase with light B ( reflected at the upper surface of the slab). This means that 2d can be written as an odd (instead of even) integral number of half wavelengths.

For the first wavelength $2d = (2n_1+1)\dfrac{\lambda_1}{2}$ (1)

whereas $n_1$ is an integral number = 0,1,2,3,.......

Substituting $\lambda_1 = 0.4\ \mu m$

yields $2d = (2n_1+1)0.2$ (2)

For the second wavelength $2d = (2n_2+1)\dfrac{\lambda_2}{2}$ (3)

whereas $n_2$ is an integral number = 0,1,2,3,........

Substituting 2d from (2) in (3) gives

$$\lambda_2 = \frac{2n_1+1}{2n_2+1} \quad (4)$$

From the information given in the problem

$$0.4 < \lambda_2 < 1.15\ \mu m \quad (5)$$

Substituting in (4)

| | | | |
|---|---|---|---|
| $n_1 = 0$ | $n_2 = 0$ | $\lambda_2 = 0.4\ \mu m$ | $( = \lambda_1 )$ |
| $n_1 = 0$ | $n_2 = 1$ | $\lambda_2 = \frac{1}{3}\times 0.4$ | $= 0.13\ \mu m$ |
| $n_1 = 1$ | $n_2 = 0$ | $\lambda_2 = 3\times 0.4$ | $= 1.2\ \mu m$ |
| $n_1 = 1$ | $n_2 = 1$ | $\lambda_2 = 0.4$ | $\mu m$ |
| $n_1 = 2$ | $n_2 = 0$ | $\lambda_2 = 5\times 0.4$ | $= 2.0\ \mu m$ |
| $n_1 = 2$ | $n_2 = 1$ | $\lambda_2 = \frac{5}{3}\times 0.4$ | $= .67\ \mu m$ |

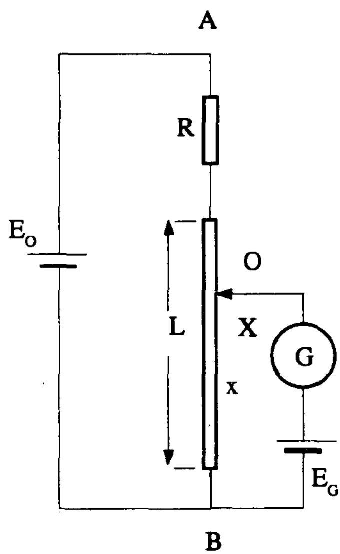
The value of $\lambda_2 = .67\ \mu m$ fits condition (5) and the problem indicates that besides $\lambda_1$ there is only one more wavelength. The second wavelength is therefore 0.67 µm **Ans**

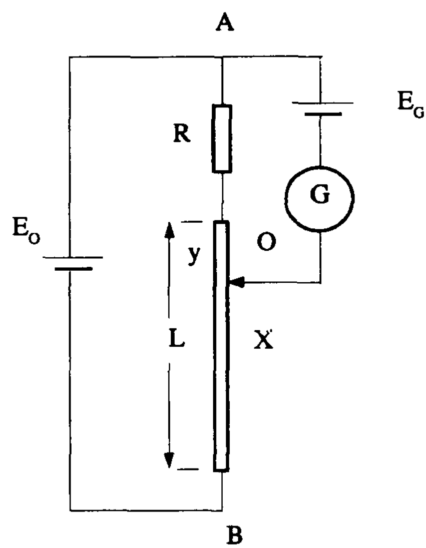
# Problem 1

Each contestant is provided with a DC voltage souce $E_0$ ( unknown) of negligible internal resistance, a resistor of known resistance R, a resistor of unknown resistance X in the form of a long thin wire placed along a scale fitted with a sliding contact (thu s X can be adjusted), and a zero-centred galvanometer and another dry cell of known voltage $E_g$.

Connect a circuit consisting of voltage source, and resistors R and X in series. The dry cell, the zero galvanometer together with the sliding contact are connected to the main circuit in order to locate the position of zero reading on the galvanometer.(See Figs 3.5 and 3.6)

Determine voltage $E_0$ and resistance X.




## Solution

Fig. 3.5

Fig. 3.6

Connect a circuit as shown in Figs. 3.4 and 3.5 as required by the needs of the problem.

Let the length of resistor X be L and resistance per unit length be ρ .

From the diagram of Fig. 3.5

Let x be the length of the wire resistance which results in 0 reading of the galvanometer.

In this position $$\frac{E_G}{E_O} = \frac{\rho x}{R + \rho L}$$

In this position $$\frac{E_G}{E_O} = \frac{\rho x}{R + \rho L} \tag{1}$$

From the diagram of Fig.3.6

Let y be the length of the wire resistance which results in 0 reading of the galvanometer.

In this position $$\frac{E_G}{E_O} = \frac{R + \rho y}{R + \rho L} \tag{2}$$

$$\frac{\rho x}{R + \rho y} = \frac{R + \rho y}{R + \rho L}$$

$$\rho\,(x - y) = R$$

$$\rho = \frac{R}{(x - y)} \tag{3}$$

$$X = \rho L$$

$$X = \frac{RL}{(x - y)} \tag{4}$$

Substituting R = r(x-y) from (3) in (1) gives

$$\frac{E_G}{E_O} = \frac{\rho x}{\rho(x - y + L)}$$

$$E_O = \frac{(x - y + L)}{x} E_G \tag{5}$$

Substituting R, L, x and y known or measued in the experiment in (4) to obtain X.

Substituting $\rho = \frac{X}{L}$ , x , y and $E_G$ in (5) to obtain $E_0$.

# International Physics Olympiad IV

# 1970

# Moscow, USSR

## Theory

### Problem 1

In Figs.4.1 and 4.2 below, a rectangular wooden block of mass $m = 0.1$ kg rests on a wooden board of mass $M = 1$ kg . A powered winch on the block pulls a string and thus the wooden block with veocity $v_0 = 0.1$ m/s towards the wall.
Given are:

The friction between the wooden board and the surface of the supporting table is zero.

The coefficient of kinetic friction between the rectangular block and the wooden board $\mu = 0.02$.

The experiment begins with the wooden board held at rest, and the motor for the winch is switched on. As soon as the rectangular block reaches the velocity $v_0 = 0.1$ m/s , the wooden board is released. If the moment the wooden board is let go, the length of the string between its fixed end and the rectangular wooden block $L = 0.5$ m
Describe the motion of the rectangular wooden block and thewooden board when:

1.1 the string from the winch is attached to the wall (Fig.4.1)

1.2 the string from the winch is attached to a post that is fixed to the wooden board. (Fig.4.2). In this case calculate the time for the wooden rectangular block to reach the edge of the wooden board.

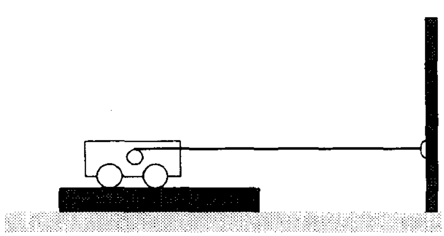


Fig. 4.1

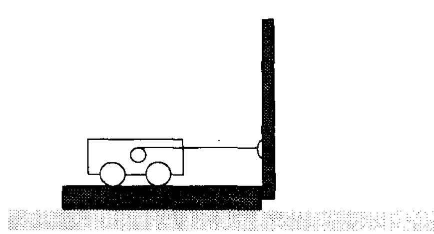


Fig.4 .2

## Solution

1.1 In the first case (Fig. 4.1) the wooden rectangular block moves with constant velocity, but the block and the supporting wooden board do not move with the same velocity. Under such situation the friction between the block and the board is maximum and equal to μmg. The friction on the wooden block is directed towards the left (opposite to the direction of motion), while the friction acting on the board is directed towards the right and equal in magnitude to the friction on the wooden block.

Let the acceleration of the board towards the wall be a

Hence $Ma = \mu m$

Subsitution gives $a = 0.02 \times 0.1 \times 9.81$

$= 0.02 \quad m/s^2$

Suppose it takes time $t_0$ for the board to reach velocity $v_0$

From the formula $v = u + at$

Subsitution gives $v_0 = 0 + a t_0$

$$t_0 = \frac{v_0 M}{\mu m g}$$

$$= \frac{0.1}{0.2 \times 0.1 \times 9.81}$$

$= 5.1 \quad s$

Within the time interval of 5.1 s, the wooden board covers the distanceof $\frac{1}{2} \times 0.02 \times 5.1^2$ m

$= 0.25$ m

Since there is no friction between the wooden board and the supporting table, the wooden block and the wooden board move together with the same velocity $v_0$, The motor pulling the winch no longer dispenses any power even though the winch still keeps turning. The role of the motor is reduced to reeling the string. Also the block never reaches the edge of the board. **Ans**

1.2 In this latter case the end of the string is attached to the post fixed to the wooden board as depicted in Fig. 4.2. As soon as the board is released, it immediatelyaquires momentum $mv_0$. As there is friction between the block the wooden board but no friction between the wooden boardand the supporting table. The system consisting of mass m and M moves with the net momentum of $mv_0$.

Let V be the velocity of the board of mass M when it is released.

Hence $MV + m(v_0+V) = mv_0$

$V(M+m) = 0$

$V = 0$

This implies that the board remains at rest while the block moves on with the velocity of 0.1 m/s until reaching the end of the board within the time interval of $\frac{0.5}{0.1} = 5$ s **Ans**

After the time interval of 5 s, the motor ends its function, and the block and the board move together with velocity $V_f$. The value of $V_f$ is determined in the following way.

$$V_f(M+m) = mv_0$$
$$1.1V_f = 0.1 \times 0.1 \text{ m/s}$$
$$V_f = 0.1 \text{ m/s}$$

As soon as the block reaches the edge of the board, the two bodies move together with final velocity 0.1 m/s **Ans**

## Problem 2.

The crystal structure of NaCl consists of elementary cubes each of which has Na atom at each of the 8 corners and also Na atom at the centre of each of the 6 surfaces. ( face-centred cubic). The length of the elementary cube is $5.6 \times 10^{-8}$ cm.

Determine the mass of hydrogen atom.
Atomic weight of Na = 23 Atomic weight of Cl = 35.5. Density of NaCl = 2.2 g/cm$^{-3}$

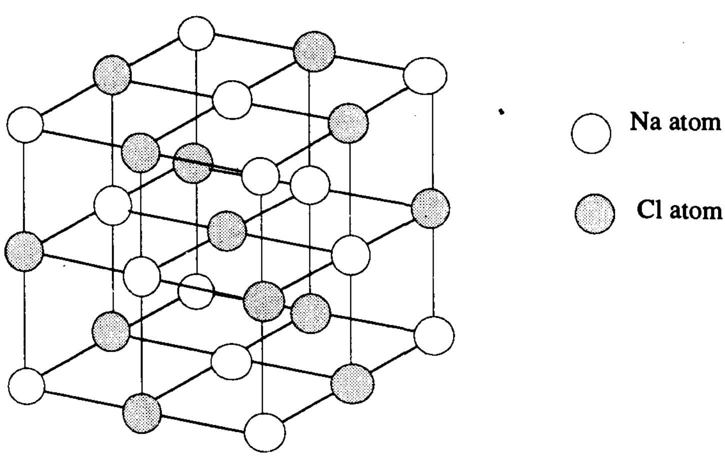



Fig. 4.3

## Solution

**Determination of the number of Na atoms in the elementary cube.**

Each Na atom at the cornor of the elementary cube does not belong to the cube under discussion alone but is shared with its seven immediate neighbours. Likewise Na atom on each side of the cube must also be shared with its neighbour.

Na atoms at the eight cornors are reduced to $\frac{1}{8} \times 8 = 1$ Na atoms.

Na atoms at the six surfaces are reduced to $\frac{1}{6} \times 6 = 3$ Na atoms.

In 1 elementary cube there are 3 +1 = 4 Na atoms.

If m is the mass of one nucleon or $^1H$ expressed in g

Density of NaCl crystal is given by

$$\frac{4 \times 23 \times m + 4 \times 35 \times m}{(5.6 \times 10^{-8})^3} = 2.2 \text{ g/cm}^{-3}$$

( There are 23 $^1H$ or 23 nucleons in Na atom and 35 $^1H$ or nucleons in Cl atom )

$$\frac{4 \times 58m}{(5.6 \times 10^{-8})^3} = 2.2$$

$$m = 1.66 \times 10^{-24} \text{ g}$$

Atomic mass of hydrgen atom is $1.66 \times 10^{-27}$ kg **Ans**

## Problem 3

A metal sphere of radius r = 10 cm is placed concentrically inside another thin metal hollow sphere of radius R = 20 cm. Both spheres are separated from one another. The inner sphere is earthed by a long wire through an opening of the outer sphere. (See Fig. 4.4) ] If the outer sphere carries charge $Q = 10^{-9}$ C , find potential V of the outer sphere

Given: $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N.m./ $C^2$

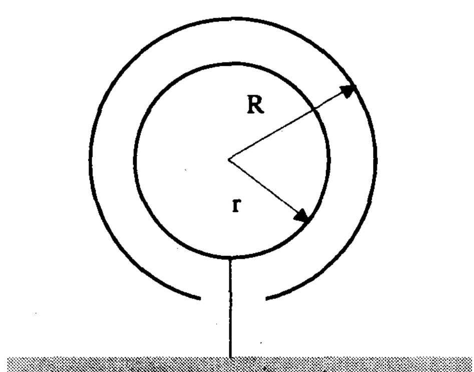



Fig. 4.4

# Solution

It follows from the diagram in Fig 4.4 that the outer sphere functions as a capacitor with the outside surface as one plate and the ground as other plate. Also the outer sphere and inner sphere together constitute another capacitor. This reasoning becomes obvious once the electric lines of force are drawn.

The two capacitors are connected in paralell.

Capacitance of the first capacitor $C_1 = \frac{R}{k}$

Capacitance of the second capacitor $C_2 = \frac{Q}{V}$

Where V is the electric potential difference between the two spheres.

Thus
$$V = \frac{kQ}{(\frac{1}{r} - \frac{1}{R})}$$
$$= kQ\frac{(R-r)}{rR}$$
$$C_2 = \frac{rR}{k(R-r)}$$

Since the two capacitors are conneced in parallel, resultant capacitance
$$C = C_1 + C_2$$
$$= \frac{1}{k}\left[R + \frac{rR}{R-r}\right]$$
$$= \frac{R^2}{k(R-r)}$$

Substitute R = 0.2 m
t = 0.1 m
k = $9 \times 10^9$ N.m./$C^2$

$$C = \frac{1}{9 \times 10^9} \times \frac{0.2^2}{(0.2 - 0.1)}$$
$$= 44.4 \times 10^{-12} \quad F$$

Electric potential of the outer sphere
$$V = \frac{Q}{C}$$
$$= \frac{10^{-8}}{44.4 \times 10^{-12}}$$
$$= 227 \text{ V} \quad \textbf{Ans}$$

## Solution

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Each Na atom at the cornor of the elementary cube does not belong to the cube under discussion alone but is shared with its seven immediate neighbours. Likewise Na atom on each side of the cube must also be shared with its neighbour.

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If m is the mass of one nucleon or $^1H$ expressed in g
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$$\frac{4 \times 23 \times m + 4 \times 35 \times m}{(5.6 \times 10^{-8})^3} = 2.2 \text{ g/cm}^{-3}$$

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If the outer sphere carries charge $Q = 10^{-9}$ C , find potential V of the outer sphere

Given: $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N.m./ C²

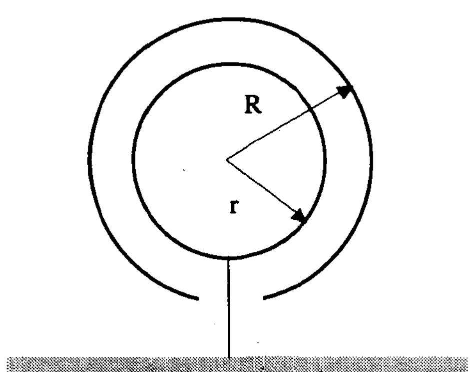


Fig. 4.4

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Where V is the electric potential difference between the two spheres.

Thus
$$V = \frac{kQ}{(\frac{1}{r} - \frac{1}{R})}$$

$$= kQ\frac{(R-r)}{rR}$$

$$C_2 = \frac{rR}{k(R-r)}$$

Since the two capacitors are conneced in parallel, resultant capacitance

$$C = C_1 + C_2$$

$$= \frac{1}{k}\left[R + \frac{rR}{R-r}\right]$$

$$= \frac{R^2}{k(R-r)}$$

Substitute $R = 0.2$ m

$t = 0.1$ m

$k = 9 \times 10^9$ N.m./ $C^2$

$$C = \frac{1}{9\times 10^9} \times \frac{0.2^2}{(0.2-0.1)}$$

$$= 44.4 \times 10^{-12} \quad F$$

Electric potential of the outer sphere

$$V = \frac{Q}{C}$$

$$= \frac{10^{-8}}{44.4\times 10^{-12}}$$

$$= 227 \text{ V} \quad \textbf{Ans}$$

## Problem 4

A concave mirror with radius r = 2 m has the diameter of its circular edge d = 0.5 m. At its focal point, a circular screen is placed normal to the principal axis so that all incident beams paralell to the axis after relection by the mirror should fall on the screen. Find the diameter of the screen. What part of the light reaches the screen at the same position if the diameter of the screen is only of the original value?

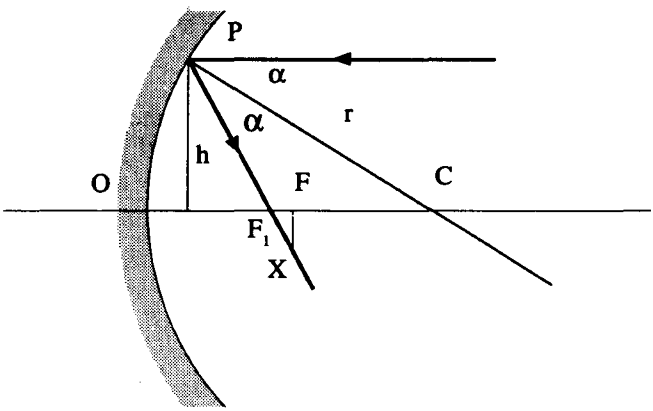



Fig. 4.5

## Solution

According to the elementary principle of the reflection at the spherical surface, the focal point should be at a mid-point between the centre of curvature C and the pole of the mirror O. However this is true only for the beams close to the axis (paraxial rays) only.

Consider a parallel! beam incident on the concave mirror at its edge at distance h from the axis.

After reflection, the beam intersects the axis at $F_1$.

If we place the screenat F, FX constitutes the radius of smallest possible screen.

$$FX = FF_1 \tan 2\alpha$$
$$EF_1 = CF_1 - C$$
$$FX \sim \left(\frac{r}{2\cos\alpha} - \frac{r}{2}\right)\tan 2\alpha$$

For small α $\quad \tan 2\alpha \sim \sin 2\alpha$

$$\sin\alpha \sim \frac{h}{r}$$

$$\cos\alpha \sim 1 - \frac{h^2}{2r^2}$$

$$\sin 2\alpha \sim (\frac{2h}{r})(1 - \frac{h^2}{2r^2})$$

$$\frac{1}{\cos\alpha} \sim \frac{1}{\left(1 - \frac{h^2}{2r^2}\right)}$$

$$FX \sim \frac{r}{2}\left(1 + \frac{h^2}{2r^2} - 1\right)\frac{2h}{r}$$

$$\sim \frac{h^3}{2r^2}$$

Substituting h = 20 cm r = 200 cm

$$FX \sim \frac{25^3}{2 \times 200^2}$$

$$\sim 0.195 \quad \text{cm}$$

The diameter of the screen weshould have is 0.39 cm **Ans**

The intensity of light is proportional to the area of the screen which in turn is proportional to $h^{2}$.

Let $FX_1$ be radius of the original screen

$FX_2$ radius of the new screen

$h_1$ vertical distance of the position where the beam incident on the mirror and falls on the original screen

$h_2$ vertical distanc of the position where the beam incident on the mirror and falls on the new screen

Thus $$\left(\frac{h_2}{h_1}\right)^2 = \left(\frac{FX_2}{FX_1}\right)^{\frac{2}{3}}$$

$$= \left(\frac{1}{8}\right)^{\frac{2}{3}} = \left(\frac{1}{4}\right)$$

The intensity of light is 1/4 of the original intensity if the radius of the new screen is 1/8 of that of the original screen **Ans**

# Experiment

## Problem 1

Each contestant is provided with 3 lenses mounted on stands, a screen with affixed geometricl design, a stick, a meassuring tape. Determine focal distance of each lens by different methods using only apparatus provided.

## Solution

1. Determine roughly the focal length of the convex lens by means of focusing distant objects ( $u = \infty$ ): focal length f may be measured by the tape.

2. Use a stick as an object for the convex lens, locate the position of the image on the screen . Measure v and u and calculate the foca l length from the formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$.

In the case of the concave lens, or virtual image is formed instead of real image, the location of the image may be carried out by means of parallax.

3. Combine 2 lenses ( convex and concave lenses or two convex lenses together) and conduct experiment similar to that described under 2.

If F is the focal length of the combined lenses
$f_1$ the focal length of the first lens in the combined lenses
and $f_2$ the focal length of the second lens also in the combined lenses

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

With known values of $f_1$ and F, $f_2$ may be calculated from the above formula.

# International Physics Olympiad V

# 1971

# Sofia, Bulgaria

## Theory

### Problem 1.

Two blocks of masses $m_1$ and $m_2$ are secured to a light string one to each end. The sring is lobbed over a small pulley on top of the inclined plane. (See Fig.5.1) The plane has mass m, while the angle of inclination on one side being $\alpha_1$ and other side $\alpha_2$. The whole ensemble is placed on the table which is frictionless. The experiment begins with masses $m_1$ and $m_2$ are held at rest and then set free.

Determine:

1.1 acceleration of the inclined plane
1.2 accelerations of masses $m_1$ and $m_2$
1.3 condition for the inclined plane to remain stationary.

( The surfaces of the inclined plane are frictionless. Also assume that block of mass $m_1$ moves downward.)

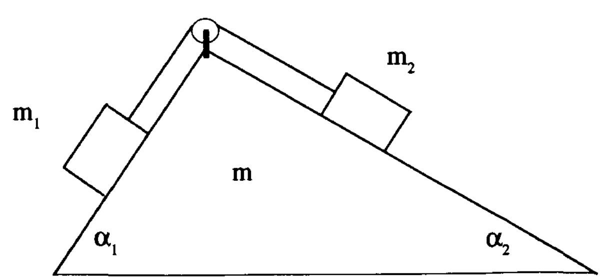



Fig. 5.1

# Solution

Observe that the motion of $m_1$ and $m_2$ is such that $m_1$ moves down the plane while $m_2$ up the plane.

Let **a** be the acceleration of the inclined plane directed towards the right
$\mathbf{a}_{10}$ acceleration of $m_1$ in the direction of motion along its slope ( relative to the plane)
$\mathbf{a}_{20}$ acceleration of $m_2$ in the direction of motion along its slope( relative to the plane)

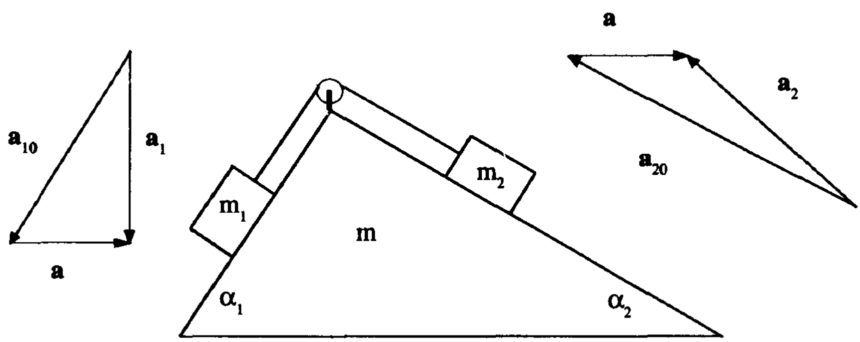



Fig. 5.2

Observe that $m_1$ and $m_2$ move together along their respective slopes.

$$|\mathbf{a}_{10}| = |\mathbf{a}_{20}| = a_0$$

For the observer at rest $m_1$ and $m_2$ have the acceleration values of $a_1$ and $a_2$ respectively.

$$\mathbf{a}_1 = \mathbf{a}_{10} + \mathbf{a}$$
$$\mathbf{a}_2 = \mathbf{a}_{20} + \mathbf{a}$$

The component of **a**, resolved along the slope on the left side = $a \cos\alpha_1$ (pointing up the slope)

The component of **a**, resolved along the slope on the right side = $a \cos\alpha_2$ (pointing down the slope)

For the observer art rest, the equations of motion of $m_1$ and $m_2$ are:

(For $m_1$) $m_1 g \sin\alpha_1 - T = m_1( a_0 - a \cos\alpha_1)$ (1)

(For $m_2$) $T - m_2 g \sin \alpha_2 = m_2( a_0 - a \cos\alpha_2)$ (2)

Where T is tension in the string.

(1)+(2) $(m_1 \sin\alpha_1 - m_2 \sin\alpha_2)g = (m_1 + m_2)a_0 + (m_1 \cos\alpha_1 + m_2 \cos\alpha_2)\, a$ (3)

**Calculation of velocity and momentum of the system of masse m of the inclined plane plus mass $m_1$ and mass $m_2$ (both are at rest with respect to the inclined plane)** ·

At any point in time, let the values of the velocitiy of $m_1$ and $m_2$ along their respective inclined slopes be $|\mathbf{v}_0|$, the magnitude of the velocity of the inclined plane along the horizontal direction be $|\mathbf{v}|$

Hence, the magnitude of the velocity of $m_1$ along the horizon is $v_0 \cos\alpha_1 - v$

likewise, the magnitude of the velocity of $m_2$ along the horizon is $v_0 \cos\alpha_2 - v$.

From the principle of the conservation of momentum

$$m_1(v_0 \cos\alpha_1 - v) + m_2(v_0 \cos\alpha_2 - v) = mv \qquad (4)$$

For motion of constant acceleration that begins from the rest, the magnitude of the velocity is proportional to the magnitude of the acceleration along the same direction.
From (4)

$$m_1(a_0 \cos\alpha_1 - a) + m_2(a_0 \cos\alpha_2 - a) = ma$$

$$a = \left[\frac{m_1 \cos\alpha_1 + m_2 \cos\alpha_2}{m_1 + m_2 + m}\right] a_0 \qquad (5)$$

Substituting a from (5) in (3) yields

$$a_0 = \left[\frac{(m_1 + m_2 + m)(m_1 \sin\alpha_1 - m_2 \sin\alpha_2)}{(m_1 + m_2)(m_1 + m_2 + m) - (m_1 \cos\alpha_1 + m_2 \cos\alpha_2)}\right] g$$

$$a = \left[\frac{(m_1 \cos\alpha_1 + m_2 \cos\alpha_2)(m_1 \sin\alpha_1 - m_2 \sin\alpha_2)}{(m_1 + m_2)(m_1 + m_2 + m) - (m_1 \cos\alpha_1 + m_2 \cos\alpha_2)}\right] g$$

Inclined planes are at rest $a = 0$

From (5) it follows that $a_0 = 0$

From (6) and (7) $\dfrac{m_1}{m_2} = \dfrac{\sin\alpha_1}{\sin\alpha_2}$

Which is the condition for $m_1$ and $m_2$ to stay at rest **Ans**

## Problem 2

Torricelli apparatus is simply a glass tube filled with mercury dipped vertically into the vessel filled also with mercury. In this problem the space on top of themercury-filled glass tube contains pure hydrogen. The apparatus is assembled inside a cylinder fitted with a piston, as illustrated in Fig. 5.3. The space inside the cylinder contains air.

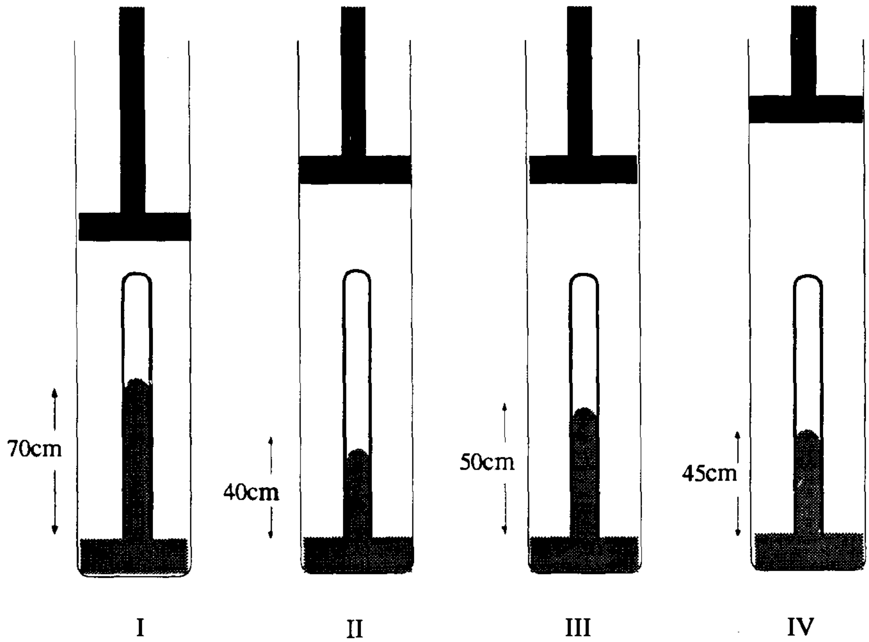



Fig. 5.3

| | | |
|---|---|---|
| In stage I | Height of mercury column | 70 cm |
| | Air pressure | 100 cm Hg ( = 1.314 atm = 133.4 kPa ) |
| | Temperature | 273 K |

In stage II The piston is pulled upwards until:
Height of mercury column is 40 cm
Air pressure 60 cm Hg

In stage III The volume of the cylinder is held constant, and heat is given to the ensemble until : Temperature is $T_3$ K
Height of mercury column 50 cm Hg

In stage IV Temperature is $T_4$ K
Height of mercury column 45 cm H
Air pressure is the same as air pressure in Stage III

Determine pressure and temperature of hydrogen gas contained in the space at the top of the glass tube.

## Solution

Construct a table listing the values of hydrogen volume, air pressure, air volume and the common temperature of air and hydrogen gas in all 4 stages.

| | I | II | III | IV |
|---|---|---|---|---|
| Pressure of Hydrogen gas | $P_{H1}$ | $P_{H2}$ | $P_{H}$ | $P_{H4}$ |
| Volume of Hydrogen gas | $V_{H1}$ | $V_{H2}$ | $V_{H3}$ | $V_{H4}$ |
| Pressure of Air | $P_{A1}$ | $P_{A2}$ | $P_{A3}$ | $P_{A4}$ |
| Volume of Air | $V_{A1}$ | $V_{A2}$ | $V_{A3}$ | $V_{A4}$ |
| Common Temperature of mixture | $T_1$ | $T_2$ | $T_3$ | $T_4$ |

Air pressure is measured from the length of mercury inside the glass tube expressed in the unit of cm Hg.

Volume of hydrogen gas is measured from the length of column in the space on top of the glass tube also expressed in cm.

Let the length of glass tube above the mercury level in the cylinder be L cm.

Apply Boyle's Law to investigate changes from Stages I to II

$$P_1V_1 = P_2V_2$$

$P_1 = P_{H1} = (100 - 70)$ cm Hg $\qquad P_2 = P_{H2} = 60 - 40$ cm Hg

$V_1 = V_{H1} = (L-70)$ cm $\qquad V_2 = V_{H2} = L - 40$ cm

$$(100-70)(L-70) = 20(L-20)$$

$$30L - 2100 = 20L - 800$$

$$L = 130 \text{ cm}$$

Thus $V_{H1} = 60$ cm
$V_{H2} = 90$ cm
$V_{H3} = 80$ cm
$V_{H4} = 85$ cm

Investigate changes of hydrogen gas from Stages II to III

From the equation of state $\frac{V_2P_2}{T_2}=\frac{V_3P_3}{T_3}$

$V_2 = V_{H2} = 130 - 40 = 90$ cm  $V_3 = 130 - 50 = 80$ cm
$P_2 = P_{H2} = 60 - 40 = 20$ cm  $P_3 = P_{H3} = P_{A3} - 50$ cm Hg
$T_2 = 273$ K  $T_3 = ?$ K

$$\frac{90.2}{273}=\frac{80\times(P_{A3}-50)}{T_3} \qquad (1)$$

Investigate changes of hydrogen gas from Stages III to IV

From $\frac{P_3V_3}{T_3}=\frac{P_4V_4}{T_4}$

$P_3 = P_{H3} = (P_{A3}-50)$ cm Hg  $P_4 = P_{H4}$ cm Hg
$V_3 = V_{H3}$ cm  $V_4 = V_{H4}$
$T_3 = ?$  $T_4 = ?$

Substitution yields

$$\frac{80\times(P_{A3}-50)}{T_3}=\frac{(P_{A3}-45)\times 75}{T_4} \qquad (2)$$

Investigate changes of air between Stages II and IV (constant volume)

From $\frac{P_2}{T_2}=\frac{P_3}{T_3}$

Substitution gives $\frac{60}{273}=\frac{P_{A3}}{T_3}$

$$P_{A3}=\frac{60}{273}T_3$$

Substituting $P_{A3}$ in (1)

$$\frac{180}{273}=8\left(\frac{60}{273}-\frac{50}{T_3}\right)$$

$$\frac{400}{T_3}=\frac{300}{273}$$

$$T_3 = \frac{4}{3}\times 2$$
$$= 364 \text{ K}$$

$$P_{A3} = \frac{60}{273} \times \frac{4}{3} \times 273$$
$$= 80 \text{ cm Hg}$$
$$P_{H4} = P_{A3} - 45$$
$$= 35 \text{ cm Hg}$$

Substituting $P_{A3}$ and $T_3$ in (2) gives

$$(80\text{-}50) \times \frac{80}{364} = 35 \times \frac{75}{T_4}$$
$$T_4 = 451 \text{ K}$$

**Conclusion** In the final stage the pressure and temperature of hydrogen ga s is 35 cm Hg and 451 K respectively. **Ans**

# Problem 3

Four resistors of the same value i.e. R Ω, 4 capacitors all of which have the same capacitance C = 1 μF, 4 batteries having voltages $U_1$ = 4 V, $U_2$ = 8 V, $U_3$ = 12 V, $U_4$ = 16 V respectively, (internal resistance of each battery is 0 Ω.) are connected to form a cube as shown in Fig. 5.4

Determine

3.1 Voltage and charge on each capacitor.

3.2 Charge on $C_2$ if H and B are short-circuited.

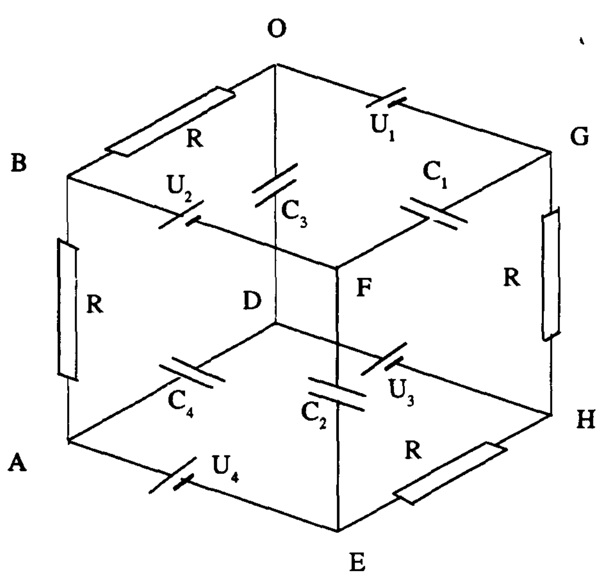



Fig. 5.4

## Solution

To facilitate anyalysis of the circuit, the 3-dimensional circuit is redrawn as a two dimensional circuit as demonstrated in Fig. 5.5 below.

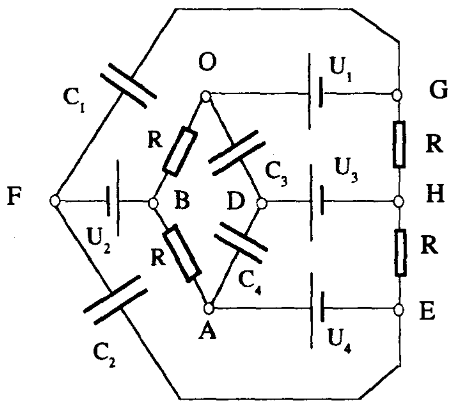



Fig.5.5

3.1 Since all voltage sources are of DC kind, we may draw a circuit that is limited to the routes ciculated by DC current. as shown in Fig. 5.6.

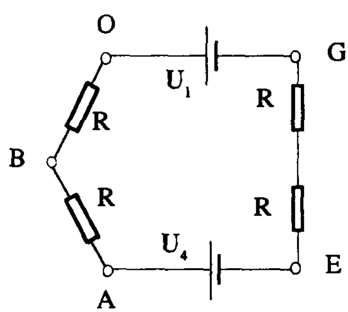



Fig.5.6

In the circuit above, the resultant emf is given by U = 16 - 4 = 12 V.

Net current in the circuit $I = \dfrac{U_4 - U_1}{4R}$

$$= \frac{12}{4R}$$

$$= \frac{3}{R} \quad A$$

We may choose A as a reference point for the purpose of calculating voltage drops.

Voltage across AB $= \frac{3}{R} \times R \quad = 3\ V$

Voltage across AO $= \frac{3}{R} \times 2R \quad = 6\ V$

Voltage across AG $= \frac{3}{R} \times 2R + U_1 = 10\ V$

Voltage across AH = Voltage across AH + 3 = 16 V

Voltage across AF = Voltage across AB + 8 = 11 V

From the value of the voltage across each capacitor in Fig. 5.5, we are able to calculate charge on each capacitor in the following manner:

Voltage across $C_1$ =Voltage across GA-Voltage across FA = 10 -11 =-1V, $Q=1\times10^{-6}$ C

Voltage across $C_2$ =Voltage across AE- Voltage across AF = 16 -11 = 5 V, $Q = 5\times10^{-6}$ C

Voltage across $C_3$ = Voltage across AO - Voltage across AD = 6 -1 = 5 V, $Q = 5\times10^{-6}$ C

Voltage across $C_4$ =Voltage across AF = 1V, $Q = 1\times10^{-6}$ C

**Ans**

3.2 If B and H are short-circuited, there are 2 circuits which function independent of each other. The situation is demonstrated in Fig. 5.6

The first circuit consists of $U_1$ and 2 resistors.
The second circuit consists of $U_4$ and 2 resistors.

Current in the second circuit is given by $I = \frac{U_4 - U_1}{4R} \frac{16}{2R} = \frac{8}{R}$ A

Voltage across BA $= \frac{8}{R} \times R = 8\ \Omega$

Voltage across FA $= 8 + U_2 = 16\ \Omega$

Voltage across EA $= \frac{8}{R} \times 2R = 16\ \Omega$

Voltage across $C_2$ is 0,hence there is no net charge on $C_2$ **Ans**

## Problem 4

A spherical fish bowl of raduis R is placed in front of a plane vertical mirror. The thickness of the wall of the fish bowl is very thin. The centre of the spherical bowl is at a distance of 3 R from the plane mirror. The bowl is filled with water and contains a fish.
One observer looks at the fish along the line that goes through the centre of the bowl and perpendicular to the plane mirror. A tiny fish is seen swimming with velocity $v_0$ along the wall of the bowl at the position nearest to the plane mirror. Find the relative velocities of the images of the fish as seen by the observer. (See Fig. 5.7)

The refractive index of water is $\frac{4}{3}$.

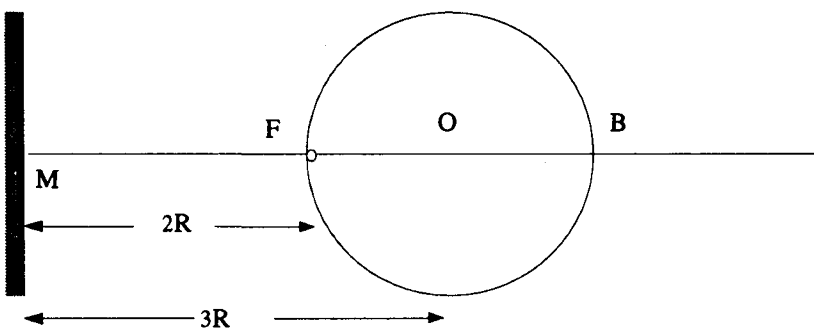



Fig. 5.7

## Solution

In Fig. 5.7

O is the centre of the spherical glass fish-bowl.

F is position of the fish

M is the position of the plane mirror.

BOFM is the line of sight of the observer

Investigate the formation of the image of the fish at the first spherical surface of the fish bowl.

From the formula $$\frac{n'}{l'}-\frac{n}{l}=\frac{n'-n}{r}$$

$n'$ is refractive index of air n refractive index of water
$l'$ image distance measured from B $l$ object distance measured from B = - 2R
r radius of curvatue of the first surface = - R

Substitution gives
$$\frac{1}{l'} - \frac{4}{3\times(-2R)} = \frac{\left(1-\frac{4}{3}\right)}{-R}$$
$$\frac{1}{l'} = \frac{1}{3R} - \frac{2}{3R}$$
$$l' = -3R$$

Magnification
$$= \frac{\frac{l'}{n'}}{\frac{l}{n}} = \frac{l'}{l}\times\frac{n}{n'}$$
$$= \frac{-3R}{-2R}\times\frac{4}{3}$$
$$= 2$$

Investigation of the formation of image of the fish at the plane mirror, the second surface.and finally the first surface.

Image formation of F at the plane mirror
image distance = 2R directed to the left of M

The image serves as an object for the formation of image at the second surface of lthe fish bowl.

Apply the formula
$$\frac{n'}{l'} - \frac{n}{l} = \frac{n'-n}{r}$$

$l'$ image distance = ? $l$ object distance= - 4R
$n'$ refractive index of water = $\frac{4}{3}$ n refractive index of air = 1
r radius of curvature of the second surface of the fish bowl = R

Substitution gives
$$\frac{4}{3\times l'} - \frac{1}{-4R} = \frac{\left(\frac{4}{3}-1\right)}{R}$$
$$\frac{4}{3\times l'} = \frac{1}{3R} - \frac{1}{4R}$$

$$l' = 16R$$

The image is formed at a distance of 16R to the right of F.

Magnification $= \dfrac{\frac{l'}{n'}}{\frac{l}{n}} \qquad = \dfrac{l'}{l} \times \dfrac{n}{n'}$

$$= \frac{16R}{-4R} \times \frac{3}{4}$$

$$= -3$$

Consider formation of the image at the second surface

Apply $\dfrac{n'}{l'} - \dfrac{n}{l} = \dfrac{n'-n}{r}$

$n'$ refractive index of air = 1 n refractive index of water $= \dfrac{4}{3}$

$l'$ image distance = ? $l$ object distance = 16R - 12R = 14R

r radius of curvature of the first surface = - R

Substitution gives $\dfrac{1}{l'} - \dfrac{4}{3 \times 14R} = \dfrac{\left(1 - \frac{4}{3}\right)}{-R}$

$$\frac{1}{l'} - \frac{2}{21R} = \frac{1}{3R}$$

$$\frac{1}{l'} = \frac{(2+7)}{21R}$$

$$l' = \frac{7R}{3}$$

Magnification $= \dfrac{l'}{l} \times \dfrac{n}{n'}$

$$= \frac{7R}{3 \times 14R} \times \frac{4}{3}$$

$$= \frac{2}{9}$$

Resultant magnification $= -3 \times \dfrac{2}{9}$

This implies that while the the image of the fish due to the first spherical surface appears to be swimming upwards, the image of the fish due to the plane mirror, the second and first spherical surface appears to be swimming downwards.

The velocity of the image in the first case = $2v_0$

The velocity of the image in the second case = $-\dfrac{2}{3}v_0$

Relative velocity of the two images = $2v_0 + \dfrac{2}{3}v_0 = \dfrac{8}{3}v_0$ **Ans**

# Experiment

## Problem 1

A DC voltage source $E_0$ with internal resistance r is connected to an appliance of resistance $R_L$

1.1 Sketch useful power as a function of current.
1.2 Determine internal resistance of voltage source.
1.3 Sketch:
    1.3.1 Total power as a function of $R_L$
    1.3.2 useful power as a function of $R_L$
    1.3.3. Circuit efficiency as a function of $R_L$.

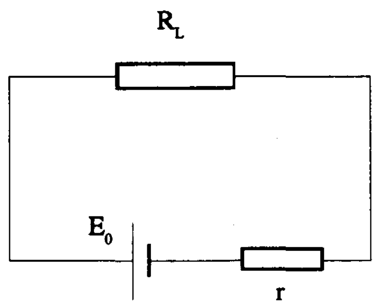



Fig. 5.8

## Solution

1.1 Connect a circuit as shown in Fig. 5.8

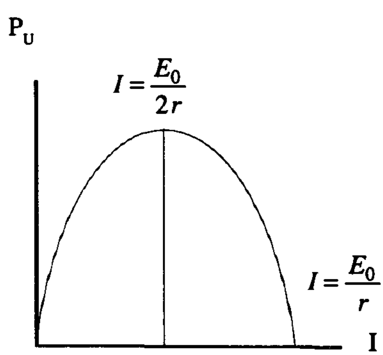



Fig. 5.9

Let I be current in the circuit
$P_T$ is the total power of the circuit $= E_0I$
$P_U$ is useful power

$P_U = E_0I - I^2r$ **Ans** 1.1

1.2 From $$I = \frac{E_0}{(r + R_L)}$$

$$IR_L + Ir = E_0$$

$$U = -Ir + E_0$$

where U is voltage across $R_L$

Vary $R_L$ and measure U for each I.

Plot a graph of U as a function of I. The slope of the curve corresponds to r and the intercept on U axis i.e. I = 0 gives $E_0$ **Ans** 1.3.2

1..3

1.3.1

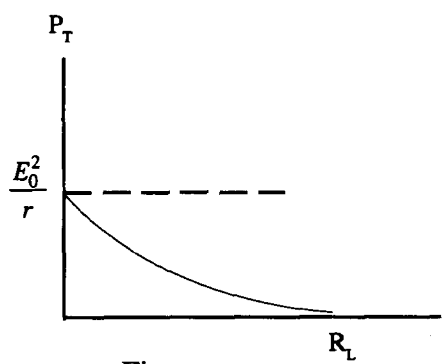
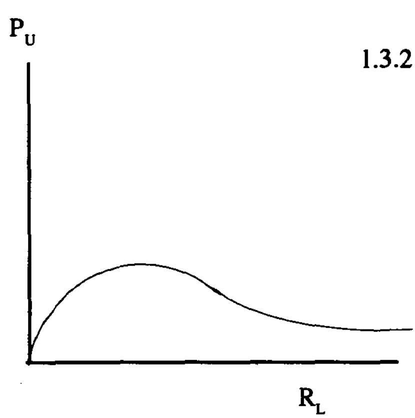



Fig. 5.10

$$P_T = I(R_L + r)$$
$$= \frac{E_0^2}{(R_L + r)}$$

1.3.2




Fi g. 5.11

$$P_U = I^2 R_L$$
$$= \frac{E_0^2 R_L}{(R_L + r)^2}$$

1.3.3 Efficiency $\eta = \frac{P_U}{P_T}$

$$= \frac{E_0^2 R_L}{(R_L + r)} \times \frac{(R_L + r)}{E_0}$$

$$= \frac{R_L}{R_L + r}$$

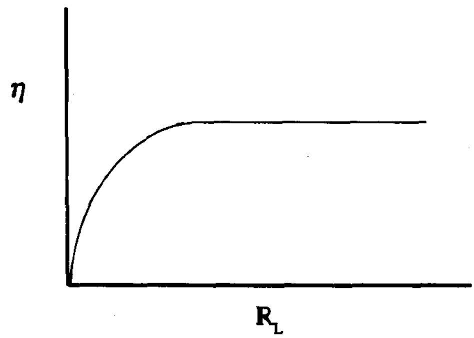



Fig. 5.12

# International Physics Olympiad VI

# 1972

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# Theory

## Problem 1

Given are three cylinders all of which have the same length, equal outer radius and the same mass. The first cylinder is a solid object of uniform density, the second is hollow with a finite shell thickness and the third cylinder is also hollow with the same shell thickness of the second cylinder but filled with liquid which has the same density as that of the wall of the third cylinder. (There are of course thin lids covering both ends of the cylinder)

Compare linear and angular accelerations of the three cylinders when the cylinders roll down the inclined plane which makes an angle $\alpha$ with the horizon and $\mu$ is the coefficient of friction between the inclined plane and the cylinders. Consider the cases when the cylinders roll without sliding and when the cylinders skid.
Given the ratio between the values of the density of the first and second cylinders be n.

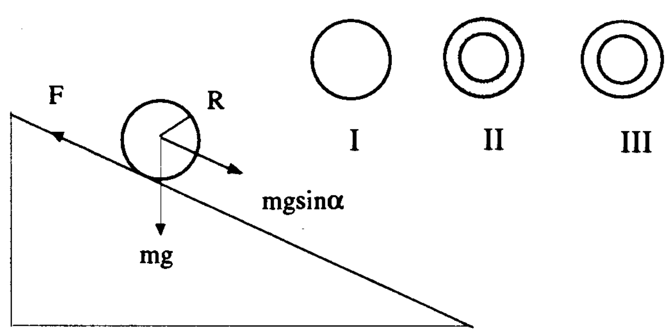



Fig. 6.1

# Solution

Investigation of the motion of the cylinders along the inclined plane.
Let **F** be friction between the cylinder and the inlined plane.

$$|\mathbf{F}| \le \mu mg\cos\alpha$$

Equation of linear motion of the cylinder along the inclined plane is

$$ma = mg\sin\alpha - F \qquad (1)$$

And equation of rotational motion of the cylinder is

$$|\mathbf{R}\times\mathbf{F}| = I\dot{\omega} = I\dot{\theta} \qquad (2)$$

where R is radius of the cylinder
I moment of inertia of the cylinder
$\dot{\omega} = \ddot{\theta}$ = angular acceleration of the cylinder

From (2) $$|\mathbf{R}\times\mathbf{F}| = \frac{|\mathbf{a}|}{R}$$

$$a = \frac{R^2F}{I} \qquad (3)$$

Substituting a in (1) to get

$$\frac{mR^2}{I}F = mg\sin\alpha - F$$

$$\left(1+\frac{mR^2}{I}\right)F = mg\sin\alpha$$

$$F = \frac{mg\sin\alpha}{\left(1+\frac{mR^2}{I}\right)} \qquad (4)$$

$$a = \frac{mR^2}{I}\frac{g\sin\alpha}{\left(1+\frac{mR^2}{I}\right)} \qquad (5)$$

The condition for the cylinders to go into pure rotation down the inclined plane is

$$mg\sin\alpha \le F \le \mu mg\cos\alpha$$ **Ans**

The cylinder begins to slide when friction is maximum i.e.

$$F = \mu mg\cos\alpha$$

$$\frac{mg\sin\alpha}{\left(1+\frac{mR^2}{I}\right)} = \mu mg\cos\alpha \qquad (6)$$

Equation (6) above is the condition for cylinders I , II and III to slide. **Ans**

$(M.I.)_I$ = Moment of inertia of cylinder I = $\frac{1}{2}mR^2$

$(M.I.)_{II}$ = Moment of inertia of cylinder II = $\frac{1}{2}m(R^2+R_2^2)$

Where $R_2$ isthe inner radius of cylinder II

Let ρ be density of solid cylinder I

nρ be density of of cylinder II

L be length of all three cylinders.

$$(\pi R^2 - \pi R_2^2)L\,n\rho = m$$

$$(\pi R^2 - \pi R_2^2)Ln\rho = \pi R^2 L\rho$$

$$\frac{R^2}{R_2^2} = \frac{n-1}{n} \qquad (7)$$

$$(M.I.)_{II} = \frac{1}{2}\,m\left[R^2+R^2\left(\frac{n-1}{n}\right)\right]$$

Let $(M.I.)_{III}$ be moment of inertia of cylinder III pertaining to the part of its solid shell

$= \frac{1}{2}\,m_0(R^2+R_3^2)$

where $m_0$ is mass of the solid shell of the third cylinder.
$R_3$ inner radius of the shell of the third cylinder.

Volume of the shell ( 3rd cylinder) $= (\pi R^2 - \pi R_3^2)L$

Mass of the shell ( 3rd cylinder) $m_0 = (R^2 - R_3^2)L \times \frac{m}{\pi R^2 L} = \frac{m(R^2 - R_3^2)}{R^2}$

$$(M.I.)_{III} = \frac{1}{2}\left[\frac{m(R^2-R_3^2)}{R^2}\right](R^2+R_3^2)$$

$$= \frac{1}{2}\frac{m(R^4-R_3^4)}{R^2}$$

$$= \frac{1}{2}mR^2\left(1-\frac{R_3^4}{R^4}\right)$$

Substituting $\frac{R_3^4}{R^4}$ from (7) gives

$$(M.I.)_{III} = \frac{1}{2}mR^2[1-\frac{(1-n)^2}{n^2}]$$

$$= \frac{1}{2}mR^2[1-\frac{(2n-1)}{n^2}]$$

For cylinder I to roll without sliding is

$$I = \frac{1}{2}mR^2 \quad \text{or}$$

$$\frac{mR^2}{I} = 2$$

Hence $$a = \frac{2g}{3}\sin\alpha$$

From equation of motion

$$mg\sin\alpha - F = mg\sin\alpha$$

$$F = \frac{1}{3}mg\sin\alpha$$

For the cylinder to roll without slipping

$$\frac{1}{3}mg\sin\alpha \leq \mu mg\cos\alpha$$

$$\tan\alpha \leq 3\mu$$

Linear acceleration is $a = \frac{2}{3}g\sin\alpha$

Angular acceleration is $\frac{a}{R} = \frac{2}{3R}mg\sin\alpha$

and $\tan\alpha \leq 3\mu$ **Ans**

For cylinder II to roll without sliding is

$$I = \frac{1}{2}\frac{mR^2(2n-1)}{n}$$

$$\frac{mR^2}{I} = \frac{2n}{2n-1}$$

From (4) $$a = \frac{2n}{2n-1}\frac{g\sin\alpha}{\left(1+\frac{2n}{2n-1}\right)}$$

$$a = \frac{2ng\sin\alpha}{4n-1}$$

Equation of motion is

$$mg \sin\alpha - F = \frac{2nmg \sin \alpha}{4n-1}$$

$$F = mg \sin\alpha - \frac{2nmg \sin \alpha}{4n-1}$$

$$= \frac{(2n-1)mg \sin \alpha}{4n-1}$$

$$\frac{(2n-1)mg \sin \alpha}{4n-1} \leq \mu\, mg\cos\alpha$$

$$\tan \alpha \leq \frac{\mu(4n-1)}{(2n-1)}$$

Linear acceleration is given by $a = \dfrac{2ng \sin \alpha}{(4n-1)}$

and angular acceleration by $\dfrac{a}{R} = \dfrac{2ng \sin \alpha}{R(4n-1)}$

$$\tan\alpha \leq \frac{\mu(4n-1)}{(2n-1)} \qquad \textbf{Ans}$$

Take the case of cylinder III.

Since there is no friction between the liquid and the cylinder, we need not bring in the moment of inertia of the liquid into calculation.

$$I = \frac{1}{2} \frac{mR^2(2n-1)}{n^2},$$

$$\frac{mR^2}{I} = \frac{2n^2}{2n-1}$$

Linear acceleration is $a = \dfrac{2n^2}{2n-1} \dfrac{g \sin \alpha}{\left(1 + \dfrac{2n^2}{2n-1}\right)} = 2n^2 \dfrac{g \sin \alpha}{\left(2n^2 + 2n - 1\right)}$

And angular acceleration is $\dfrac{a}{R} = \dfrac{2n^2}{R} \dfrac{g \sin \alpha}{\left(2n^2 + 2n - 1\right)}$

The equation of motion in this case is

$$mg \sin\alpha - F = 2mn^2 \frac{g \sin \alpha}{\left(2n^2 + 2n - 1\right)}$$

$$F = \frac{(2n-1)mg \sin \alpha}{\left(2n^2 + 2n - 1\right)}$$

$$\frac{(2n-1)mg\sin\alpha}{\left(2n^2+2n-1\right)} \leq \mu mg\cos\alpha$$

$$\tan\alpha \leq \frac{\left(2n^2+2n-1\right)\mu}{(2n-1)} \qquad \textbf{Ans}$$

Ratio betweeen linear (and also angular) accelerations of cylinders I,II and III

$$= \frac{2}{3}g\sin\alpha : \frac{2ng\sin\alpha}{4n-1} : \frac{2n^2 g\sin\alpha}{\left(2n^2+2n-1\right)}$$

$$= 1 : \frac{3n}{4n-1} : \frac{3n^2}{\left(2n^2+2n-1\right)}$$

Ratio of the values of $\tan\alpha_{MAX}$ for cases 1,2 and 3 is

$$= 3\mu : \frac{(4n-1)\mu}{(2n-1)} : \frac{(2n^2+2n-1)\mu}{(2n-1)}$$

$$= 1 : \frac{(4n-1)}{3(2n-1)} : \frac{3(2n^2+2n-1)}{(2n-1)} \qquad \textbf{Ans}$$

When cylinders I, II and III begin to slide and roll, friction in all cases is maximum i.e. $\mu$ mg cos$\alpha$.

Also the **linear** accelerations in all cases are the same a = g(sin$\alpha$. - $\mu$ cos$\alpha$.)

The **angular** acceleration for each case can be found from torque about the centre of the cylinder.

$$I\ddot{\theta} = R\mu\, mg\cos\alpha.$$

Angular acceleration of cylinder I $= \frac{R}{I}\mu\, mg\cos\alpha. = \frac{2\mu\cos\alpha}{R}$

Angular acceleration of cylinder II $= \frac{n}{2n-1}\frac{2\mu\cos\alpha}{R}$

Angular acceleration of cylinder III $= \frac{n^2}{2n-1}\frac{2\mu\cos\alpha}{R}$ **Ans**

## Problem 2

In the figure below, there are two sets of cylinder and piston joined by a common piston axis in the horizontal direction. The cross section of each cylinder is 1 $dm^2$. The cylinder on the right hand side contains a gas of mass 4 g, volume 22.4 $dm^3$ (litre) at 1 atm pressure and 0° C. The cylinder on the right- hand side also contain the same kind of gas of mass 7.44 g, volume 22.4 $dm^3$ at 0° C. The walls of the cylinder on the left hand side are insulated against heat loss, while the walls of the cylinder on the right- hand side are made from good heat conductor.
The cylinder on the right- hand side is kept at 0° C in a good heat reservoir. The whole system is in a vacuum. The pistons and their axle after being released come to rest in equilibrium after moving a distance of 5 dm.

Find the amount of heat gained by the cylinder on the right-hand side.
(Specific heat of gas at constant volume $C_V$ = 0.75 cal/g.K)

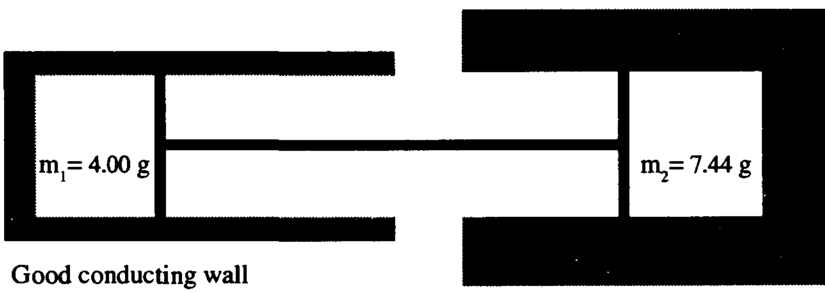



Fig. 6.2

## Solution

In the beginning stage, gases in the two cylinders have the same volume and are at the same temperature. The pressure in each cylinder is therefore proportional to the quantity of mass in its respective cylinder. i.e.

$$\frac{P_1}{m_1} = \frac{P_2}{m_2}$$

Whereas $P_1$ = 1 atm  $m_1$ = 4 g
$P_2$ = ?  $m_2$ = 7.44 g

Substitution gives $P_2 = 7.44 \times \frac{1}{4}$
$= 1.86$ atm

After the pistons are released, gas in the cylinder on the left- hand side undergoes adiabatic change,i.e. $\Delta Q = 0$
while gas in the cylinder on the right- hand side undergoes isothermal change i.e.
$\Delta T = 0$

For the set of cylinder and piston on the left- hand side,

| | | | | | |
|---|---|---|---|---|---|
| $P_{i1}$ | = initial pressure | = 1 atm | $P_{f1}$ | = | final pressure |
| $V_{i1}$ | = initial volume | = 22.4 $dm^3$ | $V_{f1}$ | = | final volume |
| $T_{i1}$ | = initial temperature | = 273 K | $T_{f1}$ | = | final temperature |

For the set of cylinder and piston on the right- hand side,

$P_{i2}$ = initial pressure = 1.86 atm  $P_{f2}$ = final pressure
$V_{i2}$ = initial volume = 22.4 $dm^3$
$V_{f2}$ = final volume = 22.4+ 5.0 = 27.4 $dm^3$
$T_{i2}$ = initial temperature = 273 K
$T_{f2}$ = final temperature

From $$P_{f2}V_{f2} = P_{i1}V_{i1}$$

Substitution gives $$P_{f2} = 22.4 \times \frac{1.86}{27.4}$$
$$= 1.52 \text{ atm}$$

Find $T_{f1}$ by applying $\frac{P_{i1}V_{i1}}{T_{i1}} = \frac{P_{f1}V_{f1}}{T_{f1}}$

Substitute $P_{f1} = P_{f2} = 1.52$ atm

$$1 \times \frac{22.4}{273} = \frac{1.52 \times 17.4}{T_{f1}}$$

$$T_{f1} = 322 \text{ K}$$

Increase in temperaure of gas in the cylinder on the left- hand side

$$= T_{f1} - T_{i1}$$
$$= 322 - 273$$
$$= 49.5 \text{ K} \quad \textbf{Ans}$$

Since $$\Delta Q = \Delta U + P\Delta V = 0$$

$$\Delta U = mC_V \Delta T$$
$$= 0.75 \times 4 \times 49.5$$
$$= 148 \text{ cal}$$

This amount of heat is obtained from the heat reservoir. The cylinder on the right -hand side extracts from the reservoir 148 cal and delivers it to the cylinder on the left- hand side and at the same time extracts another 148 cal in order to maintain the temperature of the cylinder on the right- hand side at 0° C **Ans**

## Problem 3

A convex lens having focal length given by f is sliced into two parts along the line perpendicular to its plane. The two half- lenses are then moved apart by a small distance d. If a monchromatic light source is placed at distance t (t > f) to the left of the two half lenses, determine the number of interference bands on a screen placed at distance H to the right of the half lenses. (See Fig.6.3)

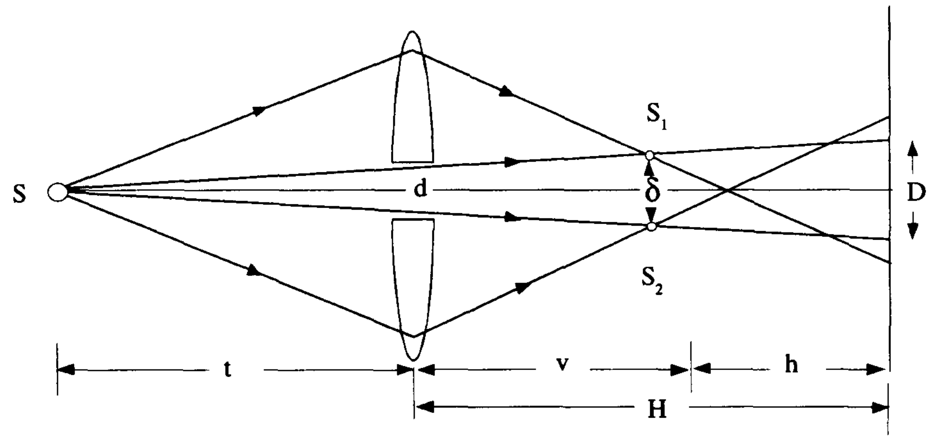



Fig. 6.3

## Solution

From the diagram, the monochromatic light source S causes its image S to be formed at $S_1$ and $S_2$. These two images serve as coherent sources for the formation of an interference pattern.

Let h be the distance of $S_1$ and $S_2$ from the screen.

Calculation of the separation between $S_1$ and $S_2$

Apply the formula $$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

in which v is the image distance = ?

u object distance = - t

f focal distance = f

Substitution gives $$\frac{1}{v}-\frac{1}{-t}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{t-f}{tf}$$

$$v=\frac{tf}{t-f} \tag{1}$$

Let δ represent distance between $S_1$ and $S_2$

From the diagram $$\frac{\delta}{d}=\frac{t+v}{t}$$

$$\delta=\frac{d}{t}(1+\frac{tf}{t-f})$$

$$=\frac{dt}{t-f} \qquad (2)$$

Also from the diagram, it is to be observed that light from $S_1$ and $S_2$ can give interference pattern in region D of the screen only.

From the diagram h = H - v

Substitute v from (1) $h = H - \frac{tf}{t-f}$

Condition for a bright band to be formed is

$$\delta \sin\alpha = 2n\frac{\lambda}{2} \qquad n = 0,1,2,3,......$$

Substituting δ $$\frac{d.t}{t-f}\times\frac{x}{h}=n\lambda \qquad (3)$$

and condition for formation of the next bright band is

$$\frac{d.t}{t-f}\times\frac{x}{h}=(n+1)\lambda \qquad (4)$$

Distance between two consecutive bright bands is obtained from (3) - (4)

$$\Delta x=\lambda h\frac{(t-f)}{d.t}$$

$$=\lambda[h-\frac{t.f}{(t-f)}]\times\frac{(t-f)}{d.t}$$

$$=\frac{\lambda}{d.t}[H.(t-f)-t.f]$$

The number of bright bands fomred in region D $=\frac{D}{\Delta x}$

From the diagram $$\frac{D}{(t+H)}=\frac{d}{t}$$

Number of bright bands $$=\frac{d.(t+h)}{t}\times\frac{d.t}{\lambda[H.(t-f)-t.f]}$$

$$=\frac{d^2.(t+H)}{\lambda[H.(t-f)-t.f]}$$ **Ans**

# Experiment

## Problem 1

Each contestant is provided with two cylinders both of which are of the same size and made from the same material. The first cylinder is solid while the second cylinder contains a hollow part in the cylindrical form. The axes of the solid and the hollow parts are not concentric but nevertheless parallel to each other. The latter cylinder has a thin lid covering made from the same material on each end of the cylinder.
Determine density of the material and the position of the axis of the cylindrical cavity.

## Solution

Let r be density of the material
$m_1$ mass of the first or solid cylinder
$m_2$ mass of the second cylinder
R radius of cylinders
L length of cylinders

Volume of the solid cylinder $= \pi R^2 L$

Density of the material $\rho = \dfrac{m_1}{\pi R^2 L}$

Vaues of $m_1$ R and L are obtained directly by measurent, and ρ can be calculated. **Ans**

The volume of the solid part of the second cylinder is given by

$$\frac{m_2}{\rho} = \frac{m_2}{m_1} \times \pi R^2 L = \pi R^2 L$$

The volume of the cavity in the second cylinder

$$= \pi R^2 L - \frac{m_2}{m_1} \times \pi R^2 L$$

$$= \pi R^2 L \left(1 - \frac{m_2}{m_1}\right)$$

Let $R_0$ be radius of the cylindrical hollow part

Hence $\pi R_0^2 L = \pi R^2 L \left(1 - \frac{m_2}{m_1}\right)$

$$R_0^2 = R^2 L \left(1 - \frac{m_2}{m_1}\right)$$

$$R_0 = R \sqrt{1 - \frac{m_2}{m_1}}$$

$R$ $m_2$ $m_1$ can be directly measured, hence $R_0$ can be calculated from the above equation **Ans**

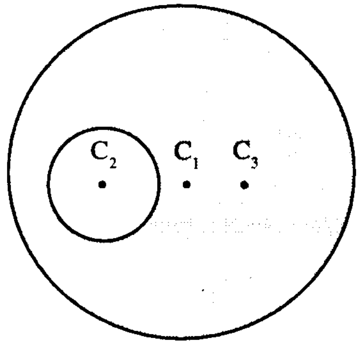



Fig. 6.4

Let $C_1$ be the centre of mass of the cross-section of the solid cylinder.
$C_2$ the centre of mass of the cross-section of the cylindrical cavity
$C_3$ the centre of mass of the cross-section of the second cylinder

The position of $C_3$ can be found by suspending the second cylinder from its edges by a chord in various positions and project the line of the chord. Where these lines intersect gives the position of $C_3$

Take moment about $C_1$

$$x_2.m_2 = (m_1 - m_2)X_3$$

where $x_3$ is the distance of $C_3$ from $C_1$
and $x_2$ the distace of $C_2$ from $C_1$

$x_2$, $m_1$ and $m_2$ are known, hence x can be calculated. **Ans**

# International Physics Olympiad VII

# 1974

# Warsaw, Poland

## Theory

### Problem1

A hydrogen atom in the ground state collides with another hydrogen atom at rest in the ground state.
Find the least possible velocity of the first hydrogen atom for which the collision becomes inelastic.
If the velocity of the first hydrogen atom is greater than the stated minimum value, emission of photons takes place. The emitted light is observed along and opposite the direction of the initial velocity of the first hydrogen atom. Determine the frequency shift from the normal frequency.
(Mass of hydrogen atom is $1.67 \times 10^{-27}$ kg. Ionization energy of hydrogen atom $E_0 = 13.6$ eV or $2.18 \times 10^{-18}$ J)

### Solution

The difference between energy of hydrogen atom in the n[th] and the ground state ( n =1) is given by

$$E = E_0 \left( \frac{1}{n^2} - \frac{1}{1^2} \right)$$

Substitute $n = \infty$ , and E becomes ionization energy i.e. $2.18 \times 10^{-18}$ J

$$E_0 = -2.18 \times 10^{-18} \text{ J}$$

The first excited state of the hydrogen atom corresponds to the state for which n =2.

Let the energy of this first excited state be $E_1$.

Hence $$E_1 = E_0 \left( \frac{1}{2^2} - \frac{1}{1^2} \right)$$

$$= -2.18\times10^{-18}\left(\frac{1}{4}-1\right)$$
$$= 2.18\times10^{-18}\times0.75$$
$$= 1.16\times10^{-18}\ \text{J}$$

The problem is to find minimum velocity for which the maximum loss of energy is equal to the first excited energy of the hydrogen atom i.e. $1.16\times10^{-18}$ J

The collision between two hydrogen atoms although inelastic, the principle of the conservation of momentum still holds. If $v_0$ is the minimum velocity for inelastic collision. The maximum loss of energy occurs when the collision is head- on and the two atoms move together with the same velocity of $\frac{v_0}{2}$

Momentum before collision $mv_0$ = momentum after collision $(2m)\frac{v_0}{2} = mv_0$

Energy loss $= \frac{1}{2}mv_0^2 - \frac{1}{2}(2m)\left(\frac{v_0}{2}\right)^2$

Substitution gives $1.16\times10^{-18} = \frac{1}{2}mv_0^2 - \frac{1}{4}mv_0^2$

$$\frac{1}{4}mv_0^2 = 1.16\times10^{-18}\ \text{J}$$

Substituting $m = 1.67\times10^{-27}$

$$v_0^2 = \frac{4\times1.6\times10^{-18}}{1.16\times10^{-27}}$$
$$v_0 = 3.13\times10^{4}\ \text{m/s}$$

Required minimum velocity is $3.13\times10^4$ m/s **Ans**

Apply the formula $v = v_0\left(1\pm\frac{v}{c}\right)$

whereas v is velocity of photon source relative to the observer

c velocity of light $= 3\times10^8$ m/s

Substitution gives $v = v_0\left(1\pm\frac{3.13\times10^4}{3\times10^8}\right)$

$$= v_0(1\pm1.04\times10^{-4})$$

If photons are observed in the direction opposite to the line of motion of the first hydrogen, the frequency observed will be higher than the normal value by $1.04\times10^{-4}$ Hz, also if photons are observed along the line of motion of the first hydrogen, the frequency observed will be lowered by $1.04\times10^{-4}$ Hz **Ans**

## Problem 2

Given a rectangular glass block of thickness d. with one side placed along x-axis. The refractive index at any point inside the glass block varies as a function of x in the form

$$n = \frac{n_0}{(1 - \frac{x}{r})}$$

in which x is distance of the position of any point in the glass measured along x-axis.
r a constant.
and $n_0$ refractive index at x = 0 (See Fig. 7.1)

The figure below depicts light entering the block at O from the air and undergoes refraction inside the block before emerging from the block at point A, at anngle α with the normal at A. Determine refractive index at A and the thickness of the glass block.

(Given $n_0 = 1.2$, $r = 13$ cm, $\alpha = 30^\circ$)

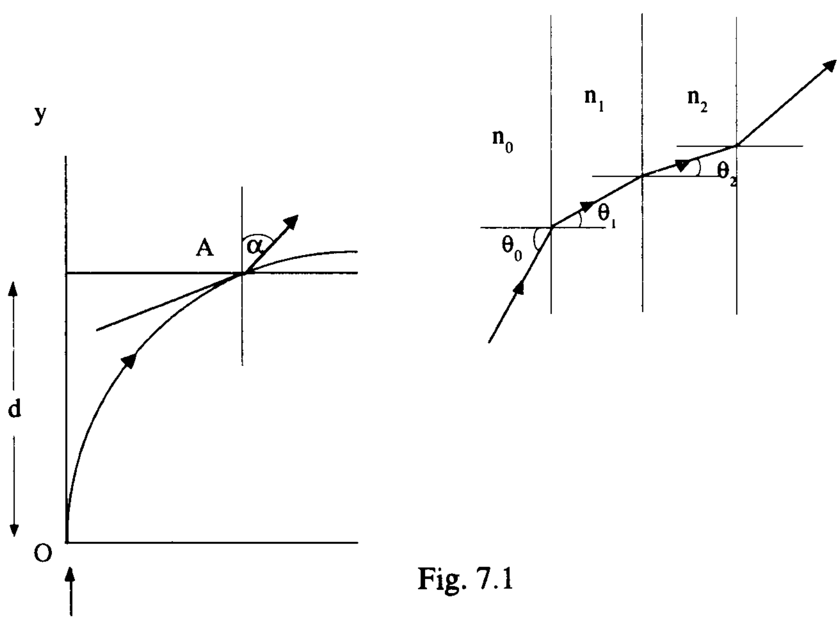



Fig. 7.1

## Solution

The refractive index n must be understood as the variable refractiv e index at various different layers of the glass block relative to air,

$$\text{or } n = {}_a\mu_g$$

Apply the law of refraction at diferent layers of glass block shown in Fig. 7.1

$$
\begin{aligned}
{}_a\mu_a \sin\theta_0 &= {}_a\mu_{g1} \sin\theta_1 \quad \text{or} \quad n_0 \sin\theta_0 = n_1 \sin\theta_1 \\
{}_a\mu_{g1} \sin\theta_1 &= {}_a\mu_{g2} \sin\theta_2 \qquad n_1\sin\theta_1 = n_2\sin\theta_2 \\
{}_a\mu_{g2} \sin\theta_2 &= {}_a\mu_{g3} \sin\theta_3 \qquad n_2\sin\theta_2 = n_3\sin\theta_3 \\
{}_a\mu_{g3} \sin\theta_3 &= {}_a\mu_{g4} \sin\theta_4 \qquad n_3\sin\theta_3 = n_4\sin\theta_4 \\
{}_a\mu_{g4} \sin\theta_4 &= {}_a\mu_{g5} \sin\theta_5 \qquad n_4\sin\theta_4 = n_5\sin\theta
\end{aligned}
$$

etc.

Hence $n_0\sin\theta_0 = n_1\sin\theta_1 = n_2\sin\theta_2 = n_3\sin\theta_3 = n_4\sin\theta_4 = n_5\sin\theta_5 \ldots\ldots\ldots\ldots = n_x\sin\theta_x$

The problem does not divide the glass block into well-defined individual layers, however we may imagine the glass block consisting of a large number of thin layers each of which is characterized by its refractive index.

$$n_0 \sin\theta_0 = n_x \sin\theta_x \qquad (1)$$

$\theta_0 = 90^\circ$ hence

$$\sin\theta_x = \frac{n_0}{n_x}$$

$$= \frac{(r-x)}{r}$$

$$(r-x) = r\sin\theta_x$$

The above equation is recognized as the equation of a circle with radius r and the centre C at (x,o)

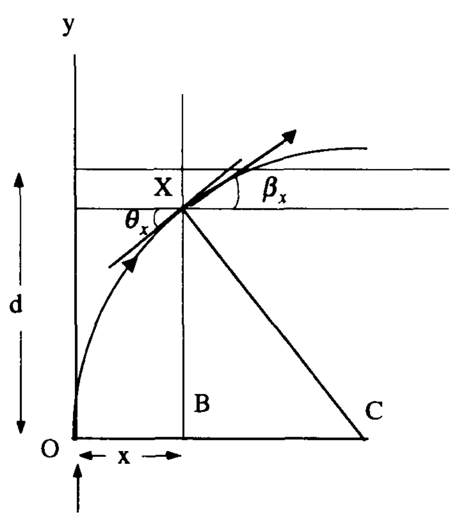



Fig. 7.2

In Fig 7.2 $\theta_x$ is the angle of incidence of the layer at X.

r-x corrsponds to OB

$\angle$ BXC is the angle of incidence $\theta_x$

Investigate the refraction at A

angle of incidence = $90° - \theta_A$

angle of refraction = $\alpha$

From the principle of refraction of light

$$n_A \sin(90° - \theta_A) = 1 \times \sin\alpha$$

$$n_A \cos\theta_A = \sin\alpha$$

(1) may be rewritten in the form

$$\sin\theta_A = \frac{n_0}{n_A} \qquad (3)$$

Hence $$\cos\theta_A = \sqrt{1-(\frac{n_0}{n_A})^2} \qquad (4)$$

Substitute cos $\theta_A$ from (5) in (3) to get

$$n_A = \frac{\sin\alpha}{\sqrt{1-(\frac{n_0}{n_A})^2}} \qquad (5)$$

$$n_A^2 = \frac{\sin^2\alpha}{1-(\frac{n_0}{n_A})^2}$$

$$n_A^2 - n_0^2 = \sin^2\alpha$$

$$n_A = \sqrt{n_0^2 + \sin^2\alpha}$$

Substitute $n_0 = 1.2$ $\sin\alpha = 0.5$

$$n_A = \sqrt{1.44 + 0.25}$$

$$= 1.3$$

The value of refractive index at A is 1.3 **Ans**

Substitute $\sin\theta_A = \sin\theta_x = \frac{n_0}{n_A}$ in (2) to get

$$\text{r - x} = \text{r}(\frac{n_0}{n_A})$$

$$\text{r} - \frac{x}{13} = \frac{1.2}{1.3}$$

$$\frac{x}{13} = 1 - \frac{1.2}{1.3}$$

x = 1 cm

From the geometry in the diagram

$$y = r\cos\theta$$

$(2)^2+(6)^2$ gives $\quad y^2 + (r - x)^2 = r^2$

which defines the path of the light beam inside the block.

Substitute x =1, r = 13, the value of y represents d that is the thickness of the block.

$$y^2 + 12^2 = 13^2$$

$$d = y = \sqrt{169 - 144}$$

$$= 5 \text{ cm } \textbf{Ans}$$

# Problem 3

A scienctific expedition party is shipwrecked on an uninhabited island. The party has no source of energy but finds a tank of chemically inert gas. This gas is heavier than air and its pressure and temperature are the same as those of the surrounding atmosphere. The party has also two kinds of membrane, one of which is permeable to the inert gas and other permeable to air only.

Suggest an idea how an engine performing useful work can be designed and built from the resouces at the disposal of the science expedition party.

# Solution

A possible design of the engine required is based on two principles:

1. In a mixture of 2 non-reactive gases in a closed container, the total pressure of the mixture is equal to the sum of the partial pressures of individual gases. By partial pressure means the pressure of the individual gas would exert if it occupies the same volume alone at the same temperature.
2. If a certain membrane is permeable to a certain gas, the partial pressure of that gas on either side is the same.

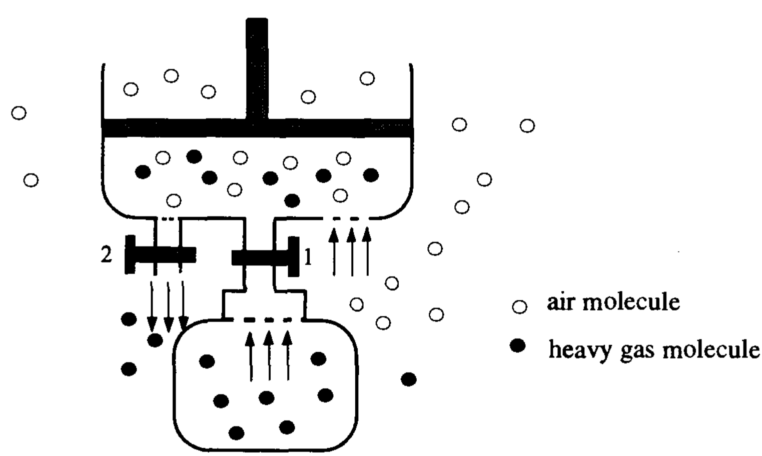



Fig. 7.3

## Design of the engine.

In the diagram depicted in Fig.7.3, the lower left side of the engine cylinder contains a membrane permeable to the heavy than air inert gas complete with a valve. In the middle of the cylinder there is also a membrane permeable to the heavy inert gas also with a valve. On the lower right side of the cylinder there is a last membrane permeable to air but with out a valve.

## Operation of the engine

1. With valve 2 closed and valve 1 open, inert gas from the container is allowed to enter the cylinder until its partial pressure inside the engine cylinder is equal to 1 atm. During this process the total pressure inside the cylinder will be graually built up until reaching the value of 2 atm. It will be observed that it is the heavy gas that performs work while air performs no work as its pressure on either side of the membrane permeable to air remains the same.

2. Close valve 1 to stop the movement of the piston.

3. While valve 1 is at close position, open valve 2 to let heavy gas into the atmosphere. The piston gradually returns to its former position at the beginning. When the piston comes to a full stop, there is only air in the cylinder.

4. The engine cycle is complete. The operation in the next cycle resumes with operation 1 and so on.

If the cylinder and the piston are made from perfect conductor, the change inside the cylinder is an isothermal process.

The work delivered by the machine is equal to work performed by the inert heavier- than air gas

i.e. $$\int PdV = \int \frac{RT}{V} dV = \ln \frac{V_2}{V_1}$$ **Ans**

# Experiment

## Problem 1

Two semi-conductor diodes of the same kind and an ohmic resistor are connected in a certain manner. The whole ensemble is contained in a closed box, fitted with two terminals for connected with incoming signals.

Determine resistance of the resistor and the two semi-conductor diodes without opening the box. The contestants are providedwith:

DC voltage source

DC ammeter and voltmeter

## Solution

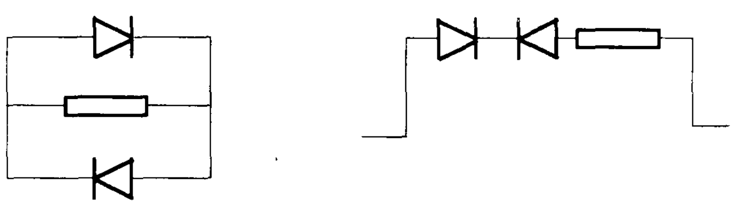
1. Connect DC voltage source across the terminals, measure current as a function of voltage.
2. Reverse the polarity of DC voltage source across the terminal. Measure current as a function of voltage.

## Experimental Results

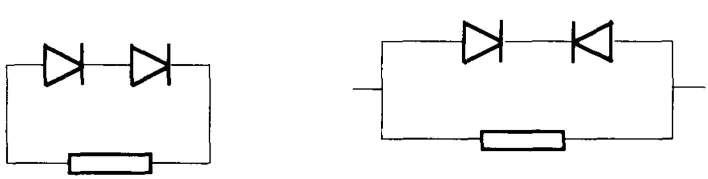
The current readings in the two directions are not the same, besides the relation between current and voltage is nonohmic. It may be inferred that:

1. The circuit lacks symmetry.
2. Resultant resistance is non-ohmic.

Not



Nor



Possible circuit is

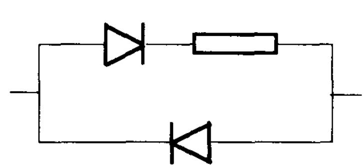


Fig. 7.4

Consider current in the upper branch of the circuit

$$I_U = \frac{V_U}{(R+r)} \qquad (1)$$

Where $I_U$ current reading in the upper branch of the circuit
$V_U$ potential across the terminals
R resistance of the ohmic resistor
r resistance of semi-conductor diode

Consider current in the lower branch of the circuit

$$I_L = \frac{V_L}{R} \qquad (2)$$

At the junction $I_U = I_L$

Hence
$$\frac{V_U}{(R+r)} = \frac{V_L}{r}$$

$$RV_L + rV_L = rV_U$$

$$R = \frac{r(V_U - V_L)}{V_L} \qquad (3)$$

Substituting
$$r = \frac{V_L}{I_L} \qquad (4)$$

to obtain
$$R = \frac{(V_U - V_L)}{I_L} \qquad (6)$$

$V_U$ $V_L$ and $I_L$ are known from measurement, thus R and r can be calculated from the above equations. **Ans**

# International Physics Olympiad VIII

# 1975

# Gustrow, German Democratic Republic

## Theory

### Problem 1

A wooden rod rotates with an angular speed $\omega$ ,while the rod is making an angle $\alpha$ with the horizon or x axis. At the rod there is a ring of mass m which can slip up and down the rod. The coeffiient of friction between the ring and the rod is equal to $\tan\theta$.

Find the condition for the ring to stay at distance L measured from the lower end of the rod, while the rod is rotating with a given angular speed $\omega$.

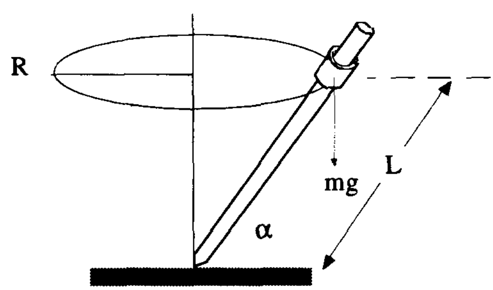



Fig. 8.1

## Solution

Static friction $F \leq \mu N$

In this case, the contibution to N on top of mg is also due to $mR\omega^2$ resolved in the direc tion normal to the wooden rod ( R is the radius of the circumference of the circle traced by the motion of the ring of mass m.)

Let $\tan\theta = \mu$

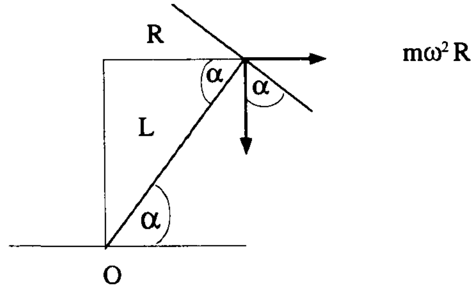



Fig. 8.2

L = distance measured from O along the rod to the ring.

N = $mg\cos\alpha + m\omega^2 R\sin\alpha$

$$\text{friction} \le \tan\theta(mg\cos\alpha + m\omega^2 R\sin\alpha)$$

In the case of the direction of friction pointing toward O (downward)

$$m\omega^2 R\cos\alpha - mg\sin\alpha \le \tan\theta(mg\cos\alpha + m\omega^2 R\sin\alpha)$$
$$m\omega^2 R\cos\alpha\cos\theta - mg\sin\alpha\cos\theta \le mg\sin\theta\cos\alpha + mR\omega^2\sin\theta\sin\alpha$$
$$m\omega^2 R(\cos\alpha\cos\theta - \sin\alpha\sin\theta) \le mg(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$$

$$m\omega^2 L\cos\alpha\cos(\alpha+\theta) \le mg\sin(\alpha+\theta)$$
$$\tan(\alpha+\theta) \ge m\omega^2\frac{L\cos\alpha}{mg}$$
$$L_{MAX} \le \frac{g\tan(\alpha+\theta)}{\omega^2 L\cos\alpha}$$

In the case of the direction of friction pointing away from O (upward)

$$mg\sin\alpha - m\omega^2 R\cos\alpha \le \tan\theta(mg\cos\alpha + m\omega^2 R\sin\alpha)$$
$$mg\sin\alpha - m\omega^2 R\cos\alpha \le \tan\theta(mg\cos\alpha + m\omega^2 R\sin\alpha)$$
$$m\omega^2 R(\sin\theta\sin\alpha + \cos\theta\cos\alpha) \ge mg(\sin\alpha\cos\theta - \cos\alpha\sin\theta)$$
$$\omega^2 L\cos\alpha\cos(\alpha-\theta) \ge g\sin(\alpha-\theta)$$

$$L_{MIN} \ge \frac{\tan(\alpha-\theta)}{\omega^2\cos\alpha}$$

The condition for the ring to stay at distance L from the lower end of the rod is

$$\frac{\tan(\alpha-\theta)}{\omega^2\cos\alpha} \le L \le \frac{\tan(\alpha+\theta)}{\omega^2\cos\alpha} \qquad \textbf{Ans}$$

# Problem 2

Find the condition for a thick lens to have the same focal length for light of two different wavelengths. Discuss possible structure of the thick lens required.

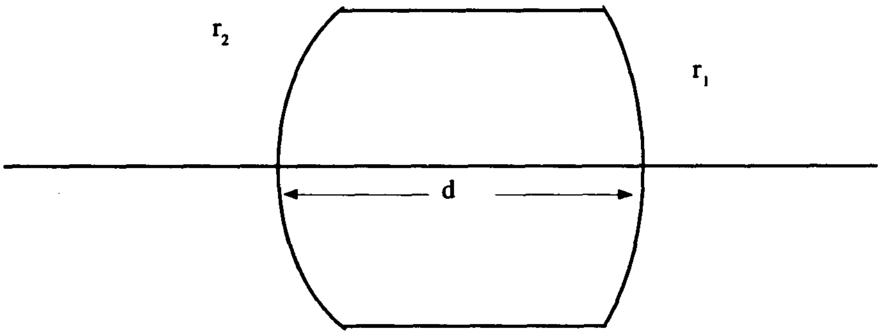



Fig. 8.3

# Solution

## First Method

The focal length of a thick lens is given by:

$$\frac{1}{f} = (n-1)\left[\frac{1}{r_1} - \frac{1}{r_2} + \frac{d.(n-1)}{n.r_1 r_2}\right]$$

where f is focal length of a thick lens measured from O as depicted in Fig 8.3
n refracrtive index of the material of which the lens is made
$r_1$ the radius of curvature of surface 1
$r_2$ the radius of curvature of surface 2
d thickness of the lens measured along the principal axis.

For light of the first wavelength, refractive index is $n_A$.

$$\frac{1}{f_A} = (n_A-1)\left[\frac{1}{r_1} - \frac{1}{r_2} + \frac{d.(n_A-1)}{n_A.r_1 r_2}\right]$$

For light of the second wavelength, refractive index is $n_B$.

$$\frac{1}{f_B} = (n_B-1)\left[\frac{1}{r_1} - \frac{1}{r_2} + \frac{d.(n_B-1)}{n_B.r_1 r_2}\right]$$

If $f_A = f_B$, then

$$(n_A-1)\left[\frac{1}{r_1} - \frac{1}{r_2} + \frac{d.(n_A-1)}{n_A.r_1 r_2}\right] = (n_B-1)\left[\frac{1}{r_1} - \frac{1}{r_2} + \frac{d.(n_B-1)}{n_B.r_1 r_2}\right]$$

Let $R = r_2 - r_1$

Hence $$(n_A-1)\left[\frac{R}{r_1r_2}+\frac{d.(n_A-1)}{n_A.r_1r_2}\right] = (n_B-1)\left[\frac{1}{r_1}-\frac{1}{r_2}+\frac{d.(n_B-1)}{n_B.r_1r_2}\right]$$

$$\frac{(n_A-1)}{r_1r_2}\left[R+\frac{d.(n_A-1)}{n_A}\right] = \frac{(n_B-1)}{r_1r_2}\left[R+\frac{d.(n_B-1)}{n_B}\right]$$

$$R(n_A-1)+\frac{d.(n_A-1)^2}{n_A} = R(n_B-1)+\frac{d.(n_B-1)^2}{n_B}$$

$$R(n_A-n_B) = d\left[\frac{(n_A-1)^2}{n_A}-\frac{(n_B-1)^2}{n_B}\right]$$

$$= d\left[\frac{n_B(n_A-1)^2-n_A(n_B-1)^2}{n_An_B}\right]$$

$$= d\left[\frac{n_B(n_A{}^2-2n_A+1)-n_A(n_B{}^2-2n_B+1)}{n_An_B}\right]$$

$$R = d\left[1-\frac{1}{n_An_B}\right]$$

Since $n_A$ and $n_B$ are both greater than 1, the right-hand side of the expression above R is positive. ( $R = r_2 - r_1$ )
The radius of curvature of the first surface may be negative or a concave surface, ( $r_1$ is negative) however the radius of curvature of the second surface must be a convex surface ( $r_2$ is positive)
In any case , $|r_1| > |r_2|$ i.e. one of the surfaces cannnot be a plane because the radius of curvature of the planar surface is $\infty$ **Ans**

## Second Method

$$\frac{1}{f}=(n-1)\left[\frac{1}{r_1}-\frac{1}{r_2}+\frac{d.(n-1)}{nr_1r_2}\right]$$

$$\Delta(\frac{1}{f})=\Delta n\left[\frac{1}{r_1}-\frac{1}{r_2}+\frac{d.(n-1)}{nr_1r_2}\right]+(n-1)\left[\frac{n.d.\Delta n-d.(n-1).\Delta n}{r_1r_2n^2}\right]=0$$

$$\Delta n\left[\frac{1}{r_1}-\frac{1}{r_2}+\frac{d.(n-1)}{nr_1r_2}\right]+(n-1).\frac{d.\Delta n}{r_1r_2n^2}=0$$

$$\frac{r_2n^2-r_1n^2+d.n.(n-1)+(n-1).d}{r_1r_2n^2}=0$$

$$r_2-r_1=\frac{d.n^2-d}{n^2} \qquad =d\left[1-\frac{1}{n^2}\right]$$

$n_A = n - \Delta n$ $\qquad$ $n_B = n + \Delta n$

$n^2 \approx n_An_B$ $\qquad$ $R = r_2 - r_1 = d\left[1-\frac{1}{n_An_B}\right]$ **Ans**

# Problem 3

An ion beam consists of ions each of which has mass m. Every ion travels from P with the same speed but in different directions. The direction of the magnetic field perpendicular to the plane of the paper forces all ions leaving P to converge on R, where PR = 2a. Ther trajectories of the ions are of course symmetrical about the symmetrical axis y. (Folding symmetry )
Determine the boundaries of the magnetic field that focuses all ions onto R.

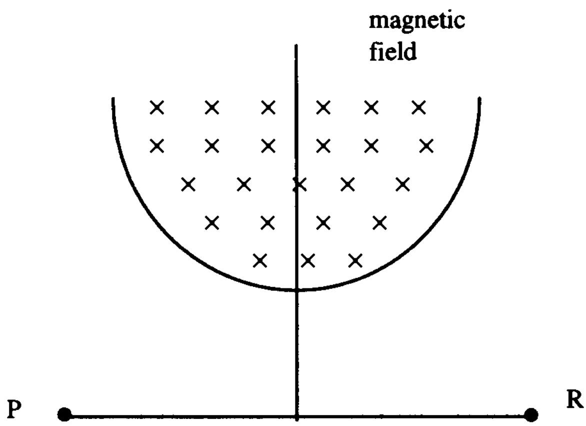



Fig. 8.5

# Solution

Lorentz force acting on an ion carrying chage q ,travelling with velocity **v** normal to the magnetic field causes the ion to move along the circumference of the circle of radius r with speed v.

Centripetal force $\quad \dfrac{mv^2}{r}\mathbf{1}_r = q\mathbf{v}\times\mathbf{B}$

where **B** is intensity of magnetic field
$\mathbf{1}_r$ a unit vector pointing in the direction of increasing **r**

Consider the upper half of the beram above **x** axis
All ions travel along the circumference of the circle of the same radiu r but with different positions of the centres depending on where the point of entry in the magnetic field is.

Let the point where an ion emerges from the magnetic field has coordinates (a, y) and ∠COD = ∠Q'RQ ( See Fig 8.6)

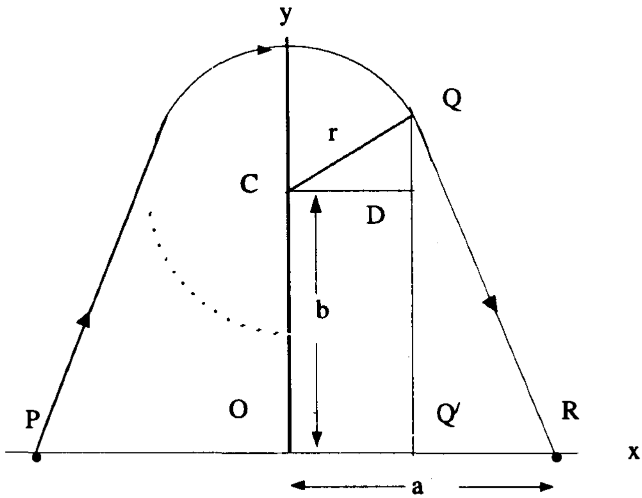



Fig. 8.6

In the diagram above

C is the centre of curvature of the circular path in the magnetic field having coordinates (0,b)
R is the point where the beam is to be focused having coordinates given by (a,0)

From the diagram $\frac{QQ'}{Q'R} = \frac{CD}{QD}$

$$\frac{y}{(a-x)} = \frac{x}{(y-b)} \tag{1}$$

The circular path of the ion is defined by

$$x^2+(y - b)^2 = r^2 \tag{2}$$

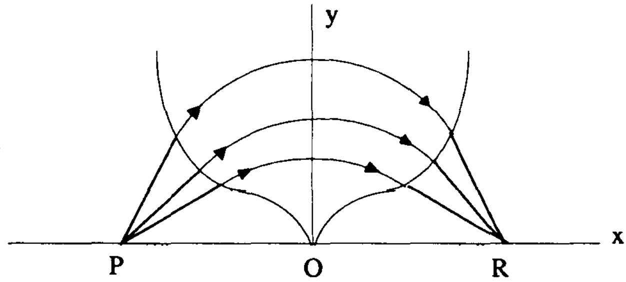
Substitute y - b from (1) in (2) to obtain

$$y = \frac{x(a-x)}{\sqrt{(r^2 - x^2}} \tag{3}$$

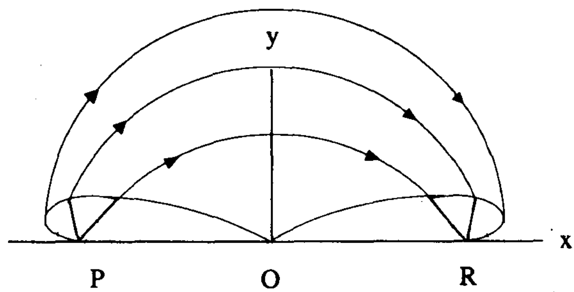
Equation (3) combining equations (1) and (2) constitutes the locus of Q defining the boundary of the magnetic field which focuses the ionic beam.

Sketches of graphs depicting the focusing of the beam under various situations are shown on the following page.

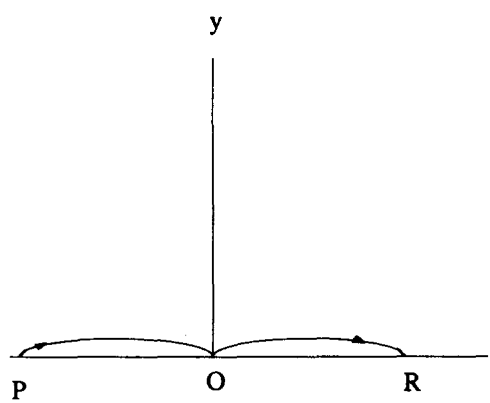
1. When r < a
High value of magnetic field and low ionic speed




2. When r = a
All ions are focused on R when magnetic field with similar configuration and boundary but different in direction is also applied to the lower half of the beam.**Ans**




3. When r > a
Only ions near the axis are focused



# Experiment

## Problem 1

A box containing a certain kind of electronic equipment has two terminals.
Find out what equipment is inside the box without opening the box. The following items of equipment are at disposal of each contestant:

9 V battery
a resistor of ixed resistance
a variable reistor
DC ammeter (with given value of resistance in each operating range)
DC voltmeter (with given value of resistance in each operating range)

( Under no circumstance should power greater than 0.25 W be used.)

## Solution

Connect a circuit as shown in Fig 8.8

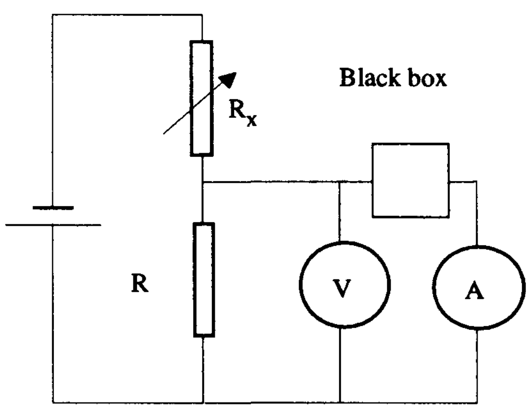



Fig. 8.8

Vary $R_x$, record current as a function of voltage.
Reverse the polarity of the terminals on the box, conduct similar experiment to the previous one.
Plot a graph of current as a function of voltage.
From the nature of the curve, it is inferred that the electronic piece of equipment inside the box is a Zener diode.

# International Physics Olympiad IX

# 1976

# Budapest , Hungary

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## Theory

### Problem1.

A hollow sphere of radius R = 0.5 m rotates about its own vertical axis with the angular speed $\omega$ = 5 rad/s. Inside the wall of the sphere at the height of $R/2$ measured along the vertical line from the lowest point of the sphere, there is a wooden block which moves along with the sphere. (See Fig. 9.1)

Determine:

1.1 The minimum value of the coefficient of friction which allows the wooden block to move with the sphere.

1.2 The condition for the wooden block to move with the sphere when $\omega$ = 8 rad/s.

1.3 Stable condition

a. if the position of the block is slightly altered.
b. if the angular spped changes slightly.

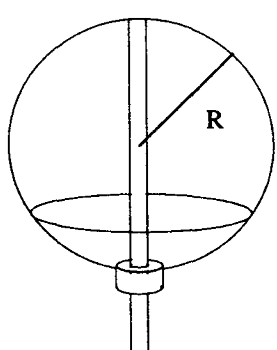


Fig. 9.1

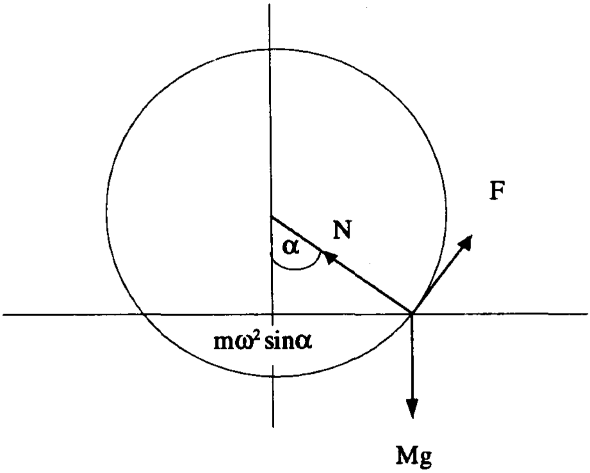


Fig. 9.2

## Solution

In the figure shown above, the block moves along a circle of radius $R\sin\alpha$.
The normal force N along PO = $mg\cos\alpha + m\omega^2 R\sin\alpha$

$$\sin\alpha = \frac{\sqrt{3}}{2}$$

$$\text{Friction} \leq \mu(mg\cos\alpha + m\omega^2 R\sin\alpha)$$

The direction of the friction is along the tangent to the spherical surface at P.

1.1 When $\omega = 5$ rad/s

$$mg\sin\alpha - m\omega^2 R\sin\alpha\cos\alpha = m\sin\alpha\,(10 - 25\times 0.5\times\frac{1}{2})$$

$$= m\sin\alpha\,(10 - 6.25)$$

$$= 3.25\, m\sin\alpha$$

The term $3.25\, m\sin\alpha$ is +ve, hence the direction of the friction points upward ($\omega = 5$ rad/s)

The condition for the block to be in the stable stage is

$$mg\sin\alpha - m\omega^2 R\sin\alpha\cos\alpha \leq \mu\,(mg\cos\alpha + m\omega^2 R\sin\alpha)$$

$$\mu \geq \sin\alpha\left[\frac{g - \omega^2\cos\alpha}{g\cos\alpha + \omega^2 R\sin^2\alpha}\right] \qquad (1)$$

Substitution gives $$\mu \geq \frac{\sqrt{3}}{2}\left[\frac{10-25\times 0.5\times 0.5}{10\times 0.5+25\times 0.5\times \frac{3}{4}}\right]$$

$$\geq \frac{3.75\times\sqrt{3}}{28.7}$$

$$\geq 0.22$$

The minimum value of coefficient of friction is 0.22 **Ans**

1.2 For ω = 8 rad/s

mgsinα - mω²Rsinα cosα= $= m\sin\alpha(10-64\times 0.5\times 1.2\ )$

= (msinα)(10 - 16)

The term -6msinα is -ve, the direction of the friction points downward. (For ω = 8 rad/s)

The condition for the block to be in the stable stage is

$m\omega^2 R\sin\alpha\cos\alpha$ - $mg\sin\alpha$ ≤ $\mu(mg\cos\alpha + m\omega^2 R\sin\alpha)$

$$\mu \geq \sin\alpha\left[\frac{\omega^2 R\cos\alpha - g}{g\cos\alpha + \omega^2 R\sin\alpha}\right] \qquad (2)$$

$$\geq \frac{\sqrt{3}}{2}\left[\frac{64\times 0.5\times 0.5-10}{5+64\times 0.5\times \frac{3}{4}}\right]$$

$$\geq \frac{6}{29}\times\frac{\sqrt{3}}{2}$$

$$\geq 0.18$$

The minimum value of the coefficient of friction is 0.18 **Ans**

1.3 To investigate stability of the block under cases a and b, graphs of $\mu_1$ and $\mu_2$ are plotted as a function of α.
From equations (1) and (2) μ is indeed the function of sinα. modified by

$$\frac{g-\omega^2 R\cos\alpha}{g\cos\alpha+\omega^2 R\sin\alpha}$$ for the case discussed under 1.1

and $$\frac{\omega^2 R\cos\alpha - g}{g\cos\alpha+\omega^2 R\sin\alpha}$$ for the case under 1.2

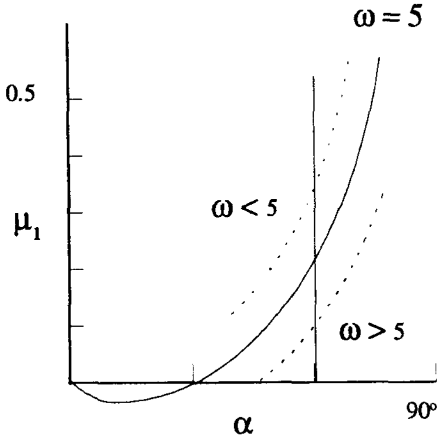


Fig. 9.3 a

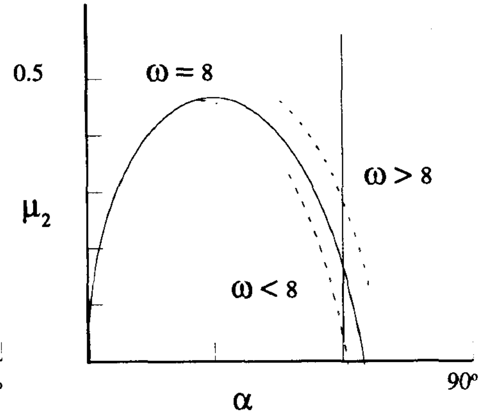


Fig. 9.3 b

From the graphs

a. if the block is slightly displaced ( α changes)

In the case of ω = 5 rad/s

If the block moves up slightly ( α increases), the required $\mu_{MIN}$ increases, and $\mu_1 < \mu_{MIN}$

This is an unstable condition, the block returns to its previous position.

If the block slips downward slightly ( α decreases), the required $\mu_{MIN}$ decreases and $\mu_1 > \mu_{MIN}$

This is a stable condition, the block does not return to its original position. **Ans**

In the case of ω = 8 rad/s

If the block moves up slightly (α increases), the required $\mu_{MIN}$ decreases and $\mu_2 > \mu_{MIN}$

This is a stable condtion, the block does not return to its original position.

If the block slips downward slightly (α decreases), the required $\mu_{MIN}$ increases and $\mu_2 < \mu_{MIN}$

This is an unstable condition, the block returns to its previous position.

b. If $\omega$ varies slightly

In the case of $\omega = 5$ rad/s

If $\omega < 5$ rad/s
(The upper curve in Fig. 9.3 a), the required $\mu_{MIN}$ increases and $\mu_1 < \mu_{MIN}$

The block slips downward to a new equilibrium position and does not return to its original position.

If $\omega > 5$ rad/s
(The lower curve in Fig. 9.3 a),the required $\mu_{MIN}$ decreases and $\mu_1 > \mu_{MIN}$

This is a stable condition, the block stays in equilibrium at its original position no change.

For $\omega = 8$ rad/s

If $\omega < 8$ rad/s (The lower curve in Fig. 9.3 b), the required $\mu_{MIN}$ decreases and $\mu_2 > .\mu_{MIN}$

This is a stable condition and the block stays inequilibriumat its original position.,no change

If $\omega > 8$ rad/s (The upper curve in Fig. 9.3 b), the required $\mu_{MIN}$ increase and $\mu_2 < \mu_{MIN}$

This is an unstable condition, the block s slips upward to a new equilibrium position and does not return to its original position. **Ans**

# Problem 2

A set of cylinder and piston having the same cross-secional area of 1 $dm^2$. Inside the cylinder there is a partition which divides the interior of the cylinder into two sections. At the partition there is a valve which opens up when the pressure in the righ-hand side is greater than the pressure in the left-hand side .(See Fig. 9.4) At the beginning, there are 12 g of helium in the left-hand side and 3 g of helium gas in the right-hand side of the cylinder. The two sections of the cylinder have the same length of 11.2 dm and the same temperature at $0^0$ C.

The pressure outside the cylinder is 10 $N/cm^2$ = 100 kPa = 1 atm.
The specific heat at constant volume of helium $C_v$ = 0.75cal/(g.K) = 3.15 J/(g.K)
The specific heat at constant pressure of helium $C_p$ = 1.25 cal/(g.K) = 5.25 J/(g.K)

The piston is slowly pushed towards the partition. As soon as the valve opens up, the pushing action stops and resumed until the piston reaches the partition. If the wall of the cylinder is made from a perfect insulator, determine work done on the helium gas.

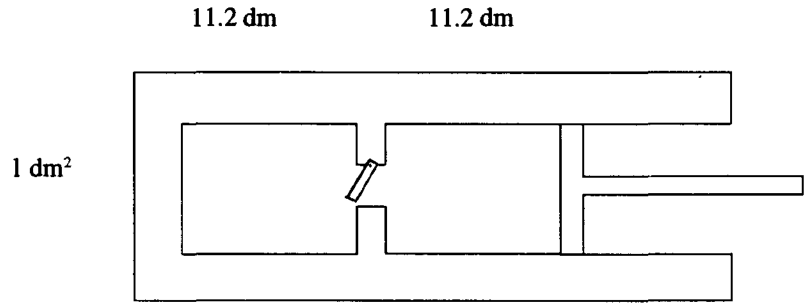


Fig. 9.4

## Solution

At the beginning, the valve is in a close position.

There are 2 g of helium in the right-hand side section of the cylinder, its volume = 11.2 dm × 1 dm = 11.2 $dm^3$ (litre) and 12 g of helium in the left-hand side of the cylinder ,its volume is also11.2 $dm^3$.

At 0° C 4 g of helium, volume 22.4 $dm^3$ , the value of pressure is 1 atm

At 0° C 12 g of helium, volume 22.4 $dm^3$, the value of pressure is 3 atm

At 0° C 12 g of helium, volum 11.2 $dm^3$, the value of the pressure is 6 atm

At 0° C 2 g of helium, volume 22.4 $dm^3$, the value of the pressure is 1/2 atm

At 0° C 2 g of helium,volume 11.2 dm3 , the value of the pressure is 1 atm

At the beginning of the experiment,
the pressure in left-hand side section of the cylinder is = 6 atm
also the pressure in the right-hand side section of piston = 1 atm.

**Stage 1** When the piston is being pushed and the valve remains close, the change is adiabatic i.e. $\Delta Q = 0$.

What we want to find out is the volume of helium gas in the right-hand side section of the cylinder when its pressure is 6 atm (Note that the valve opens up when the pressure in the right-hand side section is 6 atm)

From the formula $$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

where $$\gamma = \frac{C_P}{C_V} = \frac{1.25}{0.75} = \frac{5}{3}$$

$P_i$ = original pressure = 1 atm $P_f$ = final pressure = 6 atm
$V_i$ = original volume = 11.2 dm $V_f$ = final volume = ?

Substitution of the above values in the adiabatic equation gives

$$1 \times 11.2^{\frac{5}{3}} = 6 \times V_f^{\frac{5}{3}}$$

$$6 \times V_f^{\frac{5}{3}} \left( \frac{V_f}{11.2} \right)^{\frac{5}{3}} = \frac{1}{8}$$

$$\frac{5}{3} \times \frac{\log_{10} V_f}{11.2} = \log_{10} 6$$

$$V_f = 3.82 \text{ dm}^3$$

**Stage 2** As soon as the pressure in the right-hand side section is 6 atm (60 N/cm$^3$) the volume is decreased to 3.82 dm$^3$ while the temperaute is increased to T K.

From the relationship PV = RT

P is pressure = 60 N/$10^{-4}$m$^2$
V volume = $3.82 \times 10^{-3}$ m$^3$
R gas constant for 1 mol = $(C_P - C_V) \times 2 = (5.52 - 3.15) \times 2 = 4.20$ J/K
T = $T_0$

Substitution gives

$$60 \times 10^4 \times 3.82 \times 10^{-3} = 4.20$$

$$T_0 = 6 \times 3.82 \times 10^2 \times \frac{1}{4.2}$$

$$= 545 \text{ K}$$

**Stage 3** In this adiabatic change, the internal energy U of the gas in the righ-hand side section increases.

$$\Delta U = 2 \times C_V \Delta T$$

$$= 2 \times 0.75(545 - 273)$$

$$= 408 \text{ cal}$$

**Stage 4** As soon as the valve opens , gases in the right-hand side and the left-hand side chambers are mixed together. Let temperature of the mixture be $T_1$.

Heat lost = Heat gained

$$2 \times (545 - T_1) \times C_P = 12 \times C_P \times (T_1 - 273)$$

$$7T_1 = 545 + 1638$$

$$T_1 = 311.8 \text{ K}$$

**Stage 5** Once the gases in the two sections are completely mixed, pushing action of the piston resumes until the volume of the mixture is $11.2 \text{ dm}^3$. The change is again adiabatic.

The final temperature of the mixture is determined from,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$T_1$ = initial temperature = 311.8 K
$T_2$ = final temperature = ?
$V_1$ = original volume = 11.2+3.63 = 14.83 $\text{dm}^3$
$V_2$ = final volume = 11.2 $\text{dm}^3$

$$\gamma - 1 = \frac{5}{3} - 1 = \frac{2}{3}$$

Substitution gives

$$311\left(\frac{14.83}{11.2}\right)^{\frac{2}{3}} = T_2$$

$$T_2 = 311 \times 1.32^{\frac{2}{3}}$$
$$= 375 \text{ K}$$

**Stage 6** Increase in the internal energy

$$\Delta U = 14 \times C_P \times \Delta T$$
$$= 14 \times 0.75 \times (375 - 311) \text{ cal}$$
$$= 672 \text{ cal}$$

Total increase in the internal energy = 408 + 672 = 1080 cal

$$\text{or} \quad = 1080 \times \frac{3.15}{0.75} \text{ J}$$
$$= 4536 \text{ J}$$

**Stage 7** The atmospheric pressure outside the cylinder is 1 atm or $10/10^{-4}$ $\text{N/m}^2$ = $10^5$ $\text{N/m}^2$

Work done by the atmospheric pressure

$$P\Delta V = 10^5 \times 1 \times 11.2 \times 10^{-3} = 1120 \text{ J}$$

Total energy employed = 4536 - 1120 = 3416 J **Ans**

# Problem 3

In a glass spheroid, there is a spherical air bubble which is not concentric with the host sphere. Suggest means by which the dimension of the air bubble can be determined.

# Solution

The problem of determining the diameter of the air bubble is that light from in side the glass sphere undergoes refraction on leaving the sphere into the air. the geometricl analysis becomes very complex. More convenient methods include inter alias;

1. An X-ray photograph of the glass sphere could be taken , and from the film the dimension of the air bubble could be easily calculated. Note that X-rays do not undergo refraction as ordinary light does.

2. Place the glass sphere in a liquid which has the same value of refractive index. The wall of the container must also be transparent, thin and planar to allow light from the air bubble incident on the wall in the normal direction. The dimension of the air bubble in this setup can be easily measured using aan instrument such as a travelling microscope.

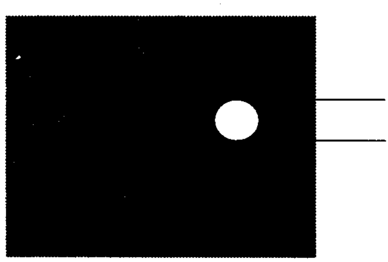

Fig. 9.5

# Experiment

## Problem 1

Given a liquid with known value of specific heat and crystal which does not dissolve in the liquid.

The following apparatuses are at disposal:

Thermometer
Test tube
Stopwatch
Electric heating coil
Determine melting point and specific heat of the crystal.

## Solution

The procedures can be taken in the following steps:

1. Divide the liquid into two equal portions.

2. Heat the first portion of the liquid using the electric heating coil. Record its temper ature as a function of time. Plot a graph of temperature of the first portion of the liquid as a function of time.

3. Drop the crystal into the second portion of the liquid. Heat the liquid using the electric heating coil.

4. Record the temperature as a function of time. Plot a graph of temperature against time.

5. From the two curves , the melting point and specific heat of the crystal can be determined.

# International Physics Olympiad X

# 1977

# Hradic,Czechoslovakia

# Theory

## Problem 1

The compression ratio of a four-stroke internal combustion engine $\varepsilon$ is e qual to 9.5. The engine takes in air and gas fuel at temperature 27° C, having volume $V_0$ and pressure of 1 atm or 100 kPa . The volume is then compressed adiabatically from state 1 to state 2 (See Fig.10.1).

The fuel mixture is ignited causing an explosion which doubles the volume(States 2-3), thus moving the piston into a position in State 3. From States 3 to 4 the gaseous mixture again expands adiabatically until the volume becomes 9.5 $V_0$ and the exhausting valve in the cylinder opens up allowing the pressure in the cylinder to return to 1 atm.

( Compression ration $\varepsilon$ = ratio between maximum and minimum values of the volume of the cylinder and $\gamma = C_p/C_V = 1.4$ )

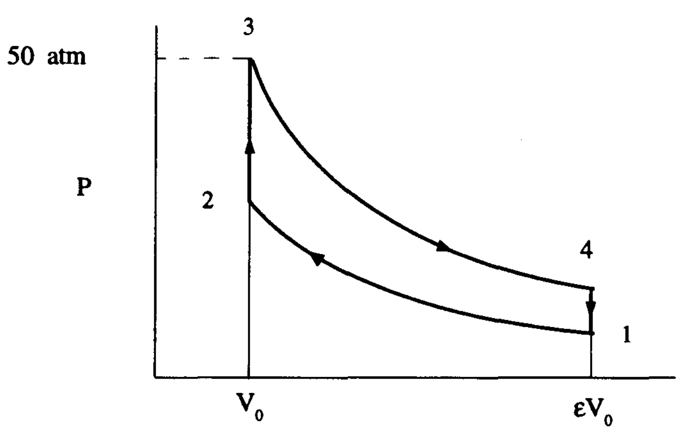


Fig. 10.1

Determine:

1.1 The pressure and temperature of the gaseous mixture in States 1, 2, 3 and 4 respectively.

1.2 The thermal efficiency of the cycle.

## Solution

1.1 For adiabatic change $$P_1V_1^{\gamma} = P_2V_2^{\gamma} \quad (1)$$

At any point in the path of the cycle PV = RT (2)

Since we have the knowledge of temperature and volume, the above equation is rewritten as

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \quad (3)$$

States $1 \rightarrow 2$

At 1 $P_1$ = 1 atm $T_1$ = 300 K $V_1 = \varepsilon V_0$
At 2 $P_2$ = ? $T_2$ = ? $V_2 = V_0$

Determine $T_2$ by substituting known values in (3)

$$T_2 = 300 \times (9.5)^{1-1.4}$$
$$= 738 \text{ K}$$

From equation $$\frac{P_2V_2}{T_2} = R = \frac{P_1V_1}{T_1}$$

Pressure at temperature $T_2$

$$P_2 = \frac{1 \times 1.95 \times 738}{300}$$
$$= 23.37 \text{ atm}$$

States $2 \rightarrow 3$

$$P_3 = 2P_2 = 46.74 \text{ atm}$$
$$\frac{P_3}{T_3} = \frac{P_2}{T_2}$$

$$T_3 = 2 \times 783 = 1476\ \text{K}$$

States 3 → 4 The change is an adiabatic process, hence $P_4$ can be found by applying

$$T_4 V_4^{\gamma-1} = T_3 V_3^{\gamma-1}$$

$$T_4 = T_3 (\frac{V_3}{V_4})^{\gamma-1}$$

$$= \frac{1476}{(9.5)^{0.4}}$$

$$= 599.7\ \text{K}$$

$P_4$ is calculated from $\frac{P_4 V_4}{T_4} = \frac{P_3 V_3}{T_3}$

$$P_4 = \frac{T_4}{T_3} \times \frac{46.74}{9.5}$$

$$= \frac{599.7 \times 46.74}{1476 \times 9.5}$$

$$= 2.00\ \text{atm}$$

## Conclusion

| State | 1 | 2 | 3 | 4 | Units |
|---|---|---|---|---|---|
| Pressure | 1 | 23.37 | 46.74 | 2 | atm |
| Temperature | 300 | 738 | 1,476 | 599.7 | K |

## Ans

1.2 Efficiency $\eta$ = Useful thermal energy ÷ Energy given by the fuel

Thermal energy from the fuel (2-3) = $C_v m(T_3 - T_2)$

Heat lost to the environment = $C_v m(T_4 - T_1)$

Useful thermal energy = $C_v m [(T_3 - T_2)-(T_4 - T_1)]$

Efficiency $\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2}$

Change $1 \to 2$ is an adiabatic process, hence

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \tag{4}$$

Likewise change $3 \to 4$ is also adiabatic, thus

$$T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1} \tag{5}$$

$$V_1 = V_4 \text{ and } V_2 = V_3$$

(4)/(5)
$$\frac{T_1}{T_4} = \frac{T_3}{T_2}$$

$$\frac{T_4 - T_1}{T_4} = \frac{T_3 - T_2}{T_3}$$

$$\eta = 1 - \frac{T_1}{T_2}$$

Sustitution of known values gives

$$\eta = 1 - \frac{300}{738}$$
$$= 0.59 \quad \text{Ans}$$

Note that in a general case $\eta = 1 - (\frac{V_2}{V_1})^{\gamma-1}$

## Problem 2

White light is incident on a thin film of soap making an angle $\alpha = 30^\circ$ with the normal. The reflected light is predominantly green light with the wavelength of 0.5 $\mu$m.

2.1 What is the minimum value of the thickness of the soap film for the phenomenon desribed to take place.

2.2 What colour is seen, if the film is viewed from the vertical position, (See Fig.10.2)

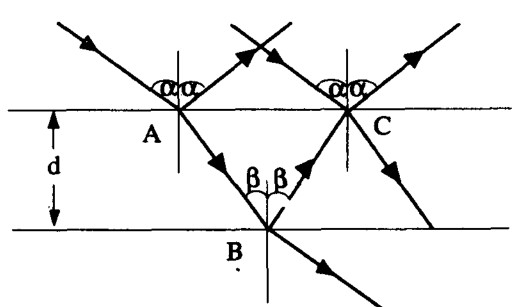


Fig. 10.2

## Solution

Let the thickness of the soap thin film be represented by d.

From the above diagram $AB + BC = \dfrac{2d}{\cos\beta}$

Optical path from A to C i.e. $ABC = \dfrac{2dn}{\cos\beta}$

Optical path from A to D $= AC\sin\alpha = 2d\tan\beta\sin\alpha$

(Note that optical path in the mediun of refractive index n is distance times n )

When light travels from a dense medium to other less dense medium, the reflected light suffers a phase change of $\pi$ corresponding to the optical path of $\lambda/2$.

The difference in optical path between that of light reflected at the upper surface of the soap film and light reflected at the lower surface of the film is $\dfrac{2dn}{\cos\beta} - 2d\tan\beta\sin\alpha$

Condition for the two beams to interfere constructively is

$$\frac{2dn}{\cos\beta} - 2d\tan\beta\sin\alpha = (2m+1)\frac{\lambda}{2} \qquad (1)$$

where m is an integers = 0,1,2,3,4,.....

The minimum thickness of the film corresponds to m = 0

$$\frac{2dn}{\cos\beta} - 2d\tan\beta\sin\alpha = \frac{\lambda}{2}$$

$$2d = \frac{\lambda}{2}\frac{\cos\beta}{(n-\sin\alpha\sin\beta)} \qquad (2)$$

From the law of refraction

$$\sin\alpha = n\sin\beta$$

$$\sin\beta = \frac{\sin\alpha}{n}$$

$$\cos\beta = \sqrt{1-\frac{\sin^2\alpha}{n^2}}$$

Substitute $\sin\beta$ and $\cos\beta$ in (2)

$$d = \frac{\lambda}{4}\sqrt{1-\frac{\sin^2\alpha}{n^2}}\cdot\frac{1}{(n-\dfrac{\sin^2\alpha}{n})}$$

$$= \frac{\lambda}{4n\sqrt{n^2 - \sin^2 \alpha}}$$

Substitute $\alpha = 30^\circ$ $n = 1.33$ $\lambda = 0.5$ m

$$d = \frac{0.5\mu m}{4\sqrt{1.76 - 0.25}}$$

$$= \frac{0.5}{4 \times 0.97}$$

$$= 0.10 \ \mu m$$

The minimum thickness of the film is 0.10 μ m **Ans**

2.2 Let $n_x$ and x represent refractive index and wavelength of the of the light beam incident on the film in the normal direction.

The optical path of light x that is reflceted at the lower surface of the film (in the normal direction) is $n_x.2d$.

The condition for the componet light x reflected at the upper and lower surfaces of the film to interfere constructively is

$$n_x 2d = \frac{\lambda_x}{2}$$

Substitute n = 1.33 d = 0.10 μm

$$\lambda_x = 1.33 \times 0.40 \ \mu m$$

$$= 0.53 \ \mu m$$

The film will be seen as a yellowish green hue characteristic of light of wavelength = 0.53 μm **Ans**

# Problem 3

An "electron gun" emits electron accelerated by potential difference U = 1000 V along direction as depicted in Fig. 10.3. We want the electrons leaving T to strike target M in the direction making an angle α with direction **a** and at a direct distance d from from T.

Given d = 5 cm, find the component of the uniform magnetic field

3.1 Perpendicular to the plane defined by line a and line d .

3.2 Parallell to TM.

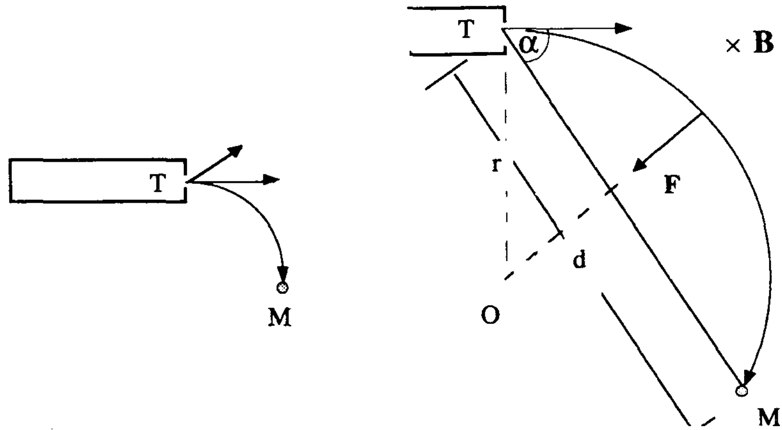


Fig. 10.3

# Solution

3.1 The path of the electron hitting M is a circumference of radius r and

$$\frac{mv^2}{r} = evB$$

where v is the magnitude of the velocity of the electron along the circumference.
( v = constant)
e electron charge
m mass of electron
B magnitude of the magnetic field required

$$r = \frac{mv}{eB} \tag{1}$$

Kinetic energy of the electron $\frac{1}{2}mv^2 = eU$

$$v = \sqrt{\frac{2eU}{m}} \tag{2}$$

Substitute v in (1) $$r = \frac{m}{eB}\sqrt{\frac{2eU}{m}} \tag{3}$$

From the geometry of the diagram

$$\frac{d}{2r} = \sin\alpha$$

Substitute r to obtain $\frac{d}{2 si\alpha n} = \frac{1}{B}\sqrt{\frac{2mU}{e}}$

$$B = \frac{2\sin\alpha}{d}\sqrt{\frac{2mU}{e}}$$

Substitute U = $10^3$ V

m = $9.11 \times 10^{-31}$ kg

e = $1.6 \times 10^{-19}$ C

d = $5 \times 10^{-2}$ m

$\alpha$ = $60°$

$$B = 2\frac{\sqrt{2 \times 10^3} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}} \frac{\sqrt{3}}{2 \times 5 \times 10^{-2}}$$

$$= 1.1 \times 10^{-4} \times \frac{\sqrt{3}}{5 \times 10^{-2}}$$

$$= 3.8 \times 10^{-3}$$ tesla **Ans**

3.2 When the required magnetic field is paralell to TM

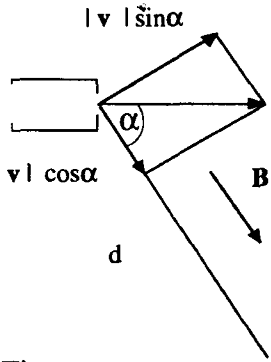


Fig.10.4

Let the component of velocity **v** normal to TM be vcos$\alpha$
the component of velocity **v** perpendicular to TM be vsin$\alpha$

The electron with component vsin$\alpha$ velocity is forced to travel along the circumference of the circleof radius r by Lorentz force, evsin$\alpha$ B along the direction directed towards the centre of curvature.

and $$\frac{mv^2 \sin^2 \alpha}{r} = evB \sin\alpha$$

$$r = \frac{mv\sin\alpha}{eB}$$

The period of the electron in uniorm circular motion with speed vsinα along the circumference of the circle of radius r is given by:

$$T = \frac{2\pi}{\omega}$$

$$\omega = \frac{v\sin\alpha}{r}$$

$$T = \frac{2\pi}{eB} m\sin\alpha$$

Since the component of the electron velocity along TM is vcosα, the path of motion of the electron takes the form of a helix of pitch h.

$$h = T v \cos\frac{2\pi}{eB} m\sin\alpha$$

Substituting $v = \sqrt{\frac{2eU}{m}}$

$$h = \frac{2\pi}{eB} m\sqrt{\frac{2eU}{m}}\cos\alpha$$

$$= \frac{2\pi}{B}\sqrt{\frac{2mU}{e}}\cos\alpha$$

In order for the electron to hit the intended taraget M, distance d needs to be a multiple of h, ie.

$$d = n h \qquad n = 1,2,3,....$$

$$\frac{d}{n} = \frac{2\pi}{B}\sqrt{\frac{2mU}{e}}\cos\alpha$$

$$B = \frac{2\pi n}{d}\sqrt{\frac{2mU}{e}}\cos\alpha$$

Substituting $U = 10^3$ V
$m = 9.11\times10^{-3}$ kg
$d = 5\times10^{-2}$ m
$\alpha = 60^{\circ}$

$$B = 2\times1.1\times10^{-4}\times\frac{1}{2\times5\times10^{-2}}$$

$$= 6.9\times10^{-3} \text{ tesla}$$ **Ans**

# Experiment

## Problem 1

Given a "black box" containing a resistor R and two capacitors connected in a star-shaped manner (See Fig.10.5). The box is fitted with three terminals, and cannot be opened. Determine the value of the resistance of the resistor and the capacity of each capacitor.

The following items of equipment are at disposal:

- -AC voltagae source, operating frequency range of 0.1 kHz to 10 kHz
- -AC voltmeter and ammeter.

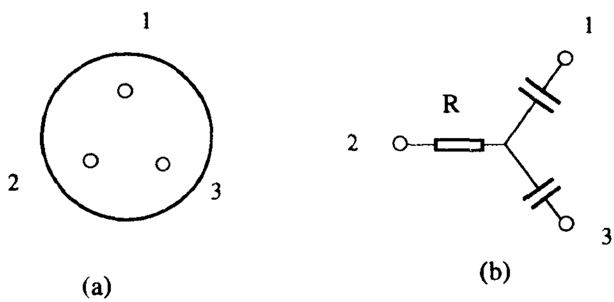


Fig. 10.5

## Solution

Based on the diagram in the fiures above, AC resistance or impedance across:

Terminals 1-2 $\quad Z_{12} =$ $\quad$ where $\quad C = \dfrac{C_1 C_2}{C_1 + C_2}$

Terminals 2-3 $\quad Z_{23} = \sqrt{R^2 + \dfrac{1}{\omega^2 C_2^2}}$

Terminals 3-1 $\quad Z_{31} = \sqrt{R^2 + \dfrac{1}{\omega^2 C_1^2}}$

Only $Z_{12}$ varies in direct proportion to $\dfrac{1}{\omega}$

Once we identify the two terminals which give $Z_{12}$, we shall be able to say right away that each of the remaining two combinations of the terminals consists of a resistaor and a capacitor in series.

In order to locate the terminals corresponding to $Z_{12}$, the following procedures are carried out:

1. Send AC signal across terminals 1-2 , take readings of frequency, voltage and current Adjust the frequency, repeat the experiment.

2. Repeat the experiment with terminals 2-3 and 3-1

3. Plot a graph of $\frac{V}{I}$ ( ie. Z) as a function of $\frac{1}{f}$ ( $\omega = 2\pi f$ ). The terminal pair which gives a linear curve is identified as one which gives $Z_{12}$ ie. two capacitors connected in series.

4. Connect AC signal to the terminals consisting of capacitor and resistor in series.

4.1 With a high frequency signal ( 100 kHz ), the impedance o r AC resistance

is mainly from R as $\frac{1}{\omega C} \sim 0$.

From V and I readings at high frequencies, calculate $R = \frac{V}{I}$

4.2 With a low freqency signal( 0.1 kHz), the impedance or AC resistance is mainly from $\frac{1}{\omega C}$ which becomes much larger than R. With known value of $\omega$ (= $2\pi f$ ) V and I from the experiment, C is calculated from

$$\frac{1}{\omega C} = \frac{V}{I}, \text{ or } C = \frac{I}{\omega V}$$

# International Physics Olympiad XI

# 1973

# Moscow,USSR

## Theory

### Problem 1

A space craft of mass m = 1,200 kg is in an orbit about the moon in the plane of the moon's equator at the altitude of 100 km above the lunar surface. In order to land the space craft on the surface of the moon, the astronaut turns on the jet engine for a brief period when the space craft is at position X . The velocity of the jet fuel firing from the nozzle is 10,000 m/s relative to the space craft.

By this means, the space craft can be brought to landing on the moon in two different methods (See Figs 11.1a and b. )

Method 1; landing at A on the opposite side with X.
Method 2; landing at B

Calculate fuel consummed in each method
Given the radius of the moon R = 1,700 km , acceleration of free fall on the surface of the moon = 1.7 m/s$^2$

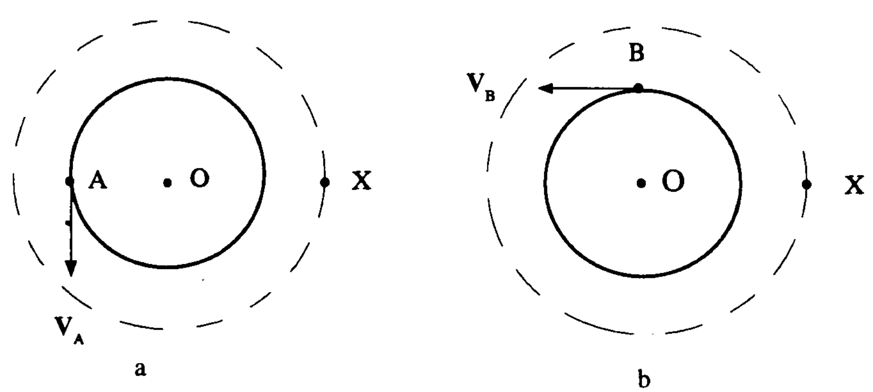


Fig. 11.1

## Solution

**First method**: The space craft has momentum in the direction tangent to the lunar surface at A. The firing of the jet engine on board must give momentum to the space craft in this direction, ie the jet fuel must be propelled in the direction opposite to the direction of added momentum.

Acceleration of free fall on the surface of the moon $g = -G\dfrac{M}{R^2}$

where G is universal gravitational constant
M mass of the moon
m mass of the space craft prior to firing jet engine for landing.

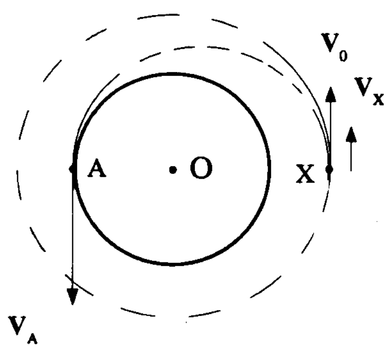


Fig.11.2

Let $v_0$ be the value of the velocity of the space craft when it is at altitude h above the lunar surface.

$$\frac{mv_0^2}{R+h} = G\frac{mM}{(R+h)^2}$$

$$mv_0^2 = g\frac{R^2 m}{(R+h)}$$

$$v_0 = R\sqrt{\frac{g}{R+h}} \qquad (1)$$

Substituting h = $10^5$ m and R = 1.7 × $10^6$ m to obtain,

$$v_0 = 1.7 \times 10^5 \sqrt{\frac{1.7}{1.7\times10^6+10^5}} \text{ m/s}$$

$$v_0 = 1.65 \times 10^3 \text{ m/s}$$

Let $v_X$ be the value of the velocity of the space craft just after the firing the jet fuel is finished. (In the direction opposite to the direction of motion of the space craft)
$v_A$ be the value of the velocity of the space craft at A.
( $mv_{A2}$ - $mv_{X2}$) mass of the space craft after firing jet fuel.

From the priciple of the conservation of energy,

$$\frac{1}{2}mv_A^2 - G\frac{mM}{R} = \frac{1}{2}mv_X^2 - G\frac{mM}{R+h}$$

$$\frac{1}{2}(mv_A^2 - mv_X^2) = Gm(\frac{1}{R} - \frac{1}{R+h})$$

$$(v_A^2 - v_X^2) = 2gR^2[\frac{R+h-R}{R(R+h)}]$$

$$= 2gR[\frac{h}{(R+h)}] \qquad (2)$$

From the priciple of the conservation of angular momentum,

$$m_0 v_A R = m_0 v_X (R+h)$$
$$v_A = \left(\frac{R+h}{R}\right) v_X$$

Sustituting $v_A$ in (2)

$$\left[\left(\frac{R+h}{R}\right)^2 - 1\right] v_X^2 = 2gR\frac{h}{R+h}$$

$$v_X^2 = 2g\frac{R^3}{(R+h)(2R+h)}$$

Substituting $g = 1.7\ m/s^2$
$R = 1.7 \times 10^6\ m$
$h = 105\ m$

$$v_X^2 = 2\times 1.7\frac{1.7\times 10^3}{(1.7\times 10^6 + 10^5)(2.17\times 10^6 + 10^5)}$$
$$= 2.65 \times 10^6 \quad (m/s)^2$$
$$v_X = 1.63 \quad m/s$$

Jet fuel must be fired until the magnitude of the velocity of the space craft is reduced to 1.63 m/s.

Let $m_f$ be mass of fuel fired from the space craft.
From the principle of the conservation of momentum,

Momentum before firing jet fuel = momentum after jet fuel is fired

$$mv_0 = (m - m_f)v_X + m_f(v_f + v_X)$$

where $v_f$ is velocity of jet fuel fired relative to the space craft = $10^4$ m/s

$$m_0 v = mv_X - m_f v_X + m_f v_f + m_f v_X$$
$$m_f v_f = m(v_0 - v_X)$$
$$m_f = \frac{m(v_0 - v_x)}{v_f}$$

Sustituting gives $\quad m_f = 1.24\times 10^{-14}\frac{(1.65-1.63)\times 10^{-13}}{10^{14}}$

$$= 24 \quad kg$$

The quantity of jet fuel fired is 24 kg **Ans**

**Second Method :** The space craft is brought to landing at B.

In this cae, the final velocityof lthe space craft $\mathbf{v}_f$ is normal to $\mathbf{v}_0$, the jet fuel must be fired in such a say that an additional momentum is given to the space çraft in the direction perpendicular to $\mathbf{v}_0$. Let $\mathbf{v}_X$ be velocity of the space craft just after the firing of jet fuel is completed. From the principle of the conservation of energy

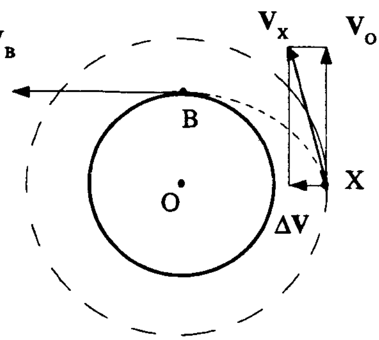


Fig. 11.3

$$(v_B^2 - v_X^2) = 2gR[\frac{h}{(R+h)}]$$

From the principle of the conservation of angular momentum,

$$m_B \mathrm{v}_B R = m_0 v_X (R+h)\cos\alpha$$

where $\alpha$ is the angle which $\mathrm{v}_X$ makes with $\mathrm{v}_0$

and $\cos\alpha = \frac{v_0}{v_X}$

$$\text{hence } v_B = \frac{R+h}{R} v_0 \qquad (3)$$

Substituting

g = 1.7 m/s²

R = 1.7 × 10⁶ m

h = 105 m

$\mathrm{v}_0$ = 1.65 × 103 m/s

$$v_B = \frac{1.8}{1.7} \times 1.65 \times 10^3$$

$$= 1.75 \times 10^3 \text{ m/s}$$

From (1) $\mathrm{v}_B^2 - \mathrm{v}_X^2 \sim 2gh$

$\sim 2 \times 1.7 \times 10^5$

$\mathrm{v}_B^2 = 3.06 \times 10^6$

$3.06 \times 10^6 - \mathrm{v}_X^2 = 3.14 \times 10^5$

$\mathrm{v}_X^2 = 3.06 \times 10^6 - 3.14 \times 10^5$

$= 2.75 \times 10^6$

$\mathrm{v}_X = 1.66 \times 10^3$ m/s

Let $\Delta\mathbf{v}$ represent the component of the velocity of the space craft along the direction pointing towards the centre of the moon just after the firing of jet fuel is completed,

$$|\Delta \mathbf{v}| = \sqrt{v_X{}^2 - v_0{}^2}$$

$$= \sqrt{(1.66 \times 10^3)^2 - (1.65 \times 10^3)^2}$$
$$= 100 \quad m/s$$

Let $m_f$ be the quantity of the jet fuel fired from the space craft,
From the principle of conservation of momentum

$$(m - m_f)\Delta v = m_f v_f$$
$$(1.22 \times 10^4 - mf)100 = m_f \times 10^4$$
$$m_f = 1.2 \times 10^2$$
$$= 120 \quad kg$$

The quantity of jet-fuel ejected is 120 kg **Ans**

## Problem 2

An aluminum ingot is weighed in the atmosphere of dry air and later in the atmosphere of moist air. The partial vapour pressure of moist air is 1/50 of the total pressure. The atmospheric pressure and temperature in both cases are the same at 700 mmHg and 20° C respectively. The balace used is a beam type with brass weights. The sensitivity of the balance is such that it can detect the difference of 0.1 mg.
What shoud be the mass of the aluminum ingot to have a difference of 0.1 mg in the two values of weight?
The values of the density of aluminum and brass are:
2.7 g/cm³ and 8.5 g/cm³ respectively.
The values of the density of dry air and water vapour are:
.00075 g/cm³ and .00120 g/cm³ respectively.

## Solution

Consider 50 unit volume of moist air.
Partial pressure of water vapour of moist air is 1/50 of the total atmospheric pressure implies that the pressure of 50 unit volume of the water vapour is equal to 1/50 of the atmopheric pressure. (Constant temperature)
Consider 50 unit volume of moist air at 20° C and 1 atmospheric pressure.
1/50 atmopheric pressure in moist air is contributed by the vapour.

From the formula $PV = \text{constant}$ (constant temperature)
Presssure varies in inverse proportion to volume but in direct proportion to density.

Density of moist air $= \dfrac{49 \times .0012 + 1 \times .00075}{50}$ g/cm³
$= .00119$ g/cm³

Differnce in density between the moist air and dry air,

$= .00120 - .00119 \quad g/cm^3$

$= .00001 \quad g/cm$

1 g. of aluminum occupies the volume of $= .370 \quad cm^3$

1 g. of brass occupies the volume of $= .117 \quad cm^3$

Forces of bouyancy of moist air on 1 g of aluminum and 1 g of brass differ by

$= (.370-.117) \times$ density of moist air

$= .253x.00001$ gramme weight

$= 2.53x10^{-5}$ gramme weight

Difference of $2.53 \times 10^{-5}$ gramme weight requires 1 g of aluminum.

Difference of $1 \times 10^{-4}$ gramme weight requires $\dfrac{1\times10^{-4}}{2.53\times10^{-6}}$ g of aluminum.

$= .395x102 \quad g$

The required mass of aluminum is 39.5 g **Ans**

# Problem 3

A telescope in the form of a concave paraboloid mirror having the diameter measured at the edge $D = 2.6$ m is equipped with a laser source giving $\lambda = 0.69\ \mu m$ at the focal point of the mirror. Laser light reflected at the surface of the paraboloid mirrror is directed toward the moon. A plane circular mirror of diameter $d = 20$ cm placed on the surface of the moon reflects the beam of laser light back to the paraboloid concave mirror, and detected by a photoelectric cell located at the focal point of the parboloid mirror. Given the distance the distance between the earth and the moon $L = 380{,}000$ km .

3.1 Calculate the range of the angle through which the telelescope can be turned and still direct light beam onto the mirror on the moon.

3.2 Calculate the ratio between the energy recieved on the earth and the energy sent out by the ensemble of laser and paraboloid.

3.3 If the enrgy sent out from the earth is 1 J, how many photons are registered by a naked eye of an observer on the earth? ( Diameter of the pupil of the eye of the observer is 5 mm., and Planck's constant $h = 6.63x10^{-34}$)

3.4 If there is no mirror on the surface of the moon, calculate the ratio between the energy received by the photocell and the energy sent out by the the system of paraboloid and laser. (The lunar surface reflects 10% of incident light uniformly in all directions.)

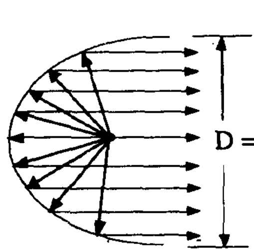


Fig.11.4 a

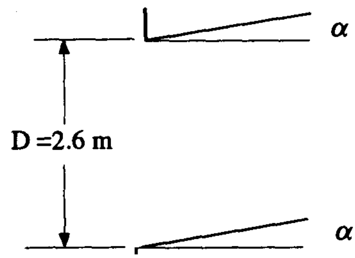


Fig.11.4 b

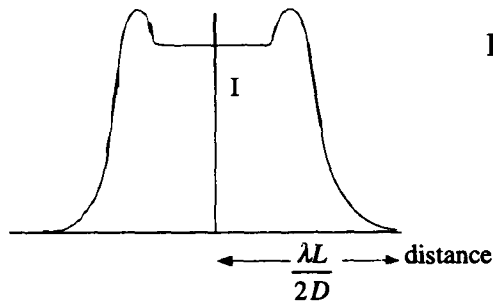


Fig.11.4 c Intensity distribution of laser light on the surface of the moon.

Light wave sent out from the paraboloid mirror is in the form of a plane wave which undergoes diffraction observed on the surface of the moon.

Zero intensity is observed when

$$D\sin\alpha = \frac{\lambda}{2}$$

For small value of $\alpha$ $\sin\alpha \sim \alpha$

$$D\alpha \sim \frac{\lambda}{2}$$

3.1 The range of the angle through which the telescope can be turned and still direct the laser beam on the mirror( and recieve the reflected signal)

$$= 2\alpha = \frac{\lambda}{2D}$$

$= 0.055''$ **Ans**

3.2 A circular area on the lunar surface hit by the laser light has the diameter given by $\frac{\lambda}{2D}$.

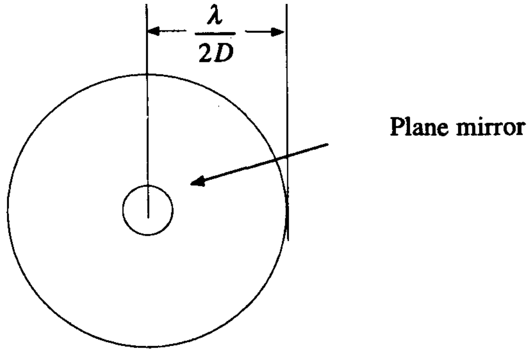


Fig.11.5

The area receiving light $= \pi(\frac{\lambda L}{2D})^2$

$= 3.14 \times (\frac{0.69 \times 10^{-6} \times 3.8 \times 10^{8}}{2 \times 2.6})$ $m^2$

$= 3.14 \times 0.25 \times 10^{4}$ $m^2$

$= 8.00 \times 10^{3}$ $m^2$

If E represents energy sent out by the laser source, the energy incident of the circular plane mirror of radius .20/2 m is given by

$= E \times 1.25 \times 10^{-4} \times \pi \times 0.10^2$ $J/m^2$

$= E \times 3.9 \times 10^{-6}$ $J/m^2$

The light beam reflected at the circular plane mirror undergoes diffraction oberved by an observer on the earth. Suppose zero intensity occurs at angle β .( See Fig.11.5)

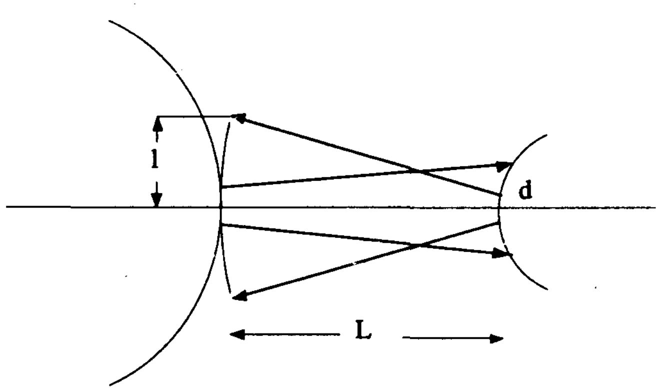


Fig 11.5 (not drawn to scale)

Let d be diameter of the circular plane mirror.

$$d\sin\beta = \frac{\lambda}{2}$$

$$\sin\beta = \frac{1}{L}$$

where 1 is radius of the circular area on the surface of the earth illuminated by laser light reflected off the lunar surface.

Substituting $$\frac{d.l}{L} = \frac{\lambda}{2}$$

$$l = \frac{0.69\times10^{-6}\times3.8\times10^{8}}{2\times20\times10^{-2}}$$

$$= 6.55\times10^{2} \quad \text{m}$$

Light energy falling on 1 square meter area on the surface of the earth is,

$$= \frac{E\times3.9\times10^{-6}}{\pi\times(6.55\times10^{2})^{2}}$$

$$= 2.89E\times10^{-12} \quad \text{J}$$

Another quantity of energy recieved by 1 square meter on the surface of the earth is due to light scattered by the lunar surface. Since the area of the tiny plane mirror is negligible compared with the illminated area, the energy of the scattered light is effective ly 10% of the energy sent out by the laser.

Energy of scattered light falling on 1 square meter area on the earth is

$$= \frac{E}{10\times2\pi\times L^{2}} \quad \text{J/m}^2$$

$$= \frac{E}{10\times2\times3.14\times3.8^{2}\times10^{16}} \quad \text{J/m}^2$$

$$= 1.10\times10^{-18}\,E \quad \text{J/m}^2$$

This value is neglible in comparison with the amount of energy reflected from the tiny mirror.

The part of the energy that enters the telescope of radius 2.62/2 m is

$$= 2.89\times10^{-12}\times1.32\ E \quad \text{J}$$

$$= 15.3\times10^{-12}E \quad \text{J}$$

Energy received at the photocell is $15.3\times10^{-12}E$ J **Ans**

3.3 Area of the pupil of the observer's eye

$$= \pi\times(2.5\times10^{-3})^2$$

$$= 1.96\times10^{-5} \quad \text{m}^2$$

Energy recieved at the eye $= 2.89 \times 10^{-12} \times 1.96 \times 10^{-5} E$ J
$= 5.6 \times 10^{-17} E$ J

Substituting $E = 1$ J

Energy recieved at the eye $= 5.6 \times 10^{-17}$ J

Energy of 1 photon $= h\nu = 6.63 \times 10^{-34} \times \frac{c}{\lambda}$

$$= \frac{6.34 \times 10^{-34} \times 3 \times 10^{8}}{0.69 \times 10^{-6}} \text{ J}$$

$= 2.89 \times 10^{-10}$ J

$$\text{Number of photons entering the eye} = \frac{5.6 \times 10^{-17}}{2.89 \times 10^{-10}}$$

$= 190$ quanta **Ans**

3.4 If there in tiny plane mirror on the moon, energy received by the telescope
$= 1.10 \times 10^{-18} E \times \pi \times 1.3^2$
$= 5.8 \times 10^{-18} E$ J

Energy received by the telescope is $5.8 \times 10^{-18}$ times the amount of energy sent out by the laser source **Ans**

# Experiment

## Problem 1

Given a black box containing items of electronic equipment in some unknown connection. There are however four elctrical terminals on the outside. Identify all electronic items in the box without opening, and also sketch a diagram depicting their connection. Items of equipment placed at disposal are;

-DC and AC voltage sources

-AC and DC voltmeters and ammeters.

-Variable resistor.

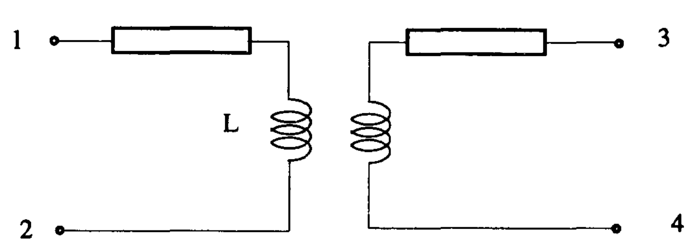


Fig. 11.6

## Solution

1. With AC and DC voltmeters, test for voltage source ( AC as well as DC ) across two terminals ( six combinations in all ), there is neither DC nor AC voltage across any two terminals tested.

2. Cónnect one terminal of DC voltage source on the terminal marked 1 . With DC voltmeter, measure voltage of the other three terminals one by one relative to the remaining terminal of DC source(1*) ie. voltage across terminals 1*-2 ,1*-3, and 1*-4. It is discovered that terminals 1- 2 are shorted . Repeat the experiment, using terminal 3 in place of terminal 1. It is discovered that terminals 3 and 4 are shorted.

3. Connect AC voltage source across terminals 1-2. With AC voltmeter, measure AC voltage across terminals 1-2 (V) and likewise with AC ammeter measure current in the circuit (I). Adjust the frequency of AC source, record the values of freqency ω, V and I.

4. At low frequencies, AC resistance or impedance $Z \sim \frac{V}{I}$ and constant.

At high frequencies, AC resistance or impedance $Z \sim \frac{V}{I}$ varies linearly with $\omega$ .

It is most likely that terminals 1 - 2 consist of resistor and induction coil connected in series.

5. Carry out testing as in 3,but using terminals 3=4 (instead of terminals 1-2).
A result similar to that of 4 is obtained i.e. terminals 3-4 consist of resistor and induc tion coil in series

6. At low freqencies, DC resistance predominates $R \sim \frac{V}{I}$

At high frequencies , AC resistance predomiinates $\omega L \sim \frac{V}{I}$

To find the value of L, a graph of frequency $\omega(= 2\pi v)$is plotted as a function of $\frac{V}{I}$.

From the slope of the linear graph, the value of L is determined.

# International Physics Olympiad XII

# 1981

# Varna, Bulgaria

## Theory

### Problem 1

A system of cylinder and piston is held in horizontal position inside a vacuum chamber. The mass of the cylinder and that of the piston are $m_1$ and $m_2$ respectively. The piston which has negligible thickness divides the volume of the cylinder into two equal parts. (See Fig.12.1)

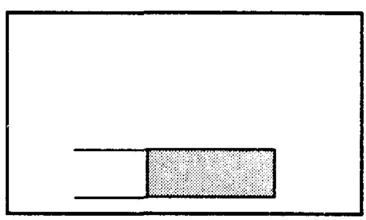

Fig. 12.1

The closed section of the clylinder contains n moles of helium which may be considered as an ideal monotomic gas at $T_0 = 273$ K. The piston is held at rest in the mid position of the cylinder as described above and then sudenly released. Determine the final velocity of the cylinder. Assume that heat transfer between cylinder and piston, and the motion of the centre of gravity are negligible.

Given:

$T_0 = 273$ K
$m_1 = 0.6$ kg ,
$m_2 = 0.3$ kg
n = number of moles of helium= 25 mol
M = Molar mass of He, = 4 g/mol

Molar specific heat at constant volume of He, $C_V = 12.6$ J/mol.K

$$\frac{C_P}{C_V} = \gamma = \frac{5}{3}$$

# Solution

Change of the system of cylinder and piston and their gaseous content in the first stage is adiabatic.

From the formula $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

where $T_1 = 273 \quad T_2 = ?$

$V_2 = 2V_1$

$\gamma = \frac{5}{3}$

Substitution gives $273 = T_2 (\frac{2V_1}{V_1})^{\frac{2}{3}}$

$$T_2 = \frac{283}{1.59}$$

$$= 172 \text{ K}$$

Drop in temperature from 273 K to 172 K results in loss of internal energy by being converted into kinetic energy of the cylinder and the piston.
From the principle of conservation of energy

$$n\, C_V (T_1 - T_2) = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$$

where n = number of moles of helium gas = 25 mole
$C_V$ = specific heat of helium at constant volume = 12.6 J/mol.K
$V_1$ = velocity of the cylinder
$V_2$ = velocity of the piston

Substitution gives

$$0.3V_1^2 + 0.15V_2^2 = 25 \times 12.6 \times 101$$
$$= 3.18 \times 10^4 \text{ J}$$

From the principle of conservation of momentum,

$$m_1 V_1 = m_2 V_2$$
$$0.6\, V_1 = 0.3 V_2$$
$$V_2 = 2\, V_1$$

Substitute $V_2$ into equation of conservation of energy

$$0.3V_1^2 + 0.6V_1^2 = 3.18 \times 10^4 \text{ J}$$

$$V_1^2 = \frac{3.18\times10^4}{0.9}$$

$$= 3.53\times10^4 \qquad \text{J}$$

$$V_1 = 1.88\times10^2 \qquad \text{m/s}$$

$$V_2 = 3.76\times10^2 \qquad \text{m/s}$$

While the piston is being pushed toward the end of the cylinder, the piston is moving toward the right hand side. The moment the piston reaches the opening end of the cylinder and becomes detached from the cylinder, the piston is moving toward the left with the velocity of $3.76\times10^2$ m/s, while the cylinder is moving toward the right with the velocity of $1.88\times10^2$ m/s, both relative to the vacuum chamber.

As soon as the piston is separated from the cylinder, the cylinder will continue to move toward the right like a rocket. This rocket-like motion is due to some portion of n mol of gas molecules( of molecular mass M, at temperature $T_2$= 172 K ) are reflected at the closed end of the cylinder and transfer momentum to the cylinder.

Kinetic energy of gas is given by

$$\frac{1}{2}mV_m^2N = \frac{3}{2}RT$$

where m is mass of a gas molecule expressed in kg

$V_m^2$ root mean square velocity of gas molecules

N Avogadro's number and $m = \frac{M}{N}$

R gas constant for 1 mol of gas $= C_P - C_V = (\gamma - 1)C_V = \frac{2}{3}C_V$

T temperature of the gas = 172.4 K

M molar mass = .004 kg

Substitute to get

$$V_m^2 = V_x^2 + V_y^2 + V_z^2 = \frac{3RT}{M}$$

$$V_x^2 = \frac{RT}{M} = \frac{2}{3}\times12.6\times\frac{172}{.004}$$

$$V_x^2 = 36.1\times10^4$$

$$\sqrt{V_x^2} = V_x = 601 \quad \text{m/s}$$

At the equilibrium,we may resolve motion of all molecules along x, y and z axes.
In this problem, we choose x axis parallel to the horizontal direction. Note that 1/6 of all molecules moving in the negative direction of x and other 1/6 of all molecules moving in the positive direction of x axis and transfer momentum to the cylinder.

From the principle of the conservation of momentum

$$\frac{0.1}{6}v_{gb} + 0.6v_{cb} = -\frac{0.1}{6}v_{ga} + +0.6v_{ca}$$

where $v_{gb}$ is x component of helium molecules just before rocket action
$v_{ga}$ x component of helium molecules just after rocket action is over
$v_{cb}$ velocity of the cylinder just before rocket action
$v_{ca}$ velocity of the cylinder just after rocket action ends.

From the principle of the conservation of energy

$$\frac{1}{2}\left(\frac{0.1}{6}v_{gb}^2 + 0.6v_{cb}^2\right) = \frac{1}{2}\left(\frac{0.1}{6}v_{ga}^2 + 0.6v_{ca}^2\right)$$

From (1) $\frac{0.1}{6}\left(v_{gb} + v_{ga}\right) = 0.6\left(v_{ca} - v_{cb}\right)$

From (2)

$$\frac{0.1}{6}\left(v_{gb} - v_{ga}\right)\left(v_{gb} + v_{ga}\right) = 0.6\left(v_{ca} - v_{cb}\right)\left(v_{ca} + v_{cb}\right)$$

Substitute $\left(v_{gb} + v_{ga}\right)\frac{0.1}{6}$ from (3) in (4) to arrive at

$$0.6\left(v_{gb} - v_{ga}\right)\left(v_{ca} - v_{cb}\right) = 0.6\left(v_{ca} - v_{cb}\right)\left(v_{ca} + v_{cb}\right)$$

$$\left(v_{gb} - v_{ga}\right) = \left(v_{ca} + v_{cb}\right)$$

From (3) $\left(v_{gb} + v_{ga}\right) = 36v_{ca} - 36v_{cb}$

(6)+(7) $2v_{gb} = 37v_{ca} - 35v_{cb}$

$$v_{ca} = \frac{2v_{gb} + 35v_{cb}}{37}$$

Substitute $v_{gb}$ = 599 m/s
$v_{cb}$= 189 m/s

$$v_{ca} = \frac{1198 + 6615}{37}$$

$$= \frac{7813}{37}$$

$$= 221 \text{ m/s}$$

Final velocity of the cylinder 221 m/s **Ans**

## Problem 2

An electric bulb having ristance R = 2 is connected across its normal operating potential difference of 4.5 V from a sliding wire rheostat. The rheostat itself is connected across a DC battery of negligible internal resistance and emf U = 6 V. (See Fig. 12.2). The bulb is connected and used in such a manner that the efficiency of the bulb 0.6.

2.1 Calculate the total resistance of the rheostat and the maximum current the rheostat shoud be able to withstand.
2.2 Find the condition for the bulb to operate at maximum efficiency and also the value of the maximum efficiency.

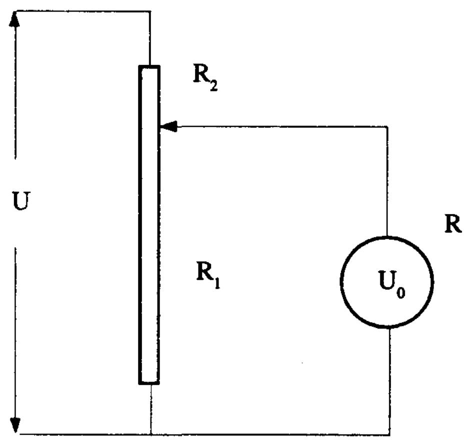


Fig.12.2

## Solution

Calculation of current $I_0$ through the bulb.

$$I_0 = \frac{U_0}{R}$$

$$= \frac{4.5}{5} \qquad \text{A}$$

Energy which the circuit delivers to the bulb per second is given by

$$= U_0 I_0$$
$$= 4.5 \times 2.25$$
$$= 10.12 \qquad \text{W}$$

Let the current through $R_2$ be $I_T$
Hence total power used by the circuit = $U I_T$
Useful power consumed = $U_0 I_0$

Efficiency $\eta = \frac{U_0 I_0}{U I_T}$

$$= \frac{4.5 \times 2.25}{6 \times 0.6}$$

From the information provided in the problem

$$0.6 \le \frac{4.5 \times 2.25}{6 \times I_T}$$

$$I_T \le 2.81 \quad A$$

$R_2$ is the resistance of the upper portion of the rheostat which must be able to withstand the current upto 2.81.

$$R_2 = \frac{U - U_0}{I_T}$$

$$= \frac{1.5}{2.81}$$

$$= 0.53 \quad \Omega$$

$$R_1 = \frac{U_0}{I_T - I_0}$$

$$= \frac{4.5}{2.81 - 2.25}$$

$$= 8 \quad \Omega$$

Total resistance of the rheostat is 8.53 Ω **Ans**

From (1) $\eta = \frac{U_0 I_0}{U I_T}$

The efficiency is maximum when $I_T$ is minimum.
The minimum posssible of the current is when $I_T = I_0$

This situation arises when $R_1$ is removed which is the same as making $R_1 = \infty$

Maximum efficiency $\eta = \frac{U_0}{U} = \frac{4.5}{6}$

$= 0.75$ **Ans**

# Problem 3

A radio astronomical observatory has its reciever installed at a height of h = 2 m above the sea level. The reciever can only pick up the horizontal component of the electric field of the electromagnetic waves. When a radio- star is rising at the horizon, it emitts an electromagnetic wave of wavelength $\lambda = 21$ cm, .the radio receiver registers maxima and minima in intensity.

3.1 Determine the direction of the electromagnetic wave when maxima and minima are observed. (Express the answer in terms of angles measured from the horizon.)

3.2 Does the intensity increase or decrease as the radio star is steadily arising from the horizon?

3.3 Determine the ratio of the intensity of adjacent maxima and minima.

**NB** The ratio of the amplitude of the incident and reflected waves is $\dfrac{n+\sin\alpha}{n-\sin\alpha}$,

where

α = the angle which the incident wave makes with the horizontal plane,

n = refractive index for electromagnetic wave travelling from the air to the water = 9.

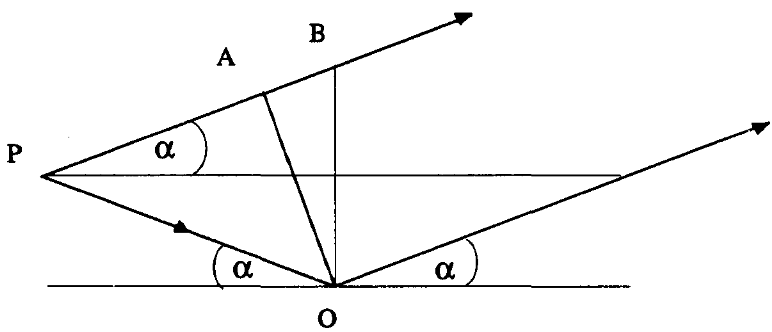


Fig.12.3

# Solution

3.1 Wave from the star can reach the reciever by two methods ie.

1. Direct from the star to the reciever at P.
2. From the star and reflected at O on the surface of the sea before being detected at P.

From the diagram, the two waves make angle $\alpha$ to the horizontal plane.
The reflected and direct waves differ in the "optical path", and the difference is given by,

$$\begin{aligned} OP - PA &= PB - PA \\ &= AB \\ &= 2h\sin\alpha \\ &\sim 2h\alpha \end{aligned}$$

Note that wave reflected at A suffers an additiona l phase change of π or in terms of optical path $\frac{\lambda}{2}$.

The condition for the two waves to reinforce each other is,

$$2h\alpha+\frac{\lambda}{2}=2k\frac{\lambda}{2}$$

$$k = 0,1,2,3...$$

$$\alpha_{MAX}=\frac{2k-1}{2h}\frac{\lambda}{2} \qquad (1)$$

The condition for the two waves to cancel each other is,

$$2h\alpha+\frac{\lambda}{2}=(2k+1)\frac{\lambda}{2}$$

$$k = 0,1,2,3.........$$

$$\alpha_{MIN}=k\frac{\lambda}{2h} \qquad (2)$$

Directions for observing maximum and minimum intensity are defined by $\alpha_{MAX}$ and $\alpha_{MIN}$ respectively. **Ans**

3.2 When $\alpha = 0$ ie. when the star is exactly at the horizon, possible condition is given by (2) with k = 0. Hence with the increase in $\alpha$, the intensity should increase from the minimum intensity. The answer is the intensity increases as the star is rising from the horizon. **Ans**

3.3 Let the component of the amplitude of the direct beam resolved parallel to the horizon be $E_0$,

and the component of the amplitude of the reflected beam resolved parallel to the horiz on be $E_r$

$$\frac{E_0}{E_r}=\frac{n+\sin\alpha}{n-\sin\alpha}$$

$$\sim\frac{n+\alpha}{n-\alpha}$$

$$E_r=\frac{n-\sin\alpha}{n+\sin\alpha}E_0$$

$$E_r \sim E_0$$

In the case of destructive interfering

Resultant amplitude is $= E_0 - E_r$

$$= E_0 - \frac{n - \sin\alpha}{n + \sin\alpha} E_0$$

$$= \frac{2\alpha}{n + \alpha} E_0$$

Substitute $\alpha = k\frac{\lambda}{2h}$ k $= 0,1,2,3,.......$

Resultant amplitude $$= \frac{2k\lambda}{2h(n + \frac{k\lambda}{2h})} E_0$$

$$= \frac{k\lambda}{h(n + \frac{k\lambda}{2h})} E_0$$

Minimum intensity is given by

$$I_{MIN} = [\frac{k\lambda}{h(n + \frac{k\lambda}{2h})}]^2 E_0^2$$

In the case of constructive interfering

Resultant amplitude $= E_0 + E_r$

$$= (\frac{n - \sin\alpha}{n + \sin\alpha}) E_0 + E_0$$

$$= \left(\frac{2n}{n + \alpha}\right) E_0$$

Maximum intensity $$I_{MAX} = \left(\frac{2n}{n + \alpha}\right)^2 E_0^2$$

Substitute $\alpha = \frac{2k - 1}{2}\frac{\lambda}{2h}$, k= 1,2,3,....

$$I_{MAX} = \left(\frac{2n}{n + \frac{\lambda(2k-1)}{4h}}\right)^2 E_0^2$$

Substitute n = 9
$\lambda = 0.21$ m
h = 2m

$$I_{MIN} = \left(\frac{0.105k}{9+0.53}\right)^2 E_0^2$$

$$\alpha_{MIN} = 0.53k$$

$$I_{MAX} = \left(\frac{18}{9+(2k-1)\times.0053}\right)^2 E_0^2$$

$$\alpha_{MAX} = .026(2k-1)$$

Different values of $\alpha_{MIN}$ $\alpha_{MAX}$ $I_{MIN}$ and $I_{MAX}$ calculated for various k's are given in the table.

| k | $a_{MIN}$ | $a_{MAX}$ | $I_{MIN}$ | $I_{MAX}$ |
|---|---|---|---|---|
| 0 | 0 | | 0 | |
| 1 | | .026 rad = 1.5° | | $3.90E_0^2$ |
| 1 | .053 rad = 3° | | $1.36\times10^{-4}E_0^2$ | |
| 2 | | .078 rad = 4.5° | | $3.93E_0^2$ |
| 2 | .13 rad = 6° | | $5.32\times10^{-4}E_0^2$ | |
| 3 | | .130 rad = 7.5° | | $3.89E_0^2$ |
| 3 | .19 rad = 9° | | $1.18\times10^{-4}E_0^2$ | |

# Experiment

## Problem 1

**Investigation of extension of a rubber band in the non-linear part**

Given the following items of equipment and apparatus:

-a piece of cut rubber band with one end suspended from a support and original length $l_0$ = 150 mm.

-a weight pan with mass m = 5 g.

-a set of bobs from 5 to 100 g

-a stop watch

-a ruler and graph paper

The following experiment is to be carried out:

1.1 Meaure the extension ε of the rubber band and the pulling force F provided by bobs from 15 to 95 g placed on the pan which is suspended from the other end of the rubber band.

1.2 Use the experimental results from 1.1 to calculate the volume of the rubber band at various values of pulling force from 35 g to 95 g weight and fill the results in an appropriate table. ( First determine the formula expressing V as a function of F and be aware of the fact that Hoke's law applied to the extension of the rubber band is only approximatly valid. Young's Modulus of the rubber band E =2 × $10^6$ N/m.

1.3 Derive the formula giving volume V of the rubber band as a funtion of time t when a weight of 60 g is suspended from the rubber band. Use the stop watch provided.

## Solution

From Hooke's law

$$\frac{F}{A} = E\frac{\varepsilon}{l_0} \qquad (1)$$

where F is force pulling the rubber band due to suspended bobs and pan.

A cross sectional area of the rubber band

E Young's modulus of the rubber band

$l_0$ initial length of the rubber band

ε extended length of the rubber band

1.1 Place bobs on the pan using the values from 15 g to 105 g.

Measure ε for every value of F.
Record F as a function of ε in the table which contains a column for calculated volume (V) of the rubber band.

**Table of extentsion and applied force and associated volumeof rubber band**

| ε (m) | F (N) | V ($m^3$) |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |

Plot a graph of F against ε .

1.2 The volume V of the rubber band at length l is given by

$$V(l) = A(l).\, l$$

From (1)

$$A(l) = \frac{Fl_0}{E\varepsilon}$$

$$V(l) = A(l) = \frac{Fl_0 l}{E\varepsilon} \qquad (2)$$

Once the values of l and ε are known, V can be calculated from the formula above.

1.3 This is the case of constant F but ε and l vary with time.

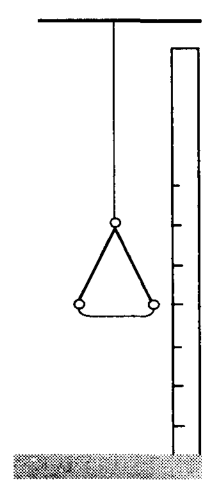

Fig. 12.4

Consider variation of Δε and ΔA under a quasi-static condition.

From (1) $$\Delta F = \frac{E}{l_0}(\Delta A.\varepsilon + A.\Delta\varepsilon)$$

In this case A represent the cross sectional area of the rubber band when the total weight suspended from the rubber band is 60 g at time t = 0 s.
Since $\Delta F = 0$, hence $\Delta A.\varepsilon + A.\Delta\varepsilon = 0$
When 60 g weight is suspended from the rubber band at t =0

$$F - \frac{E.A.\Delta\varepsilon}{l_0} = 0 \qquad (3)$$

The fact that A changes into A+ ΔA and ε into ε +Δε makes the term onthe left hand side of (3) no longer zero. The quantity of the left had side is $m\ddot{\varepsilon}$ due to net force .

$$mg - \frac{E.A.\varepsilon}{l_0} = m\ddot{\varepsilon} \qquad (4)$$

$$mg - \frac{E.(A_0 + \Delta A)(\varepsilon_0 + \Delta\varepsilon)}{l_0} = m\ddot{\varepsilon} \qquad (5)$$

Where $A_0$ and $\varepsilon_0$ are cross- sectional area and extension of the rubber band at t = 0 s.

$$\frac{E.(A_0 + \Delta A)(\varepsilon_0 + \Delta\varepsilon)}{l_0} = \frac{E.(A_0\varepsilon_0 + \Delta A.\varepsilon_0 + A.\Delta\varepsilon + \Delta A.\Delta\varepsilon)}{l_0} \qquad (6)$$

Apply the results from (2) and ignore $\Delta A.\Delta\varepsilon$ to obtain

$$\frac{E.(A_0 + \Delta A)(\varepsilon_0 + \Delta\varepsilon)}{l_0} \sim \frac{E.A_0.\varepsilon_0}{l_0}$$

(4) thus becomes

$$mg - \frac{E.A_0.\varepsilon_0}{l_0} \sim m\ddot{\varepsilon}$$

( $\frac{E.A_0.\varepsilon 0}{l_0}$ may be thought of as the maximum restoring force of the rubber band )

The acceleration $\ddot{\varepsilon}$ of the system of bobs and the pan is thus constant and will be represented by a.

From (4) $$A = m(g-a)\frac{l_0}{E\varepsilon}$$

$$V = A.l$$

$$= \frac{m(g-a)l.l_0}{E.\varepsilon}$$

$$= \frac{m(g-a)(l_0+\varepsilon).l_0}{E.\varepsilon}$$

$$V = \frac{m(g-a)}{E}\left[\frac{l_0}{\varepsilon_0 + \frac{1}{2}at^2} + 1\right]$$ **Ans**

# International Physics Olympiad XIII

# 1982

# Malente, Federal Republic of Germany

## Theory

### Problem 1

An AC voltage source of 50 Hz is connected across a fluorescent lamp and accessories as shown in Fig.13.1.
Given the following data:

| | | | | |
|---|---|---|---|---|
| Voltage (main) | $U$ | = | 228.5 | V |
| Current | $I$ | = | 0.6 | A |
| Partial voltage across the fulorescent lamp | $U_f$ | = | 84 | V |
| Ohmic(DC) resistance of the ballast in series | $R_B$ | = | 26.3 | Ω |

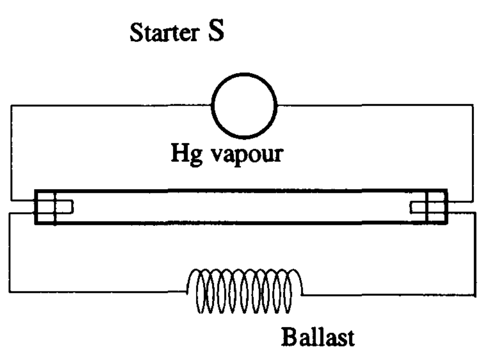


Fig. 13.1

In all calculations of this problem, the fluorescent lamp can be treated as a simple ohmic resistor .

1.1 Determine

1.1.1 Inductance( AC resistance) of the ballast connected in series with the fluorescent lamp.

1.1.2 Phase difference between voltage and current.

1.1.3 Active or useful power consumed and transformed by the fluorescent lamp.

1.2 The ballast has other function besides controlling the current, describe this function and explain the principle behind.
(**NB.** Starter S is a bi-metal contact which connects the circuit as soon as the lamp is switched on and disconnects the circuit a few moments later.)

1.3 Draw a diagram depicting variation of the luminous flux as a function of operating time.

1.4 Why is the lamp ignited only once although the applied AC voltage goes through zero values at ragular intervals?

1.5 According to the maufacturer of the lamp, the operation of this lamp may use an additional capacitor of about 4.7 $\mu$F capacity in series with the ballast. For what purpose does the addition of the capcitor serve? And does the capacitor alter the function of the lamp in any way at all?

1.6 With the spectroscope provided, examine light from both halves of the fluorescent lamp set up for this particular purpose. Explain the difference in the two spectra observed.

# Solution

1.1

1.1.2 Impedance or total AC resistance of the lamp circuit is given by

$$Z = \frac{228.5V}{0.6A} = 280.0 \quad \Omega$$

Ohmic or DC resistance of the fluorescent lamp is given by,

$$R = \frac{84V}{0.6A} = 140 \quad \Omega$$

Total DC resistance of the fluorescent lamp circuit = 140 + 26.3 = 166.3 Ω

From the formula $Z^2 = R^2 + \omega L^2$

Substitution gives $280.8^2 = 166^2 + (2 \times \pi \times 50)^2 L^2$

$$7.88 \times 10^4 = 2.76 \times 10^4 + 9.78 \times 10^4 \times L^2$$

$$L = 0.72 \quad H \quad \textbf{Ans}$$

1.1..2 Let phase of the florescent lamp circuit be given by ,

$$\tan\phi = \frac{\omega L}{R}$$

$$= \frac{2\pi \times 50 \times 0.72}{166.3}$$

$$= 53.6^\circ$$

Current lags behind the voltage by 53.6° or 53° 36′ Ans

1.1.3 Useful power $P = UI\cos\alpha$

$$= 228.5 \times 0.6 \times \cos 53.6^\circ$$

$= 228.5 \times 0.6 \times 0.59$

$= 80$ W **Ans**

1.2 Apart from the function of controlling the current, the ballast is to induce voltage when S is cut off from the circuit. As soon as the starter S is disconnected,voltage induced across the lamp is large enough to cause conduction through the lamp. The function of the ballast is also as an ignitor . The subsidiary role of the ballast is to set phase and hence the value of cos $\alpha$ which determines the value of useful or active power. **Ans**

1.3

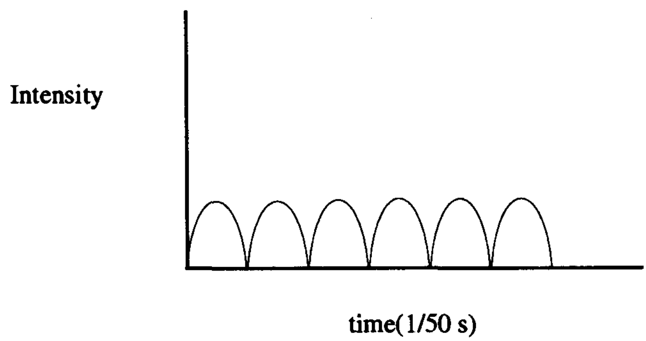


Fig. 13.2 Luminous flux as a function of operating time

1.4 Even though AC takes on 0 value every 1/100 s, ions and electrons still retain their charges. This is made possible by the fact that the recombination time between ions and electrons in the ionized gas mixture is much larger than 1/100 s. Under this situation, the gaseous discharge can conduct electricity at its operating voltagae. Hence there is no need to re-ignite the lamp. **Ans**

1.5 Capacitance or AC resistance of the capacitor

$$= \frac{1}{\omega C}$$

$$= \frac{1}{100\pi \times 4.7 \times 10^{-6}}$$

$$= 6.77 \times 10^{2}$$

Net reactance or resultant AC resistance of ballast and capacitor combined

$$= \omega L - \frac{1}{\omega C}$$

$$= 100\pi \times 7.2 \times 10^{-1} - 6.77 \times 10^{-2}$$

$$= 2.25 \times 10^{2} - 6.77 \times 10^{2}$$

$= \; -4.51\times10^2$

Total AC resistance or impedance is

$$= \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$= \sqrt{166^2 + (4.51\times10^2)^2}$$

$$= \; 4.8\times10^2 \quad \Omega$$

The fluorescent lamp still operates as before, but the active or useful power changes due to phase change ie. $\cos\phi$ decreases with increse in $\phi$.

$$\tan\phi = \frac{(\omega L - \frac{1}{\omega C})}{R}$$

$$= \frac{-4.51\times10^2}{166.3} =$$

$$= \; -69.7^\circ$$

$$\cos\phi = \; 0.34$$

Useful or active power $= \; UI\cos\phi$

$= \; 228.5\times0.6\times0.34$ W

$= \; 46.6$ W **Ans**

The role of the capacitor is to reduce reactance and thus reactive current. Large reactive current is undesirable, as the power of the AC source has to be designed larger than it would be necessary. In such a case, the transfer power loss has to be added and not paid for by the consumers if the useful or active power meter is used. **Ans**

1.6 The fluorescent lamp set up for this purpose has phosphor coating on one section and no phosphor coating on the other. The spectrum obtained from the nonphosphor side is that of mercury vapour consisting of dark blue and purple emission lines(toward ultraviolet region) characteristic of mercury, while the spectrum from the phosphor-coated side is a continuous spectrum in the visble range. This spectrum is due to the absorption of ultra-violet light from the mercury vapour and re-emission of visible light bythe phosphor substance. **Ans**

# Problem 2

A wire coat hanger supported from a certain point can perform oscillations of small ampli tude in the plane which contains that coat hanger. The coat hanger is suspended from three different points as shown in Fig. 13.3 a, b and c. In positions a and b, the longest side is along horizontal diirection, and the turning point is in the axis of symmetry. The periodic

time of the oscillation in all three cases is the same.
Find:

2.1 The position of the centre of mass of the coat hanger.
2.2 The period of oscillation.

(Note Fig 13.3 merely provides information on the dimensions of the coat hanger only and interpretaion beyond this is not intended.)

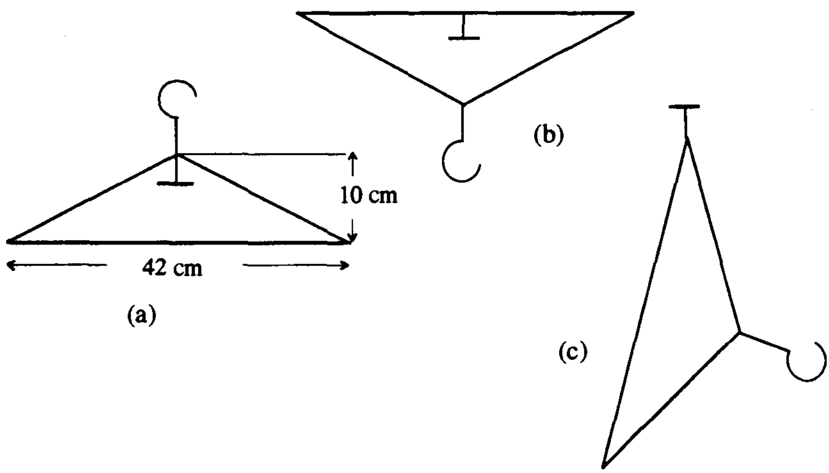


Fig. 13.3

## Solution

Let s be the distance between the suspending point and the centre of mass.

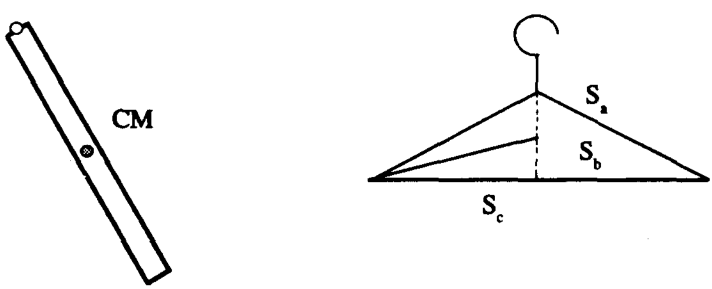


Fig. 13.4

$$\text{torque} = \text{rate of change of angular momentum}$$

$$-smg\sin\theta = \left|\frac{dL}{dt}\right|$$

(The negative sign indicates that the direction of torque is opposite to the direction of the angle, ie. torqe increases with the decrease in the magnitude of the angle)

where θ is the angle between the vector position defining the centre of mass and the vertical.

L angular momentum $= I\ddot{\theta}$

I moment of inertia of the coat hanger about the axis through the suspending point.

$$-smg\sin\theta = I\ddot{\theta}$$

For small values of θ $\sin\theta \approx \theta$

$$I\ddot{\theta} \approx -smg\theta$$

Angular frequency $\omega = \sqrt{\frac{smg}{I}}$

Period $T = 2\pi\sqrt{\frac{I}{mgs}}$ (1)

If $I_0$ is moment of inertia about the axis through the centre of mass

$$I = I_0 + ms^2$$

$$T = 2\pi\sqrt{\frac{I_0 + ms^2}{mgs}}\text{T} \qquad (2)$$

For case a, let $s_a$ be the distance between the suspending point and the centre of mass.
For case b, let $s_b$ be the distance between the suspending point and the centre of mass.
For case c, let $s_c$ be the distance between the suspending point and the centre of mass.

From (2) $mgs\,T^2 = 4\pi^2 I_0 + 4\pi^2 ms^2$

$$4\pi^2 ms^2 - mgT^2 s + 4\pi^2 I_0 = 0 \qquad (3)$$

Equation (3) is quadratic and has two roots.
From the information provided by the problem $T_a = T_b$ and $s_a = s_b$
From the diagram $s_a + s_b = 10$

Distances $s_a$ and $s_b$ give the same period of oscillation,thus it implies ,

$$s_a = s_b = 5 \text{ cm}$$

ie. the centre of mass is at mid-point between the top and the longest side of the coat hanger.

From the geometry $s_c = \sqrt{5^2 + 21^2}$

$= 21.5$ cm

This answer of $s_c = 21.5$ cm can be arrived at from the consideration of the simple geometry of the coat hanger. **Ans**

Fiom (3)

The sum of the two roots $= \frac{T^2}{4\pi^2} g$

$$s_a + s_c = \frac{T^2}{4\pi^2} g$$

$$(21.5 + 5) \times 10^{-2} = \frac{T^2}{4\pi^2} g \times 9.81$$

$$T^2 = \frac{27.6 \times 10^{-2} \times (2\pi)^2}{9.81}$$

$$T = 1.03 \text{ s}$$

The period of the oscillation for all three cases is 1.03 s **Ans 2.2**

# Problem 3

The hot air balloon favoured by most hot air balloonists has an opening at the bottom where the air in the airbag of the balloon is heated. The hot air ballon to be considered here has a constant volume $V_1 = 1.1$ m$^3$. The mass of the envelop is $m_B = 0.187$ kg and its volume is negligible and needs not be brought into calculation.The initial temperature of the air is $T_1 = 20^\circ$ C, and the atmospheric pressure outsid e the balloon $P_0 = 1.013$ bar. Under the atmopheric condition described, the density of the air, $\rho = 1.2$ kg/m$^3$.

3.1 To what temperature must the air in the ballon be heated in order for the balloon just begins to float?

3.2 The balloon is held fast to the ground and the air in the ballon is heated to a steady state temperature 110$^\circ$C. What is the initial acceleration of the balloon when it is released ?

3.3 The balloon is held fast to the ground and the air inside the balloon is heated to a steady state temperature of 110$^\circ$C. and released. If the ballon rises isothermally in the atmosphere at temperature of 20$^\circ$ C, and the atmospheric pressure at the ground $p_0$ is 1.013 bar, determine the height attained by the balloon under the conditions described.

3.4 In consequence of the event in 3.3, at altitude h, the balloon is pulled from its equili brium position by $\Delta h = 10$ m and released. Describe the type of motion assumed by the balloon.

## .Solution

3.1 Analysis of the density of the air inside the balloon when the balloon begins to float off the ground.

Condition for floating (Archimedes' principle)

$$m_1 g = m_2 g + m_B g \qquad (1)$$

where $m_1$ mass of the air having the same volume as the balloon at 20° C.
$m_2$ mass of the air inside the balloon which changes with temperature.
$m_B$ mass of the envelop of the balloon plus of the whole load in the balloon

From

$$m_1 = \rho_1 V_1 \qquad (2)$$

$$m_2 = \rho_2 V_2 \qquad (3)$$

where $\rho_1$ and $\rho_2$ are the values of the density at 20° C and T °C respectively.

and $V_1 = V_2 =$ volume of the air bag

Substitute $m_1$ and $m_2$ in (1) to obtain

$$\rho_2 V_1 + m_B g = \rho_1 V_1$$

$$\rho_2 = \rho_1 - m_B V_1$$

Substitute $\rho_1 = 1.2 \text{ kg/m}^3$
$m_B = 0.187$ kg

$$\rho_2 = 1.2 - \frac{0.187}{1.1}$$

$$= 1.03 \quad \text{kg/m}^3$$

Calculation of the temperature of the air inside the balloon which causes the balloon to have the density of 1.03 kg/m³ and begin to rise.

From the equation $P_1 V_1 = nRT_1$

where $P_1$ is the pressure inside the balloon which is equal to the atmospheric pressure
$P_0$ = 1.103 bar
$V_1$ volume of the air balloon = 1.1 m³

n the number of mole of the air inside the balloon = $\frac{m_2}{M}$

M molcular mass ot the air
$m_2$ masss of the air inside tha balloon at tmeperature $T_1$
R gas constant
$T_1$ temperature of the air inside the balloon

Hence $P_1 = P_0 = \frac{m_2}{V_1}\frac{RT_1}{M} = \frac{\rho_1 RT_1}{M}$ (4)

Likewise

$$P_2 = P_0 = \frac{m_2}{V_2}\frac{RT_2}{M} = \frac{\rho_2 RT_2}{M} \quad (5)$$

where $m_2$ is mass of the air at $T_2$.

(4) = (5) $\rho_1 T_1 = \rho_2 T_2$

Substitution gives

$$1.2 \times (273+20) = 1.03 \ T2$$
$$= 341.3 \quad K$$

The air inside the balloon must be heated to reach the temperature of 341.3 K or 68.3° C before it begins to float. **Ans**

3.2 consider the density of the air inside tha balloon at temperature 110° C .

$$P_3 = P_0 = \frac{m_1}{V_1}\frac{RT_3}{M} = \frac{\rho_3 RT_3}{M} \quad (6)$$

where $\rho_3$ is density of the air inside the balloon at $T_3$

(4) = (6) $\rho_1 T_1 = \rho_3 T_3$ (7)

Substitution gives

$$1.2 \times (273+20) = \rho_2 \times 383$$
$$= 1.2 \times \frac{293}{383}$$
$$= 0.198 \quad kg/m^3$$

The net force acting on the balloon, when the air inside the balloon is 110° C or 383 K, is:

$$F = m_1 g - m_2 g - m_B g$$
$$= gV_1 \rho_1 - gV_1 \rho_3 - m_B g$$
$$= gV_1 (\rho_1 - \rho_3) - m_B g$$
$$= 1.1 \times 9.81 \times (1.2\text{-}0.918) - 0.187 \times 9.81$$
$$= 3.04\text{-}1.83$$
$$= 1.21 \quad N$$

Initial acceleration of the air balloon $a = \frac{F}{m_B}$

$$= \frac{1.21}{0.187}$$
$$= 6.47 \quad m/s^2$$ **Ans**

3.3 If the balloon is allowed to float freely under the condition in 1.2 and the temperature of the air outside the balloon remains constant at 20° C, let the atmospheric layer reached by the balloon has density given by $\rho_a$ (when the bouyancy of the air is equal to the weight of the balloon.)

At the asmospheric density of $\rho_a$,

$$m_2 g + m_B g = \rho_a V_1$$

$$\rho_3 V_1 + m_B = \rho_a V_1$$

$$\rho_a = \rho_3 + \frac{m_B}{V_1}$$

$$= 0.918 + \frac{0.187}{1.1}$$

$$= 1.088 \quad kg/m^3$$

From the knowledge that the number of molecules per unit volume varies with altitude measured from the ground, we write;

$$n = n_0 e^{-\frac{m_0 gh}{kT}} \tag{8}$$

where $n$ number of molecules per unit volume at altitude h
$n_0$ number of molecules per unit volume at the ground level
$m_0$ mass of an air molecule
$k$ Boltzman's constant

Write (7) in the form $n = n_0 e^{-\frac{Nm_0 gh}{NkT}}$

where N is Avogadro's number

hence $n = n_0 e^{-\frac{m_a gh}{NkT}}$

where $m_a$ mass of 1 mol of air

$$n = n_0 e^{-\frac{m_a gh}{PV}}$$

Substitute the values of P and V at the ground level

$$P = P_0 \qquad V = V_1 \text{ and } \qquad \frac{m_a}{V} = \rho_1$$

to get $n = n_0 e^{-\frac{\rho_1 gh}{P_0}}$

or $\rho_a = \rho_0 e^{-\frac{\rho_1 gh}{P_0}}$

where $\rho_a$ and $\rho_0$ are values of the density of the air at a latitude h and ground level respectively.

$$-\frac{\rho_0 g h}{P_0} = \ln\frac{\rho_a}{\rho_0}$$

$$h = -\frac{P_0}{\rho_1 g}\ln\frac{\rho_a}{\rho_0}$$

Substitute $P_0 = 1.013\times10^5$ N

$\rho_a = 1.088$ Kg/m$^3$

$\rho_0 = \rho_1 = 1.2$ kg/m$^3$

to obtain

$$h = -\frac{1.03\times10^5}{1.2\times9.81}\ln\frac{1.088}{1.2}$$

$$= 8.6\times10^5\times9.78\times10^{-2} \text{ m}$$

$$= 8.43\times10^2 \text{ m}$$

The balloon will reach altitude of 843 m **Ans**

3.4 If the balloon is pulled out of its equilibrium position by $\Delta h$ =10 m , it will undergo simple harmonic motion dampened by air resistance **Ans**

# Experiment

## Problem 1

The following apparatus and equipment items are at disposal of the contesrants:

- Double convex lens of equal radius of curvature
- Plane mirror
- water
- meter stick
- stand and clamps
- pencils
- etc.

Find:

1.1 focal length of the convex lens.(with error of not more than 1 %)
1.2 refractive index of the glass of which the lens is made.

Given : the refractive index of water $n_w$ is 1.33
The formula of the folcal length of thin lens is

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where n is the refractive index of the material of the lens
$r_1$ and $r_2$ radii of curvature of the two surfaces.
(Using the convention that distance measured to the right of the lens is positive, and to the left negative.)

## Solution

1.1 Experimental procedures can be undertaken in the following order:

- Determine the rough value of the focal length by focusing distant objects such as trees walls etc.
- Place the convex lens over the plane mirror at the base of the stand.
- With the clamp, hold the pencil in the position directly above the lens at the distance of the rough value of the focal length determined earlier. Adjust the position of the tip of the pencil until no paralax is observed between the pencil and its image form by the system of the plane mirror and the convex lens. (See Fig.13.5)
- Measure the disntance between the tip of the pencil to the centre of the convex lens to obtain the value of the focal length.

From the formula $$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad (1)$$

$r_1$ radius of curvature of the upper surface of the lens(pointing downward hence negative sign)

$r_1 = -|r| = -r$

$r_2$ radius of curvature of the lower surface of the lens(pointing upward hence positive sign)

$r_2 = r$

n refractive index of the lens = $n_g$

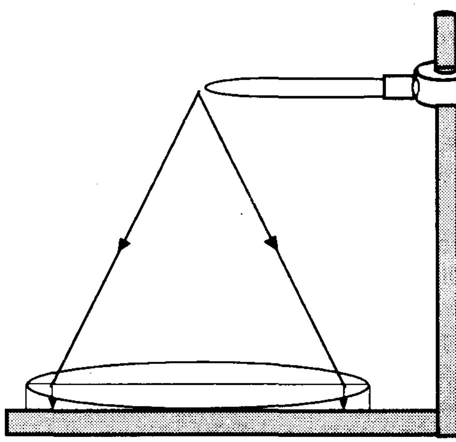

Fig.13.5

The focal length of the convex lens given carries negative sign and is on the opposite side of the object.

In order to find $n_g$, in addition to knowing f, we must also know r.
In order to find r, we build up a composite lens from the given convex lens and water lens as follows:

- Fill in the gap between the lens and the plane mirror with water to obtain a water lens
- Carry out the experiment in the same way as above to determine the focal ength of the composite lens.

Let F be the focal length of the composite lens
$f_1$ focal length of the convex lens provided.
$f_2$ focal length of the concave water lens.

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2} \tag{2}$$

$$\frac{1}{f_2}=\left(n_g-1\right)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

In this case $r_1$ is the radius of curvature of the first surface of the convex lens = r

$r_2$ is the radius of curvature of the second surface of the concave water lens = ∞

$$\frac{1}{f_2}=(n_w-1)(\frac{1}{r}) \tag{3}$$

The value of $f_2$ is calculated from (2) and r from (3)
The value of $n_g$ is calculated from(1).

## Problem 2

The rotational motion of a cylinder to be considered in this problem also involves linear motion of the centre of gravity of the cylinder along the horizontal direction. In this experiment, only the values associated with linear motion of the centre of gravity of the cylinder and force causing such motion (change of motion) are directly measured.

The cylinder provided has mass M, radius R resting on a horizontal board. At distance $r_i$ ( i = 1, 2, 3,.. ....6 ) measured from the axis of the cylinder, tension T in the light string wrapped around the cylinder acts on the cylinder. (See Fig. 13.6 below) When mass m suspended from the other end of the string is released, the cylinder begins to rotate and at the sme time move along the horizontal direction with constant acceleration.

2.1 Determine linear acceleration $a_i$ ( i = 1,2,3,...6)

2.2 From $a_i$ ( i = 1, 2, 3,...6) determined in 2.1, calculate frictional force $F_i$ acting in the horizontal direction between the cylinder and the horizontal surface.

2.3 Plot a graph of $F_i$ as a function of $r_i$ and comment on the results on the basis of mechanical principles.

2.4 In this experiment, the board is adjusted so that the board is in the horizontal direction before the experiment can begin. If the board is not in the horizontal position, what will happen?

2.5 Describe how other auxiliary parameters of motion are determined and what effect will it has on the motion if these parameters are not properly adjusted?

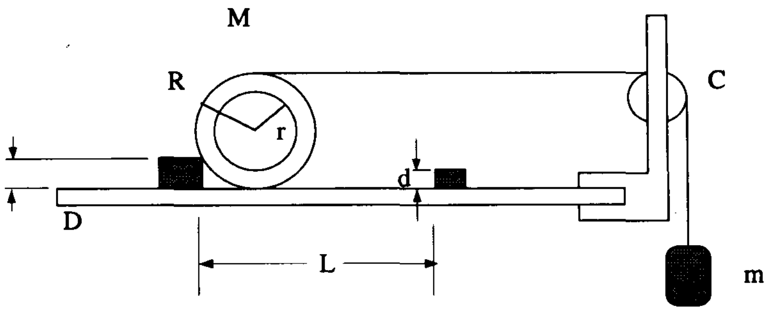


Fig. 13.6

Given:

| | |
|---|---|
| R = 5.00 cm | $r_1$ = 0.75 cm |
| M = 3.275 kg | $r_2$ = 1.50 cm |
| m = 250 g | $r_3$ = 2.25 cm |
| D = 150 cm | $r_4$ = 3.00 cm |
| d = 0.1 m | $r_5$ = 3.75 cm |
| | $r_6$ = 4.50 cm |

NB Mass of pulley C and friction are negligible and need not be introduced into the calculation. The measurement of distance is to be made using the measuring tape provided, likewise the measurement of time is to be made using the timer provided for this particular purpose.

## Solution

Adjust the board so that it lies in the horizontal position, and the string pulling the cylinder so that it is parallel to the board for every value of $r_i$. Without the spirit level, adjustment must be made based on sighting. If the string is not in the horizontal position and makes an angle α with the horizontal, the pulling force in the string is no longer mg, and there will be an additional force mgcosα pushing the cylinder downward along the vertical direction.

Referring to Fig 13.6 above, distance s along the horizontal direction is

$$s = L - \sqrt{R^2-(R-D)^2} - \sqrt{R^2-(R-d)^2}$$
$$= L - -\sqrt{2RD-D^2} - \sqrt{2RD-d^2}$$

L,D and d are known from the experiment, hance s can be determined from the equation above.

For each $r_i$ ( the string in the horizontal direction), measure the time for the cylinder to cover distance s.

In each case, calculate acceleration from the formula,

$$s = \frac{1}{2}at^2$$

Record the results in the table below:

| r(cm) | t (s) | | | t (averaged) |
|---|---|---|---|---|
| | First trial | Second trial | Third trial | |
| 0.75 | | | | |
| 1.5 | | | | |
| 2.25 | | | | |
| 3 | | | | |
| 3.75 | | | | |
| 4.5 | | | | |

Analysis of the relationship between rotational and linear motions of the cylinder:

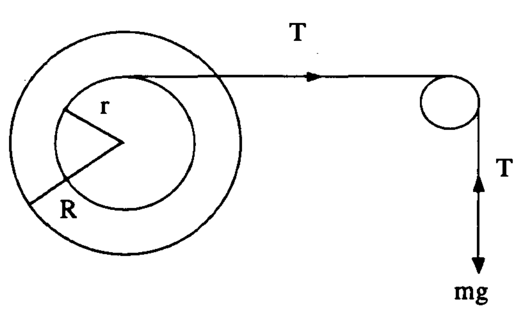


Fig. 13.7

Let $a_m$ be linear acceleration of mass m
$a_M$ be linar acceleration of mass M

Equation of motion of mass m is $ma_m = mg - T$ (1)
Equation of motion of mass M is $Ma_M = T - F$ (2)

$$I\ddot{\theta} = Tr + FR \qquad (3)$$

where $\ddot{\theta}$ is angular or rotational acceleration
I moment of inertia of the cylinder
F frictional force which the board exerts on the cylinder

$$R^2\ddot{\theta} = a_M \qquad (4)$$

(1)+(2) $ma_m + Ma_M = mg - F$

$$F = mg - ma_m - Ma_M \qquad (5)$$

$$T = mg - ma_m \qquad (6)$$

From the principle of the conservation of energy

$$\frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}MV_M^2 + \frac{1}{2}mv_m^2 = mgs$$

where s is the distance of falling of m mass,
$V_M$ velocity of mass M
$v_m$ velocity of mass m.

Substitute s from the relation $v^2 = 2as$ in the above formula to obtain

$$\frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}MV_M^2 + \frac{1}{2}mv_m^2 = mg\frac{v_m^2}{2a_m} \qquad (7)$$

Substitute $I = \frac{1}{2}MR^2$ and $\dot{\theta} = \frac{V_M}{R}$ in (7)

$$\frac{1}{4}MV_M^2 + \frac{1}{2}MV_M^2 + \frac{1}{2}mv_m^2 = mg\frac{v_m^2}{2a_m}$$

$$3MV_M^2 + 2mv_m^2 = mg\frac{v_m^2}{2a_m}$$

$$3MV_M^2 = 2m(g-a)\frac{v_m^2}{2a_m}$$

Note that M and m begin motion from rest, hence their initial velocity is 0, and

$V_M = \frac{1}{2}a_M t^2$ , and $v_m = \frac{1}{2}a_m t^2$

$$3MV_M^2 = 2m(g - a_m)\frac{v_m^2}{2a_m} \qquad (8)$$

From (3) $$\frac{1}{2}MRa_M = Tr + FR$$

$$\frac{1}{2}MRa_M = mr(g - a_m)a_m(r + R)$$

$$\frac{3}{2}MRa_M = m(a_M - a_m)a_m(r + R) \qquad (9)$$

Substitute( g - $a_m$)$a_m$ from (8) into (9) to obtain

$$\frac{3}{2}MRa_M = \frac{3}{2}\frac{M}{m}a_m(r + R)$$

$$a_M = \frac{(r + R)}{R}a_m \qquad (10)$$

This relationship can be directly obtained from the diagram without going through lengthy calculation.

Since $a_M$ is obtained or known from the experiment $a_m$ can be calculated from Equation (10). **Ans**

Substitute $a_m$ from (10) into (5) to get

$$F = mg\left[M + m\left(1 + \frac{r}{R}\right)\right]a_M$$

For each pair of $r_i$ and $a_M$, F is calculated and recorded in the table provided

From the values of F and $r_i$ in the table , a graph of F is plotted against $r_i$.

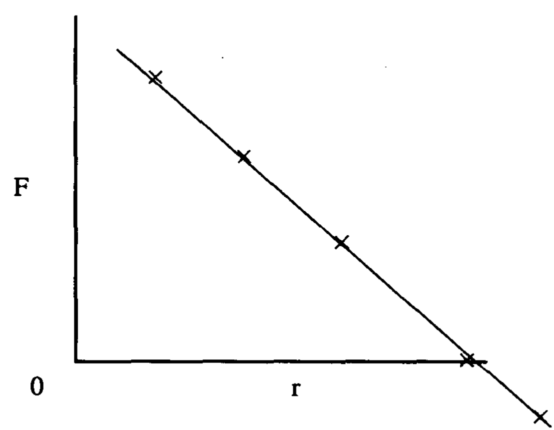


Fig. 13.8 A graph of frictional force as a function of radius r

Note that **F** changes sign from positiveto negative as r increases. Negative **F** tells us that frictional force serves as a pulling force in the direction of motion as r increases. **Ans**

# International Physics Olympiad XIV

# 1988
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## Theory

### Problem 1

A particle moves along the positive x axis, OX as shown in Fig. 14.1 a. One of the forces acting on the particle is f(x) as shown in Fig.14.1b. The wall at x = 0 is a perfect reflecting surface

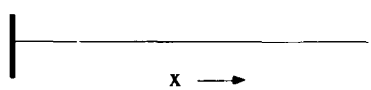
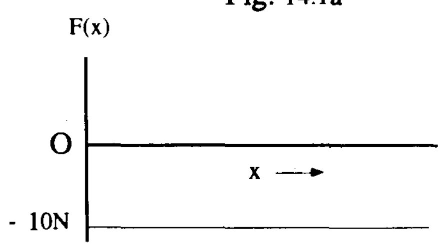



Fig. 14.1a




$f(x) = -10\,N$ when $0 \le x$

Fig 14.1b

During the motion of the particle, friction force F = 1 N acts on the particle in the direction opposite to the direction of motion. If the particle starts from point $x = x_0$ with initial kintetic energy $E_0 = 10.0$ J

1.1 Find the expression for the distance travelled or traced by the particle before coming to rest.

1.2 Sketch the graph of potential energy of the particle U(x) in the field of force f(x).

1.3 Sketch a graph of the velocity of the particle as a function of x.

# Solution

1.1 From the graph, force acting on the particle has the magnitude of 10 N having direction along positive x axis.
In the case of the particle moving in the direction of positive x, friction is negative or directed along the direction of negative x( i.e.direction of decreasing x.)
In the case of the particle is moving in the direction of negative x, friction is positive or directed along the direction of positive x( i.e. direction of increasing x.)
The general description of the motion of the particle is, $|v|$ the magnitude of the particle velocity keeps decreasing while undergoing reflection at the wall until the velocity becomes zero.

The particle comes to a full stop at the wall ie. at x = 0.

From the problem $$f(x) = -\frac{dU(x)}{dx} = -10$$

$$U(x) = 10x + c$$

Choose $U(x) = 0$ when $x = 0$

Hence $U(x) = 10x$

Energy lost in the form of heat = Initial potential energy + Initial kinetic energy

$$\mathbf{F.x} = 10\,x_0 + 10$$

If s is distance traced by the particle until it comes to a complete rest

$$Fs = 10x_0 + 10$$

Substitute F = 1 N $s = 10x_0 + 10$ **Ans**

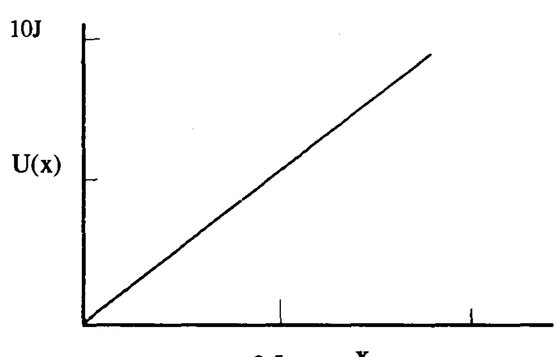



Fig.14.2

1.3 Acceleration $\mathbf{a} = \frac{1}{m}\left[\mathbf{F} - \frac{\mathbf{v}}{|\mathbf{v}|}\right]$

The sign - in front of $\frac{\mathbf{v}}{|\mathbf{v}|}$ indicates that friction always in the opposite direction to that of v

$$\mathbf{a} = \frac{1}{m}\left[-10\mathbf{1}_X - \frac{\mathbf{v}}{|\mathbf{v}|}\right]$$

From the formula $v^2 = u^2 + 2a\,x$

where v is velocity at time t
u is initial velocity
x is distance of the particle measured from 0.

and $\frac{1}{2}mu^2 = 10J$

$(2)\times\frac{m}{2}$ $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + a\,mx$

$$\frac{1}{2}mv^2 = 10 + [-10 - \frac{\mathbf{v}.\mathbf{1}_x}{|\mathbf{v}|}]\,x$$

If v is positive, then $\frac{\mathbf{v}.\mathbf{1}_x}{|\mathbf{v}|} = 1$ and $v^2 = \frac{1}{m}[20 - 11x]$ (1)

If v is negative, then $\frac{\mathbf{v}.\mathbf{1}_x}{|\mathbf{v}|} = -1$ and $v^2 = \frac{1}{m}[20 - 9x]$ (2)

Sketches of v as a function of x are shown in Fig 14,4
Note When the particle moves away from O, the trajectory of v is a parabola defined by (3)

$$v^2 = \frac{1}{m}(20 - 11x)$$

while the particle moves towards O, the trajectory of v is defined by (4)

$$v^2 = \frac{1}{m}(20 - 9x.)$$

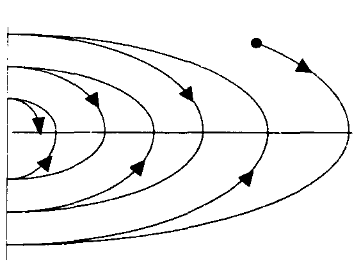


Fig. 14.3 Particle begins by moving away from O

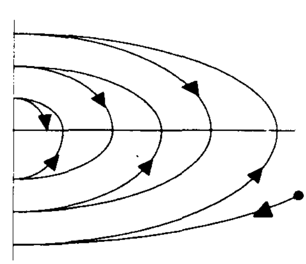


Fig. 14.4 Particle begins by moving toward O.

# Problem 2

In the circuit shown in Fig. 14.5

| | | | |
|---|---|---|---|
| $L_1$ = 10 mH | | $L_2$ = 20 mH | |
| $C_1$ = 10 F | | $C_2$ = 5 F | |
| R = 100 Ω | | | |

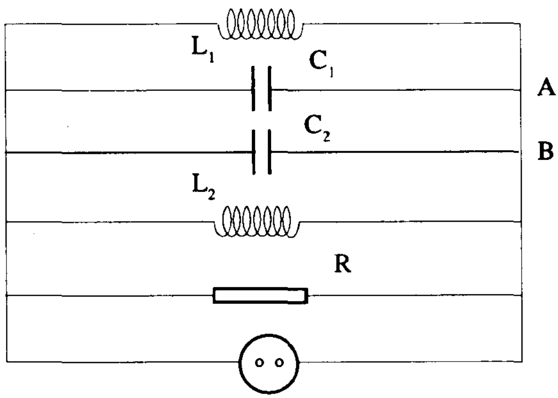



Fig.14.5

Switch K is closed for a sufficienlty long interval. The frequency of the sinusoidal wave from the source may be varied, but the amplitude is always kept constant.

2.1 Given:

the frequency pertaining to the maximum effective power ($P_m$) be $f_m$

and the frequency associated with $\frac{P_m}{2}$ be $f_+$ and f respectively

Determine ratio $\frac{f_m}{\Delta f}$, where $\Delta f = f_+ - f$

2.2 Switch K is opened. At time $t_0$ after the switch is opened,the current through $L_1$ and $L_2$ are $i_{01}$ = 0.1 A and $i_{02}$ = 0.2 A, respectively and voltage U = 40 V.

2.2.1 Calculate f, the natural frequency of the circuit (oscillating circuit) that consists of $L_1$ $C_1$ $C_2$ and $L_2$.

2.2.2 Determine the current in conductor AB.

2.2.3 Calculate the amplitude of the oscillation of the current in coil $L_1$.

# Solution

2.1 From the figure, $L_1$ $C_1C_2$ $L_2$ and R are connected in parallel.
Let $\omega L$ be resultant AC resistance due to inductance of the coils,

$\frac{1}{\omega C}$ be resultant AC resistance due to capacitance of the capacitors

$$\frac{1}{\omega L} = \frac{1}{\omega L_1} + \frac{1}{\omega L_2}$$

$$C = C_1 + C_2$$

Let Z be total resistance or impedance

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

( where $j = \sqrt{-1}$ )

$$(\frac{1}{Z})^2 = (\frac{1}{R})^2 + (\omega C - \frac{1}{\omega L})^2$$

Effective or useful power through load R is

$$P = I_R^2 R$$

where I is the current through R and $I_R = \frac{U}{R}$ where U is voltage across R.

Hence $$P = \frac{U^2}{R}$$

$$P = \frac{I_T^2 Z^2}{R}$$ where $I_T$ is net current.

$$P = \frac{I^2}{R[\frac{1}{R^2} + (\omega C - \frac{1}{\omega L})^2]}$$

Maximum useful power $P_m$ occurs when

$$(\omega C - \frac{1}{\omega L}) = 0$$

$$\omega^2 = \frac{1}{LC}$$

$$f_m = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

One half of maximum useful power occurs when

$$(\frac{1}{R})^2 = (\omega C - \frac{1}{\omega L})^2$$

thus $$\frac{1}{R} = (\omega_+ C - \frac{1}{\omega_+ L})$$

and $$-\frac{1}{R} = (\omega_- C - \frac{1}{\omega_- L})$$

It can be shown by straight- forward algebra that

$$\omega_+ - \omega_- = \frac{1}{RC}$$

$$\Delta f = \frac{1}{2\pi}(\omega_+ - \omega_-)$$

$$\frac{f}{\Delta f} = \frac{1}{2\pi\sqrt{LC}} 2\pi RC$$

$$= R\sqrt{\frac{C}{L}}$$

$$= 150$$

2.2 When the switch is opened;

$L_1C_1$ = 10 mH × 10 F = 100 $s^{-2}$

$L_2C_2$ = 20 mH × 5 F = 100 $s^{-2}$

$L_1C_1$ = $L_2C_2$

Oscillators $L_1C_1$ and $L_2C_2$ oscillate with the same frequency i.e.

$$= \frac{1}{2\pi\sqrt{L_1C_1}} = \frac{1}{2\pi\sqrt{L_2C_2}} = \frac{1}{2\pi\sqrt{100\times10^{-2}}} = 15.9 \text{ kHz}$$ **Ans**

2.2.1 Since the oscillating circuits $L_1C_1$ and $L_2C_2$ oscillate independent of each other, there is no AC signal in AB.

Let $I_{C1}$ be current from $C_1$ into AB

$I_{C2}$ be current from $C_2$ into AB

Consider $C_1$ $\quad Q = C_1V$

$$\frac{dQ}{dt} = C_1\frac{dV}{dt}$$

$$I_{C1} = C_1\frac{dV}{dt}$$

Consider $C_2$ $\quad I_{C2} = C_2\frac{dV}{dt}$

$$\frac{I_{C1}}{I_{C2}} = \frac{C_1}{C_2} = 2$$

$$IC_1 = 2IC_2$$

Apply Kirchoff's law at junctions A and B,

$$I_{AB} = I_{01} + I_{C1} \quad (3)$$

$$I_{AB} = -I_{02} - I_{C2} \quad (4)$$

(4)×2 $$2I_{AB} = -2I_{02} - 2I_{C2} \quad (5)$$

(5)+ (3) $$3I_{A} = -2I_{02} + I_{01}$$

$$= -0.3 \quad A$$

$$I_{AB} = -0.1 \quad A$$

2.2.2 In circuit $L_1 C_1$ , let AC curent due to oscillation be $I_{01}'$

$$I_{01}' = I_{01} - I_{AB}$$

$$= 0.2 \quad A$$

From the principle of the conservation of energy,

$$\frac{1}{2} L_1 (I'_{01(MAX)})^2 = \frac{1}{2} L_1 (I'_{01(MAX)})^2 + \frac{1}{2} C_1 V_1^2$$

$$I'_{01(MAX)} = \sqrt{(I'_{01})^2 + (\frac{C_1}{L_1})^2 V_1^2}$$

Substitute $V_1 = 40$ V

$$I'_{01(MAX)} = 0.204 \quad A$$ **Ans**

# Problem 3

Two prisms having angles $A_1 = 60°$ and $A_2 = 30°$ respectively, are put together as shown in Fig. 14.6 to form a composite prism ABCD with angle $\angle BCD = 90°$

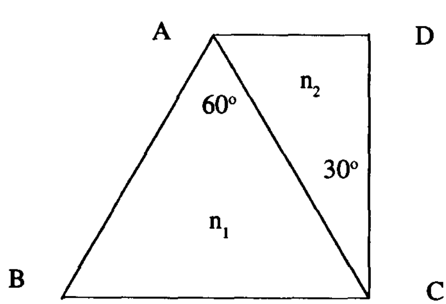



Fig.14.6

The refractive indices of the two prisms are given as a function of $\lambda$ by the following formulas:

$$n_1 = a_1 + \frac{b_1}{\lambda_1^2} \qquad n_2 = a_2 + \frac{b_2}{\lambda_2^2}$$

$$a_1 = 1.1 \qquad a_2 = 1.3$$

$$b_1 = 10^5 \text{ nm}^2 \qquad b_2 = 5\times10^4 \text{ nm}^2$$

3.1 Determine wavelength $\lambda_0$ such that light beam coming f rom any direction passes through AB to and out of AC without undergoing reflection at any surface.

3.2 Draw the paths of three different beams of wavelengths $\lambda_{RED}$, $\lambda_0$, and $\lambda_{BLUE}$ having the same angle of incidence at the side AB.

3.3 Determine the angle of minimum deviation for the composite prism.

3.4 Determine the wavelength of an incident beamwhich gives the refraction beam inside the composite prism parallel to surface BC and emerges from the composite prism in the direction also parallel to BC.

## Solution

3.1 The conditon for the light beam of wavelength $\lambda_0$ incident on AB at any angle of incidence will not be relflected at surface AC is,

$$n_1 = n_2$$

$$a_1 + \frac{b_1}{\lambda_0^2} = a_2 + \frac{b_2}{\lambda_0^2}$$

$$\lambda_0 = \sqrt{\frac{(b_1 - b_2)}{(a_1 - a_2)}}$$

$$= \sqrt{\frac{0.5\times10^5}{0.2}}$$

$$= 500 \text{ nm} \qquad \textbf{Ans}$$

Also in this case $n_1 = n_2 = 1.5$

3.2 For a red light having 500 nm wavelength, its refractive indices both $n_1$ or $n_2$ are less than 1.5.
Likewise, for a blue light having wavelength shorter than 600 nm, its refractive indices are greater than 1.5.

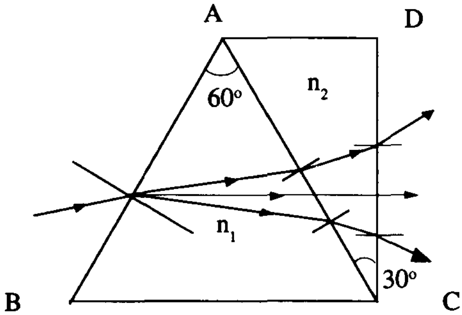



Fig.14.7

We investigate the nature of the variation of the refractive index as a function of $\Delta\lambda$ at $\lambda_o$

$$n_1 = a_1 + \frac{b_1}{\lambda_1^2} \qquad n_2 = a_2 + \frac{b_2}{\lambda_2^2}$$

$$\Delta n_1 = -\frac{2b_1\Delta\lambda}{\lambda^3} \qquad n_2 = -\frac{2b_2\Delta\lambda}{\lambda^3}$$

For red light $\lambda_{RED} > \lambda_0$ , $\Delta\lambda$ is positive

For blue light $\lambda_{BLUE} < \lambda_0$ , $\Delta\lambda$ is negative.

This is in consistence our expectation.

This means for red light the deviation becomes less in the first prism , but increses in the second prism;while for the blue light, the deviation increases in the first prism but decreases in the second prism.

3.3 For light of wavelength $\lambda_0$ , both prisms can be considered as one uniform prism.

Minimum deviation occurs when the incident and emerging beams form a folding symmetry about the line dividing the vertex if the prism is an isoceles, and also the beam inside the prism is parallel to the base of the isoceles triangle.

In Fig14.8 , lines BA and CD are projected to intersect at 0 forming the prism angle. From the law of refraction,

$$\sin i = {}_a\mu_g \sin r$$

where

- ${}_a\mu_g$ is the refractive index relative to air (light travels from air to the prrims)
- i incident angle in the air
- r refracting angle in the prism

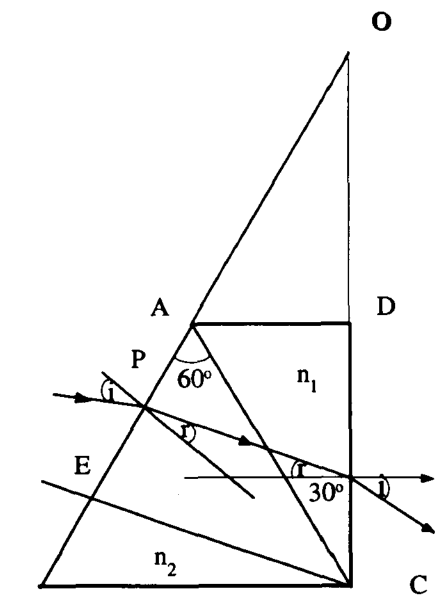



Fig.14.8

From the geometry $\angle AOD = 90° - 60° = 30°$

$\angle APQ = (180° - 30°)/2 = 75°$

Refracting angle $r = 90° - 75° = 15°$

$\sin i = 1.5 \sin 15°$

$i \approx 22° 50'$

$i - r = 6° 10'$

Angle of minimum deviation $=( 6° 10')/2 = 12° 20'$ **Ans**

3.4 From the geometry,the incident beam in the direction i = 30° parallel to BC has the angle of incidence equal to 30° and the beam refracting out of the prism on the side DC is normal to DC ( refracting angle in the air = 90° )

Apply the law of refraction at side AB

$$\sin 30° = {}_a\mu_{g1} \sin r \qquad (1)$$

at AC
$$ {}_a\mu_{g1} \sin(60° - r) = {}_a\mu_{g2} \sin 30° \qquad (2)$$

where ${}_a\mu_{g1}$ is refractive index of the first prism $= n_1$

${}_a\mu_{g2}$ is refractive index of thesecond prism $= n_2$

From (1)
$$\sin r = \frac{1}{2n_1} \qquad (3)$$

From (2)
$$2[\sin 60° \cos r - \cos 60° \sin r] = \frac{n_2}{n_1} \qquad (4)$$

Substitute sin r from (3) in (4)

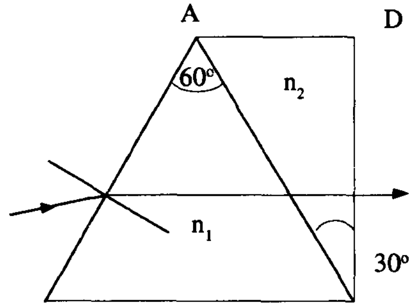



Fig.14.9

$$2[\frac{\sqrt{3}}{2}\sqrt{1-\sin^2 r} - \frac{1}{4n_1}] = \frac{n_2}{n_1}$$

$$[\frac{\sqrt{3}}{2}\sqrt{1-\sin^2 r} - \frac{1}{4n_1}] = \frac{n_2}{n_1}$$

$$\sqrt{3\left[1-\left(\frac{1}{2n_1}\right)^2\right]} - \frac{1}{2n_1} = \frac{n_2}{n_1}$$

$$3[1 - \frac{1}{4n_1^2}] = (\frac{n_2}{n_1} + \frac{1}{2n_1})^2$$

$$12n_1^2 - 3 = (2n_2 + 1)^2$$

$$n_2^2 + n^2 + (1 - 3n_1^2) = 0$$

Substitute $n_1 = 1.1 + \frac{10^5}{\lambda^2}$ $\qquad n_1^2 = 1.21 + 2.2 \times \frac{10^5}{\lambda^2} + \frac{10^4}{\lambda^4}$

$n^2 = 1.3 + \frac{0.5 \times 10^5}{\lambda^2}$ $\qquad n_2^2 = 1.69 + 1.3 \times \frac{10^5}{\lambda^2} + \frac{0.25 \times 10^4}{\lambda^4}$

To obtain $1.69 + 1.3 \times \frac{10^5}{\lambda^2} + \frac{0.25 \times 10^4}{\lambda^4} + 1.3 + \frac{0.5 \times 10^5}{\lambda^2} + 1 - 3(1.1 + \frac{10^5}{\lambda^2}) = 0$

$$2.75 \times \frac{10^{10}}{\lambda^4} + 4.8 \times \frac{10^5}{\lambda^2} - 0.36 = 0$$

$$\frac{1}{\lambda^2} = \frac{-4.8 \times 10^5 \pm \sqrt{23 \times 10^{10} + 11 \times 3.6 \times 10^5}}{5.5 \times 10^{10}}$$

$$= \frac{-4.8 \times 10^5 \pm \sqrt{26.69 \times 10^{10}}}{5.5 \times 10^{10}}$$

$$= \frac{-4.8 \times 10^5 \pm 5{,}19 \times 10^5}{5.5 \times 10^{10}}$$

$$\lambda^2 = \frac{5.5 \times 10^{10}}{3.9}$$

$$= 1.18 \times 10^3 \text{ nm}$$

$$= 1.18 \ \mu\text{m}$$ **Ans**

# Problem 4

A photon having wavelength $\lambda_0$ collides with a free electron in motion causing the elec tron to be at rest after the collision. The wavelength of the photon changes to $\lambda_0'$, and the photon continues to travel in the direction making an angle of $60^\circ$ with the initial direction of motion. Thereafter the photon of wavelength $\lambda_0'$ collides with another free electron at rest causing the photon's wavelength to change from $\lambda_0'$ to $\lambda_0'' = 1.25 \times 10^{-10}$ m ,while the direction of photon changes by $60^\circ$ from its direction before the second collision.

Determine De Broglie's wavelength of the first electron before the first collision.

Given:

Planck' s constant $h = 6.6 \times 10^{-34}$ .J.s
Electron mass $m = 9.1 \times 10^{-31}$ kg
Magnitude of the velocity of light $c = 3.0 \times 10^8$ m/s

# Solution

## First Method

The collision between a photon and a moving electron that results in the eletron being brought to rest is possible only when the electron and the photon move toward each other, but not necessary along the same line of motion.

In Fig. 14.10, the electron moves in the direction making angle $\phi$ with the initial direction of photon.

After the collision, the photon travels in the direction making an angle of $\theta = 60°$ with the initial direction of photon.

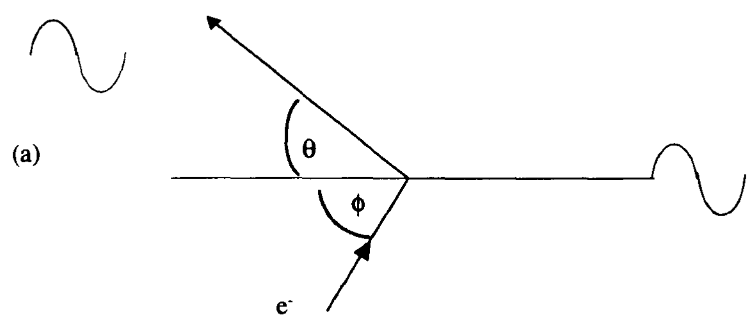



Fig.14.10

From the principle of the conservation of energy,

$$E + h\nu_0 = E_0 + h\nu_0' \tag{1}$$

where E is total energy = $mc^2$ of the moving electron before the collision
m relativistic mass
$\nu_0$ freqency of photon corresponding to wavelength $\lambda_0$
$\nu_0'$ freqency of photon after the collision corresponding to wavelength $\lambda_0'$
$E_0$ total eneregy = $m_0c^2$ of an electron at rest

From the principle of conservation of momentum applied:
Along the direction of motion,

$$\frac{h}{\lambda_0}\cos\theta + |\mathbf{p}|\cos\phi = \frac{h}{\lambda_0'} \tag{2}$$

Normal to the direction of motion

$$\frac{h}{\lambda_0}\sin\theta = |\mathbf{p}|\sin\phi \tag{3}$$

where **p** is momentum of the electron before the collision

From (1) $$E = h\nu_0' - h\nu_0 + E_0 \tag{4}$$

$(4)^2$ $$m_0^2c^4 + p^2c^2 = (h\nu_0' - h\nu_0 + E)^2 \tag{5}$$

$(2)\times c$ $$h\nu_0' - h\nu_0\cos\theta = pc\cos\phi \tag{6}$$

$(6)^2$ $$(h\nu_0')^2 + (h\nu_0)^2 \cos^2\theta - 2h^2 \nu_0 \nu_0' \cos\theta = p^2c^2 \cos^2\phi \quad (7)$$

$(3)\times c$ $$h\nu_0 \frac{\sin\theta}{pc} = \sin\phi$$

$$\cos^2\phi = 1 - (h\nu_0)^2 \frac{\sin^2\theta}{p^2c^2} \quad (8)$$

Substitute $\cos^2\phi$ from (8) in (7) to obtain,

$$(h\nu_0')^2 + (h\nu_0)^2 \cos^2\theta - 2h^2 \nu_0 \nu_0' \cos\theta = p^2c^2[1 - (h\nu_0)^2 \frac{\sin^2\theta}{p^2c^2}] \quad (9)$$

From (5)
$$m_0^2c^4 + p^2c^2 = (h\nu_0)^2 + (h\nu_0')^2 - 2h^2\nu_0\nu_0' + 2E_0(h\nu_0 - h\nu_0') + E_0^2$$
$$m_0^2c^4 + p^2c^2 = (h\nu_0)^2 + (h\nu_0')^2 - 2h^2\nu_0\nu_0' + 2E_0(h\nu_0 - h\nu_0') + m_0^2c^4$$
$$p^2c^2 = (h\nu_0)^2 + (h\nu_0')^2 - 2h^2\nu_0\nu_0' + 2E_0(h\nu_0 - h\nu_0') \quad (10)$$

From (9) $$p^2c^2 = (h\nu_0)^2 + (h\nu_0')^2 - 2h^2\nu_0\nu_0'\cos\theta \quad (11)$$
(10)- (11) $$0 = -2h^2(1-\cos\theta) + 2m_0^2c^4(h\nu_0' - h\nu_0')$$
$$m_0c^2(h\nu_0' - h\nu_0) = h^2(1-\cos\theta)$$
$$\frac{1}{\nu_0} - \frac{1}{\nu_0'} = \frac{h}{m_0c^2}(1-\cos\theta)$$
$$\lambda_0 - \lambda_0' = \frac{h}{m_0c}(1-\cos\theta) \quad (12)$$

This implies that in the first collision, the electron imparts energy to photon.

The second collision is a similar process to the first collision. Change in wavelength of the photon can be immediately written down in the following form;

$$\lambda_0' - \lambda_0'' = \frac{h}{m_0c}(1-\cos\theta) \quad (13)$$

(12)+ (13) gives $$\lambda_0 = \lambda_o'' = 1.25\times10^{-10} \text{ m}$$

substitute $\lambda_0$ and $\cos\theta$ in equation (12) to obtain,

$$\lambda_0' = 1.25\times10^{-10} - \frac{6.6\times10^{-34}}{9.10\times10^{-31}\times3\times10^8}(1-\frac{1}{2})$$
$$= 1.25\times10^{-10} - 1.2\times10^{-12}$$
$$= 1.25\times10^{-10} - 0.012\times10^{-10}$$
$$= 1.24\times10^{-10} \text{ m}$$

From (5) $$p^2c^2 = (h\nu_0' - h\nu_0 + m_0^2c^4)^2 - m_0^2c^4$$
$$= (h\nu_0' - h\nu_0)(h\nu_0' - h\nu_0 + 2m_0^2c^4$$

$$\approx (h\nu_0' - h\nu_0)2m_0c^2$$

$$\approx 2m_0c^2hc(\frac{1}{\lambda_0'} - \frac{1}{\lambda_0})$$

$$\approx 2m_0c^2hc(\frac{\lambda_0 - \lambda_0'}{\lambda_0'\lambda_0})$$

Substitute $\lambda_0 - \lambda_0' = \frac{h}{mc}(1-\cos\theta)$ in the above equation to get,

$$p^2c^2 = 2m_0c^2hc(\frac{h}{mc})(\frac{1-\cos\theta}{\lambda_0'\lambda_0})$$

$$p^2 = \frac{h^2}{1.238\times10^{-10}\times1.25\times10^{-10}}$$

$$p = \frac{h}{1.24\times10^{-10}}$$

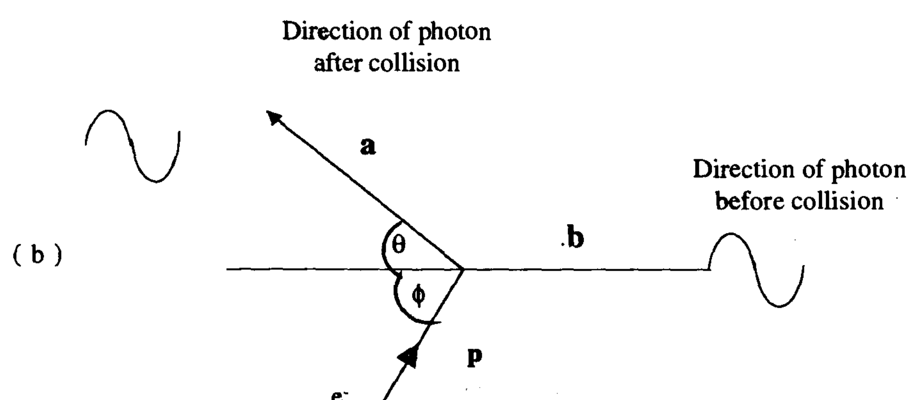
$$\lambda = \frac{h}{p}$$

$$= 1.24\times10^{-10} \text{ m}$$

De Broglie 's wavelength of the first electron is $1.24\times10^{-10}$ m **Ans**

## Second Method

The second collision can be understood as a reversed process of the first collision ie. if time runs backward, the second collision is the first collision traced back.




รูป 14.10

Fig.14.10

The angle θ is 60°.

Let **p** represent momentum of the electron in motion

**a** momentum of scattered beam ( wavelength $\lambda_0'$)

**b** momentum of incident beam( wavelength $\lambda_0$)

Vectorial consideration gives (Conservation of momentum)

$$\mathbf{p} = \mathbf{a} - \mathbf{b}$$

$$p^2 = a^2 + b^2 - 2ab\cos 60^\circ$$

$$p^2 = \left(\frac{h\nu_0}{c}\right)^2 + \left(\frac{h\nu_0'}{c}\right)^2 - \frac{2h^2\nu_0\nu_0'}{c^2}\times\frac{1}{2}$$

$$\left(\frac{p}{h}\right)^2 = \left(\frac{1}{\lambda_0}\right)^2 + \left(\frac{1}{\lambda_0'}\right)^2 - \frac{1}{\lambda_0\lambda_0'} \qquad (1)$$

Conservation of energy gives

$$h\nu_0' + E_0 = h\nu_0 + E$$

where $E_0$ is rest mass emnergy of the electron
and E is relativistic mass of the electron

$$h\nu_0' - h\nu_0 + E_0 = E$$

$$hc\left(\frac{1}{\lambda_0'} - \frac{1}{\lambda_0}\right) + E_0 = E$$

$$(hc)^2\left[\left(\frac{1}{\lambda_0'}\right)^2 + \left(\frac{1}{\lambda_0}\right)^2 - \frac{2}{\lambda_0'\lambda_0}\right] + 2hcm_0c^2\left(\frac{1}{\lambda_0'} - \frac{1}{\lambda_0}\right) + m_0^2c^4 = p^2c^2 + m_0^2c^4$$

$$\left[\left(\frac{1}{\lambda_0'}\right)^2 + \left(\frac{1}{\lambda_0}\right)^2 - \frac{2}{\lambda_0'\lambda_0}\right] + \frac{2m_0c}{h}\left(\frac{1}{\lambda_0'} - \frac{1}{\lambda_0}\right) = \frac{p^2}{h^2} \qquad (2)$$

(1)=(2) Hence

$$\left(\frac{1}{\lambda_0'}\right)^2 + \left(\frac{1}{\lambda_0}\right)^2 - \frac{2}{\lambda_0'\lambda_0} + \frac{2m_0c}{h}\left(\frac{1}{\lambda_0'} - \frac{1}{\lambda_0}\right) = \left(\frac{1}{\lambda_0'}\right)^2 + \left(\frac{1}{\lambda_0}\right)^2 - \frac{1}{\lambda_0'\lambda_0}$$

$$\frac{2m_0c}{h}\left(\frac{1}{\lambda_0'} - \frac{1}{\lambda_0}\right) = \frac{1}{\lambda_0'\lambda_0}$$

$$\lambda_0 - \lambda_0' = \frac{h}{2m_0c}$$

$\frac{h}{m_0c}$ is often referred to as Compton's wavelength represented by $\lambda_c$

Thus $$\left(\frac{p}{h}\right)^2 = \left(\frac{1}{\lambda_0}\right)^2 + \left(\frac{1}{\lambda_0 - \frac{\lambda_c}{2}}\right)^2 - \frac{1}{\lambda_0\left(\lambda_0 - \frac{\lambda_c}{2}\right)}$$

$$= \frac{\left(\lambda_0 - \frac{\lambda_c}{2}\right)^2 + \lambda_0^2 - \lambda_0\left(\lambda_0 - \frac{\lambda_c}{2}\right)}{\lambda_0^2\left(\lambda_0 - \frac{\lambda_c}{2}\right)^2}$$

Ignoring the term involving second order of $\lambda_c$

$$\left(\frac{p}{h}\right)^2 = \frac{\lambda_0^2 - \frac{\lambda_0\lambda_c}{2}}{\lambda_0^2\left(\lambda_0 - \frac{\lambda_c}{2}\right)^2}$$

$$= \frac{1}{\lambda_0\left(\lambda_0 - \frac{\lambda_c}{2}\right)}$$

$$\lambda = \frac{h}{p} = \sqrt{\lambda_0\left(\lambda_0 - \frac{\lambda_c}{2}\right)}$$

Substitute $\lambda_c = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times3\times10^8} = 0.022\times10^{-10}$

$$\lambda = \sqrt{1.25\times1.239}\times10^{-10}$$

$\lambda = 1.24\times10^{-10}$ m **Ans**

**Third Method**

As argued in the preceding method, the second collision is the time-reversed process of the the first collision.
Conservation of momentum gives

$$p^2 = \left(\frac{h\nu_0}{c}\right)^2 + \left(\frac{h\nu_0'}{c}\right)^2 - \frac{2h^2\nu_0\nu_0'}{c^2}\cos60^o$$

$$\left(\frac{p}{h}\right)^2 = \left(\frac{\nu_0}{c}\right)^2 + \left(\frac{\nu_0'}{c}\right)^2 - \frac{\nu_0\nu_0'}{c^2} \qquad (1)$$

$$\left(\frac{1}{\lambda}\right)^2 = \left(\frac{1}{\lambda_0}\right)^2 + \left(\frac{1}{\lambda_0'}\right)^2 - \frac{1}{\lambda_0\lambda_0'} \qquad (2)$$

where $\lambda$ is De Broglie's wavelength of movong electron
Conservation of energy gives

$$h\nu_0' + E_0 = h\nu_0 + E$$

where $E_0$ is rest mass energy of the electron
and E is relativistic energy of the electron.

$$E = h\nu_0' - h\nu_0 + E_0 \qquad (3$$

(3)[2]
$$p^2c^2 + m_0^2c^4 = \left[\frac{hc}{\lambda_0'} - \frac{hc}{\lambda_0} + m_0c^2\right]$$

$$\left(\frac{p}{h}\right)^2 + \left(\frac{m_0c}{h}\right)^2 = \left[\frac{1}{\lambda_0'} - \frac{1}{\lambda_0} + \frac{m_0c}{h}\right]^2$$

$$\left(\frac{1}{\lambda}\right)^2 + \left(\frac{m_0c}{h}\right)^2 = \left[\frac{1}{\lambda_0'} - \frac{1}{\lambda_0} + \frac{mc}{h}\right]^2 \qquad (4)$$

Substitue $\left(\frac{1}{\lambda}\right)^2$ from(2) in (4)

$$\left(\frac{1}{\lambda_0}\right)^2 + \left(\frac{1}{\lambda_0'}\right)^2 - \frac{1}{\lambda_0\lambda_0'} + \left(\frac{m_0c}{h}\right)^2 = \left(\frac{1}{\lambda_0'}\right)^2 + \left(\frac{1}{\lambda_0}\right)^2$$

$$- \frac{2}{\lambda_0\lambda_0'} + \frac{2m_0c}{h}\left(\frac{1}{\lambda_0'} - \frac{1}{\lambda_0}\right) + \left(\frac{m_0c}{h}\right)^2$$

$$\frac{1}{\lambda_0\lambda_0'} = \frac{2m_0c}{h}\left(\frac{1}{\lambda_0'} - \frac{1}{\lambda_0}\right)$$

$$\lambda_0' = \lambda_0 - \frac{h}{2m_0c}$$

$$= \lambda_0 - \frac{\lambda_c}{2}$$

where $\lambda_c$ is Compton's wavelength $= 022 \times 10^{-10}$ m which is very small comapred with $\lambda_0$

Substitute $\lambda_0'$ in (1) to obtain

$$\left(\frac{1}{\lambda}\right)^2 = \left(\frac{1}{\lambda_0}\right)^2 + \frac{1}{\left(\lambda_0 - \frac{\lambda_c}{2}\right)^2} + \frac{1}{\lambda_0\left(\lambda_0 - \frac{\lambda_c}{2}\right)}$$

$$\lambda = \sqrt{\lambda_0\left(\lambda_0 - \frac{h}{2m_0c}\right)}$$

As in the second method

$$\lambda = 1.24 \times 10^{-10} \text{ m } \textbf{Ans}$$

# Experiment

## Problem 1

Given are the following:

DC battery with unknown internal resistance
Two DC voltmeters
One adjustable resistor
Use of voltmeters as ammeters are not allowed.

1.1 Using a minimum number of circuits,determine the emf of the DC battery. (The adjustable resistor must not be used in this part of the experiment)

1.2 Using one voltmeter and the adjustable resistor, determine the emf of the battery and the internal resistance of the voltmeter. (The result of 1.1 cannot be used in this part of the experiment)
It is to be advisable that graphs which are theoretically linear are plotted to determine the values required by the problem.

1.3 Indicate the sources of errors. Which of these errors influence the results the most?

## Solution

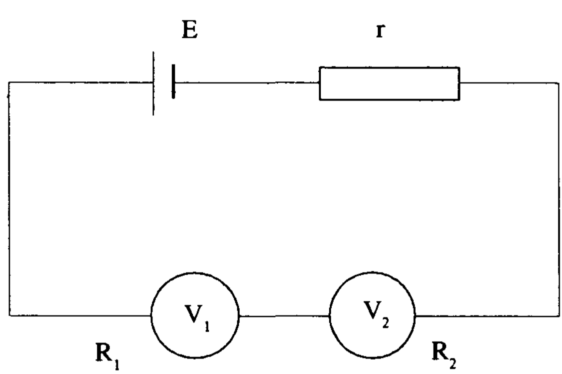



Fig.14.11

1.1 Connect a circuit consisting of two voltmeterV1andV2inseriashown in the figure above.
Let E be emf of the battery

r internal resistance of the battery

$R_1$ and $R_2$ are resistances of the firstr and second voltmeters respectively.

Current in the circuit $I = \dfrac{E}{R_1 + R_2 + r}$

Voltagae across $V_1$ $V_1 = I R_1$

$$V_1 = \frac{ER_1}{R_1 + R_2 + r} \qquad (1)$$

Voltagae across $V_2$ $V_2 = \dfrac{ER_2}{R_1 + R_2 + r}$ (2)

Connect a circuit as shown in Fig.14.12

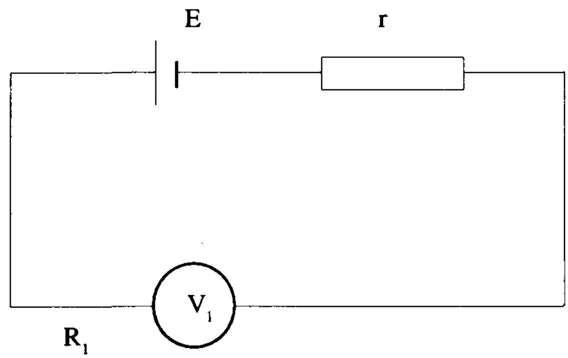



Fig.14.12

In this case, current $I = \dfrac{E}{R_1 + r}$

Voltagae across voltmeter $V_1$ $V_1' = \dfrac{ER_1}{R_1 + r}$ (3)

$V_1$, $V_2$ and $V_1'$ are obtained form the experiment, while E, $R_1$, $R_{2a}$ and r are calculated from the experimental values.

In order to calculate E, we write E as a function of $V_1$, $V_2$ and $V_1'$

(1)×(3) $V_2 V_1' = \dfrac{E^2 R_1 R_2}{(R_1 + R_2 + r)(R_1 + r)}$ (4)

$$V_1' - V_1 = \frac{ER_1(R_1 + R_2 + r) - ER_1(R_1 + r)}{[R_1 + R_2 + r + (R_1 + r)]}$$

$$V_1' - V_1 = \frac{ER_1 R_2}{(R_1 + R_2 + r)(R_1 + r)} \qquad (5)$$

(4) /(5) $\dfrac{V_2 V_1'}{V_1' - V_1} = E$

Substitute the values of $V_1$ , $V_2$ and $V_1'$ to obtain the value of E **Ans**

E r

1.2 Connect one of the voltmeters in series with the adjustable resistor as shown in Fig. 14.13

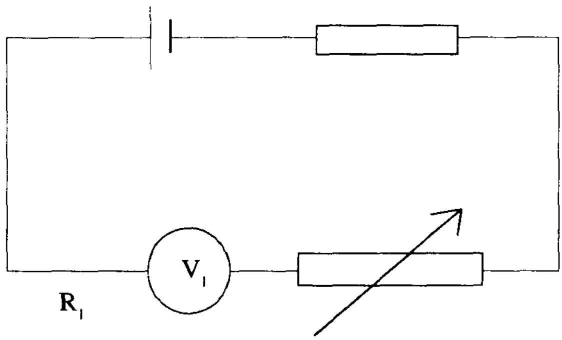



Fig14.13

From
$$I = \frac{E}{R_1 + R + r}$$

$$IR_1 = \frac{ER_1}{R_1 + R + r}$$

$$V_1 = \frac{ER}{R_1 + R + r}$$

$$R + R_1 + r = \frac{ER_1}{V_1}$$

$$R = \frac{ER_1}{V_1} - (R_1 + r)$$

Adjust R and record $V_1$

Plot a graph of R as a function of $\frac{1}{V_1}$ .

Let $m_1$ be the slope of the linear graph,

$$m_1 = ER_1 \qquad (7)$$

At $\frac{1}{V_1}$t $= 0$, intercept $b_1 = -(R_1 + r)$ (8)

Connect the voltmeter in paralell with the adjustable resistor R as shown in Fig.14.14

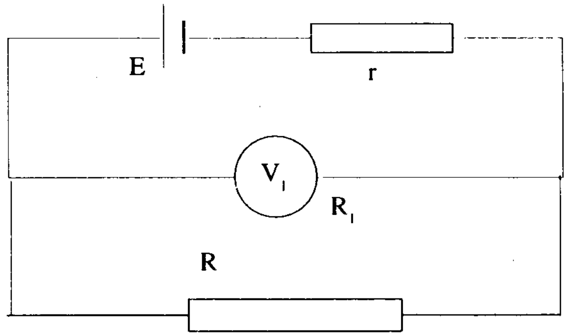



Fig. 14.14

Let $R_E$ be resultant resistance of external resistance.

$$\frac{1}{R_E} = \frac{1}{R} + \frac{1}{R_1}$$

and $\quad V_1 = IR_E \qquad (9)$

Hence $\quad V_1 = \frac{ER_E}{R_E + r}$

$$\frac{R_E + r}{R_E} = \frac{E}{V_1}$$

$$\frac{r}{R_E} = \frac{E}{V_1} - 1 \qquad (10)$$

Substitute $\frac{1}{R_E}$ from (9) in (10) to obtain

$$\frac{1}{R_1} + \frac{1}{R} = \frac{E}{rV_1} - \frac{1}{r}$$

$$\frac{1}{R} = \frac{E}{rV_1} - \left(\frac{1}{R_1} + \frac{1}{r}\right) \qquad (11)$$

Slope of the linear curve, $\quad m_2 = \frac{E}{r} \qquad (12)$

At $\frac{1}{V_1} = 0$, $\quad$ intercept $\quad b_2 = \frac{1}{R_1} + \frac{1}{r} \qquad (13)$

From equations (7), (8) ,(12) and (13) the values of r, $R_1$ and E can be calculated **Ans**

1.3 The chief source of errors is the quality of the voltmeters or the accuracy of the voltmeter readings, while the erros due to R, $R_1$ and $R_2$ have little effects on the answers **Ans**

# International Physics Olympiad XV

# 1984

# Sigtuna, Sweden

## Theory

### Problem 1

1.1 Consider a plane parallel plate which has thevalue of refractive index n varying with distance z measured from the lower surface (See Fig.15.1 )

Show that: $n_A \sin i = n_B \sin r$
where $n_A$ and $n_B$ are refractive indices of the media above and under the plate respectively.

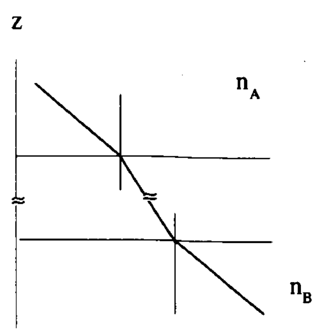



Fig.15.1

1.2 A traveller finding himself in a large flat desert. At some distance away, he sees what seems to be a water surface. As he moves towards the "water", it appears to recede so that the distance to the"water" remains the same as before. Explain the phenomenon.

1.3 Calculate temperature at the ground level for the phenomenon described under 1.2 to occcur, assuming that the traveller's eyes are at 1.60 m height above the ground and the distance to the "water" is 250 m. The refractive index of air at the temperature of 15° C and normal atmospheric pressure is 1.000276 atm. The air temperature at heights 1 m and beyond above the ground is assumed to be constant at 30° C, and the air pressure at normal atmospheric pressure is 0.1013 MPa.
Denote the refractive index by n, and assume that n -1 is proportional to the density of the air.
Discuss the accuracy of the result.

# Solution

1.1 Divide the parallel plate into k thin layers each of which has the same thickness and surfaces parallel to the plane surfaces. If k is very large, each layer may be considered to have a constant value of refractive index characteristic of that layer.

Consider the surface between k and k+1 layers,

$$n_k \sin i_k = n_{k+1} \sin r_{k+1}$$

which is true for every value of k.

Hence $n_A \sin i = n_1 \sin r_1 = n_1 \sin i_1 = n_2 \sin r_2 = n_2 \sin i_2 = n_3 \sin r_3 = n_3 \sin i_3 = \dots n_B \sin r$

and $n_A \sin i = n_B \sin r$

1.2 Since the temperature of the ground can be raised quickly by the sun, the air near the ground will be hotter than the air at an eye level. As a result, the refractive index of the air near the ground surface will be smaller than the air layers further away from the ground. When the travaeller looks down at the "water" from a distance at a small depressing angle i, total reflection occurs when $r = 90°$ i.e. when

$$n_A \sin i = n_B \sin 90°$$

where $n_A$ is the refractive index of the air layer 1 m above the ground.

When the travaeller moves towards the "water", the same phenomenon persists as the position of ther "water" also moves away from the traveller. If the temperature does not change, the phenomenon is observed from a certain angle only. **Ans**

1.3 Let $n_T$ be refractive index of the air layer in contact with the ground
T temperature of the air layer in contact with the ground

Hence $n_{30^o} \sin i = n_T$

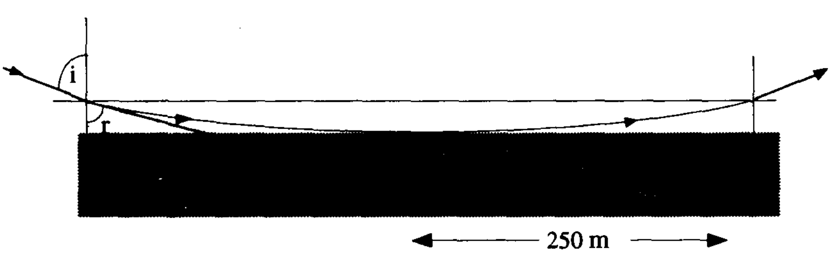



Fig. 15.2

(Not drawn to scale)

From the diagram $\tan i = \frac{250}{1.6}$

$$i = 89^{o}\ 38'$$

$$n_{30^o} \sin 89^o 38' = n_T$$

$$n_T = .99998$$

From the problem, $n_T = \rho_T$

$$\frac{n_{30^o} - 1}{n_{15^o} - 1} = \frac{\rho_{30^0}}{\rho_{15^0}}$$

Apply the equation of state at normal atmospheric pressure and constant volume, only the temperature and density are allowed to change.

$$PV = (\frac{n}{M})RT$$

where n is mass of the fixed air volume under consideration

M, molar mass

$$P = (\frac{\rho}{M})RT$$

$$\frac{PM}{R} = \rho T = cons\tan t$$

$$\rho_{15^0} T_{288} = \rho_{30^0} T_{303} = \rho_T T_{(273^0 + C^0)}$$

$$\frac{\rho_{30^o}}{\rho_{15^o}} = \frac{288}{303} \qquad (3)$$

$$\frac{\rho_T}{\rho_{15^0}} = \frac{288}{273 + C} \qquad (4)$$

Substtiute in (2) to obtain $\frac{n_{30^0} - 1}{.000276} = \frac{288}{303}$

$$n_{30^0} - 1 = .95049 \times 2.76 \times 10^{-4}$$

$$= 2.6233 \times 10^{-4}$$

$$n_{30^0} = 1.000262$$

Hence $\frac{n_T - 1}{n_{15^0} - 1} = \frac{\rho_T}{\rho_{15^0}}$

Substitute $n_T = .99998\, n_{30^0}$

$$= .99998 \times 1.000262$$

$$= 1.000242$$

$$\frac{.00242}{.000276} = \frac{288}{273 + C}$$

$$273 + C = 328.4$$

$$C = 55.4\ C$$

The accuracy of calculated temperature depends largely on the accuracy of the measured values of air temperature and refractive index of the air. **Ans**

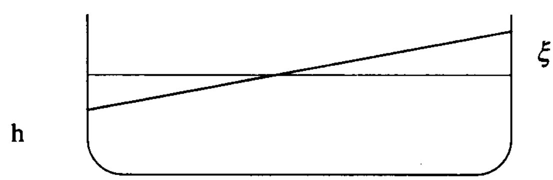
# Problem 2

In some lakes in some parts of the world, a curious phenomenon known as "seiching" (oscillation of water) can be observed occasionally. It is usually seen on shallow, long and comparatively narrow lakes. The whole mass of the water seems to move like the coffee in a cup carried to a waiting guest, and cannot be mistaken for the waves normally seen on the lakes.

A rectangular tank is used to build a model for demonstration purpose. Denote the length of the tank by L and height of water by h. Assume that the water level initially makes a small angle with the horizontal direction and the water level begins to oscillate about a horizontal axis half way along the length of the tank and also normal to the length of the tank.

Develop a model for the motion of the water and find an expression for the period of oscillation T. The initial conditions are given in Fig 15.3. Assume also that $\xi \lll h$

Fig. 15.3




| L = 479 mm | |
|---|---|
| h (mm) | T(s) |
| 30 | 1.78 |
| 50 | 1.40 |
| 69 | 1.18 |
| 88 | 1.08 |
| 107 | 1.00 |
| 124 | 0.91 |
| 142 | 0.82 |

| L = 143 mm | |
|---|---|
| h(mm) | T(s) |
| 31 | 0.52 |
| 38 | 0.48 |
| 58 | 0.43 |
| 67 | 0.35 |
| 124 | 0.28 |

Table of oscillation time for different water depths in two rectangular tanks of different lengths.

Check how well the derived formula fits the experimental data and give your opinion on the usefulness of the model.

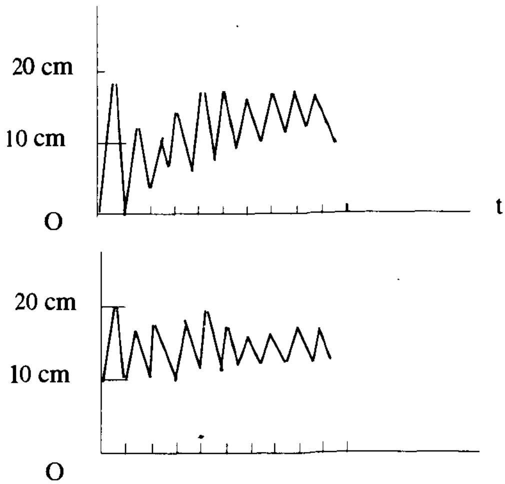



Fig15.4 Graphs of water level( measured from equilibrium position) against time

The graphs above show results from measaurements on Lake Vattern in Sweden at Balstedalen (northern end) and Jonkoping (southern end). The lake has a length of - 123 km and an average depth of 50 m .
What is the period of Lake Vattern ?

## Solution

Let the displacement of the centre of gravity of the total mass of water from its equilibrium position be x and y respectively.
The part of water that moves up occupies the area of $\Delta BPD$ .

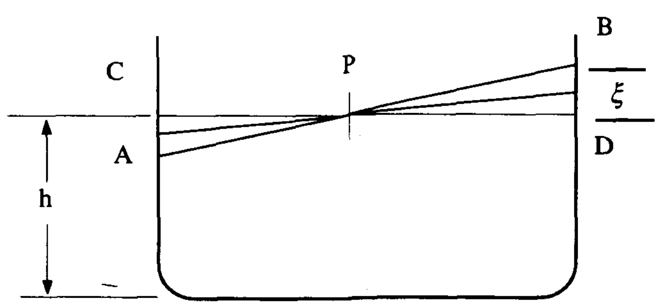



Fig. 15.5

From the geometry of the figure, the centre of gravity of ΔBPD is at distance of $\frac{\xi}{3}$ above its equilibrium position.

Likewise, the centre of gravity of ΔPCA is at distance of $\frac{\xi}{3}$ below its equilibrium position.

The centre of gravity of the total mass of water in the tank is raised from its equilibrium position by the distance,

$$y = \frac{2\xi}{3} \qquad (1)$$

Likewise, the centre of mass of the whole mass of water is shifted toward the right by a distance of

$$x = 2\times\frac{L}{3} \qquad (2)$$

Let w be width of the tank
M mass of water in the tank
ρ density of the water

Thus when the water level reaches the highest height of $\xi$.

The volume of water displaced is $= 2\times\frac{L}{2}\times\frac{\xi}{3}\times w$

which corresponds to the mass of m = $\frac{1}{2}\times\xi\times\frac{L}{2}\times w\times\frac{M}{L.h.w}$

or $$m = \xi\frac{M}{4h} \qquad (3)$$

Equations (1) and (2) imply that motion along x axis is more pronounced than motion along y axis, as both motions take place during the same time interval, the velocity along x axis >>> velocity along y axis. Hence the motion along x axis depicted by the model is that of simple harmonic.

Energy equation is $\frac{1}{2}mV_{MAX}^2 = Mgh\Delta y$

$$\frac{1}{2}\times\frac{1}{4}\xi MV_{MAX}^2 = Mgh\frac{2\xi}{3}$$

$$V_{MAX}^2 = \frac{16}{3}gh$$

Maximum amplitude $a_0 = \frac{2L}{3}$

angular frequency $$\omega = \frac{V_{MAX}}{a_0}$$

$$= \frac{3V_{MAX}}{2L}$$

$$= \frac{3\times4\sqrt{gh}}{\sqrt{3}\times2L}$$

$$= \frac{2\sqrt{3gh}}{L}$$

Period of oscillation $\quad T = \frac{2\pi}{\omega}$

$$= \frac{2\pi L}{2\sqrt{3gh}}$$

$$= \frac{\pi L}{\sqrt{3gh}}$$

Plot a graph of T as a function of $\frac{1}{\sqrt{h}}$ .

Compare T calculated from the model with actual data provided in the problem.

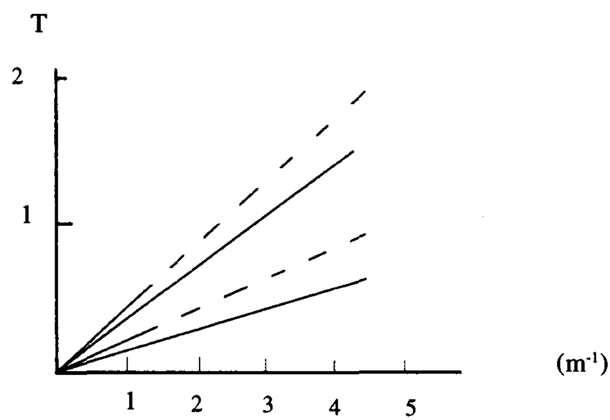



Fig.15.6

The slope of the linear curve is $T = \frac{\pi L}{3g}$

The actual data are consistent with calculated values within the accuracy error of 15 %
Hence the reasonable formula that should be used is,

$$T = 1.15\frac{\pi L}{3g}$$

From the data of Vattern, L = 123 km, h = 50 km

$$T = 3.2 \text{ hr}$$

( connected between two peaks of the curves shown in Fig. 15.4) **Ans**

# Problem 3

An electronic filter depicted in Fig.15.7 consists of 4 components. Its input impedance is negligible, while the output impedance is assumed to be infinite. The design of the filter is such that the quotient $\frac{V_i}{V_o}$ has a frequency dependence as shown in Fig.15.7 b, where $V_i$ and $V_0$ are input and output voltages respectively. At frequency $f_0$, the phase difference between $V_o$ and $V_i$ should be zero.

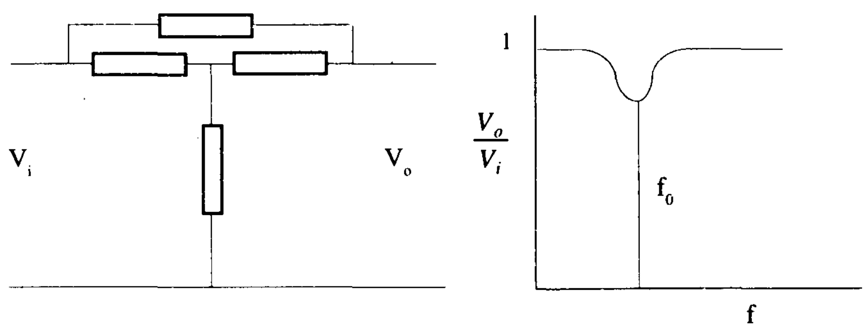



Fig.15.7

In order to build a filter, the following electronic items may be used;
- resistors 10 Ω ; 2 units
- capacitor 10 nF; 2 units
- induction coil 160 mH; 2 units

(the coil is without soft magnetic core, and its DC resistance is zero.)

# Solution

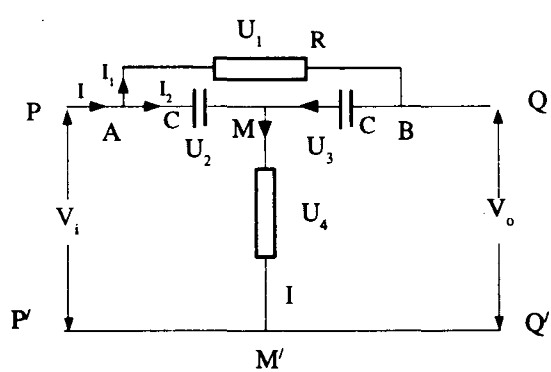



Fig.15.8

Let $A(\omega) = \frac{V_o}{V_i}$

For DC component $\omega = 0$ The capacitor has $\infty$ DC resistance.
and $A(0) = 0$

At very high frequencies The capacitor has 0 AC resistance.

$A(0) = 1$

Hence $0 \leq A(\omega) \leq 1$

Let current PA be I
current through AB in the upper branch ( $V_1$ voltage ) be I
current through AM ( $U_2$ voltage ) $I_2 = I - I_1$
current through BM ( $U_3$ voltage ) be $I_1$ ( infinite imedance across QQ′)
current through MM′ ( $U_4$ voltage) be $I = I_1 + I_2$

**First Method**

Apply Kirchhoff's law

$$(I - I_1)Z_C + IZ_R = V_1$$
$$I(Z_C + Z_R) - I_1 Z_C - V_i = 0 \quad (1)$$
$$I_1 Z_R + I_1 Z_C - (I - I_1)Z_C = 0$$
$$-IZ_C + I_1(Z_R + 2Z_C) = 0 \quad (2)$$
$$I_1 Z_C + IZ_R = V_0$$
$$IZ_R + I_1 Z_C - V_0 = 0 \quad (3)$$

In a marix form

$$\begin{pmatrix} Z_C + Z_R & -Z_C & -V_i \\ -Z_C & Z_R + 2Z_C & 0 \\ Z_R & Z_C & V_o \end{pmatrix} \begin{pmatrix} I \\ I_1 \\ 1 \end{pmatrix} = 0$$

ถ้า I และ $I_1$ จะมีค่าไม่เป็น 0 ทั้งคู่ Determinantของแมทริกส์ (3x3 )ข้างบนจะต้องเป็นศูนย์

$$\begin{vmatrix} Z_C + Z_R & -Z_C & -V_i \\ -Z_C & Z_R + 2Z_C & 0 \\ Z_R & Z_C & V_o \end{vmatrix} = 0$$

$$-V_i \begin{vmatrix} -Z_C & Z_R + 2Z_C \\ Z_R & Z_C \end{vmatrix} + V_o \begin{vmatrix} Z_C + Z_R & -Z_C \\ -Z_C & Z_R + 2Z_C \end{vmatrix} = 0$$

$$-V_i\left(-Z_C^2 - Z_R^2 - 2Z_C Z_R\right) + V_o\left(Z_R^2 + 2Z_C^2 + 3Z_C Z_R - Z_C^2\right) = 0$$

$$\frac{V_o}{V_i} = \frac{Z_C^2 + Z_R^2 + 2Z_C Z_R}{Z_C^2 + Z_R^2 + 3Z_C Z_R}$$

Substitute $Z_C = \frac{j}{\omega C}$ $Z_C^2 = \frac{-1}{\omega^2 C^2}$

$Z_R = R$ $Z_R^2 = R^2$

$$Z_R Z_R = \frac{-jR}{\omega C}$$

$$\frac{V_o}{V_i} = \frac{-\frac{1}{\omega^2 C^2} + R^2 - \frac{2jR}{\omega C}}{-\frac{1}{\omega^2 C^2} + R^2 - \frac{3jR}{\omega C}}$$

$$= \frac{(\omega^2 R^2 C^2 - 1) - 2j\omega RC}{(\omega^2 R^2 C^2 - 1) - 3j\omega RC}$$

$$\left(\frac{V_o}{V_i}\right)^2 = \frac{\left(\omega^4 R^4 C^4 - 2\omega^2 R^2 C^2 + 1 + 4\omega^2 R^2 C^2\right)}{\left(\omega^4 R^4 C^4 - 2\omega^2 R^2 C^2 + 1 + 9\omega^2 R^2 C^2\right)}$$

$$= \frac{(\omega^4 R^4 C^4 + 2\omega^2 R^2 C^2 + 1)}{(\omega^4 R^4 C^4 + 7\omega^2 R^2 C^2 + 1)}$$

$$A^2(\omega) = \frac{(\omega^4 R^4 C^4 + 2\omega^2 R^2 C^2 + 1)}{(\omega^4 R^4 C^4 + 7\omega^2 R^2 C^2 + 1)}$$

The condtion for A(ω)to be minimum is $2\omega \frac{dA(\omega)}{d\omega} = 0$

$$\frac{(\omega^4 R^4 C^4 + 7\omega^2 R^2 C^2 + 1)(4\omega^4 R^4 C^4 + 4\omega^2 R^2 C^2)}{(\omega^4 R^4 C^4 + 7\omega^2 R^2 C^2 + 1)^2}$$

$$\frac{(\omega^4 R^4 C^4 + 2\omega^2 R^2 C^2 + 1)(4\omega^4 R^4 C^4 + 14\omega^2 R^2 C^2}{(\omega^4 R^4 C^4 + 7\omega^2 R^2 C^2 + 1)^2} = 0$$

$$2\omega^2 R^2 C^2(\omega^4 R^4 C^4 + 7\omega^2 R^2 C^2 + 1)(2\omega^2 R^2 C^2 + 2)$$
$$- 2\omega^2 R^2 C^2(\omega^4 R^4 C^4 + 2\omega^2 R^2 C^2 + 1)(2\omega^2 R^2 C^2 + 7) = 0$$

$$2\omega^6 R^6 C^6 + 4\omega^4 R^4 C^4 + 2\omega^2 R^2 C^2 + 2\omega^4 R^4 C^4 + 14\omega^2 R^2 C^2 + 2$$
$$-2\omega^6 R^6 C^6 - 4\omega^4 R^4 C^4 - 2\omega^2 R^2 C^2 - 7\omega^4 R^4 C^4 - 14\omega^2 R^2 C^2 + 7 = 0$$

$$5\omega^4 R^4 C^4 - 5 = 0$$

$$\omega^4 R^4 C^4 = 1$$

$$\omega = \frac{1}{RC}$$

$$= 10^{-4}\ s^{-1}$$

$$f = \frac{\omega}{2\pi} = 1.6 \quad \text{kHz}$$

Substitute $\omega RC = 1$ to obtain

$$A(\omega) = \frac{2}{3}$$

**Second Method**

Let $I_1 = I_{m1} \sin\omega t$ where $I_{m1}$ is peak current.,

Hence $U_1 = I_{m1} R \sin\omega t$ (Voltage and current are in the same phase)

$$U_3 = \frac{I_{m1}}{\omega C}\sin(\omega t - \frac{\pi}{2}) \quad \text{(Voltage lags behind current by } 90^\circ\text{)}$$

$$U_2 = U_1 + U_3 = I_{m1}\left[R\sin\omega t + \frac{1}{\omega C}\sin(\omega t - \frac{\pi}{2})\right]$$

As current $I_2$ leads $U_2$ voltage by $90^\circ$

$$\frac{I_2}{\omega C} = I_{m1}\left[R\sin(\omega t + \frac{\pi}{2}) + \frac{1}{\omega C}\sin\omega t\right]$$

$$I_2 = I_{m1}\left[\omega CR\sin(\omega t + \frac{\pi}{2}) + \sin\omega t\right]$$

$$U_4 = (I_1 + I_2)R = I_{m1}\left[2R\sin\omega t + \omega R^2 C\cos\omega t\right]$$

$$V_o = U_3 + U_4 = I_{m1}\left[2R\sin\omega t + (\omega R^2 C - \frac{1}{\omega C})\cos\omega t\right]$$

$$V_i = U_2 + U_4 = I_{m1}\left[3R\sin\omega t + (\omega R^2 C - \frac{1}{\omega C})\cos\omega t\right]$$

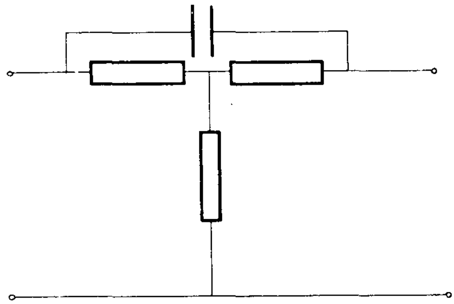
$$A(\omega) = \frac{V_o}{V_i} = \frac{2R\sin\omega t + (\omega R^2 C - \frac{1}{\omega C})\cos\omega t}{3R\sin\omega t + (\omega R^2 C - \frac{1}{\omega C})\cos\omega t}$$

From data provided by the problem A(ω) = 1at low and high frequencies .

When $\omega R^2 C - \frac{1}{\omega C} = 0$ or $\omega = \frac{1}{RC}$

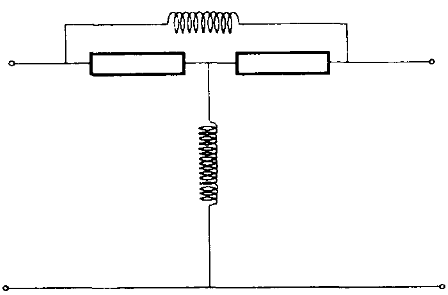
$A(\omega) = \frac{2}{3}$ is minimum, and $V_o$ $V_i$ are in phase..

Other forms of filters are:



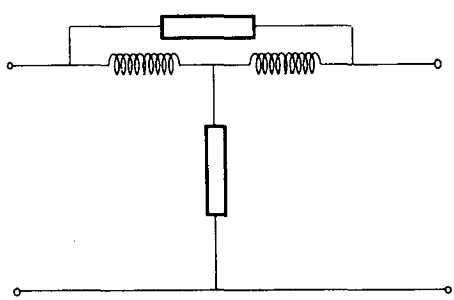
$$\omega = \frac{1}{RC}$$

$$f = \frac{1}{2\pi RC}$$



$$\omega = \frac{L}{R}$$

$$f = \frac{R}{2\pi L}$$



$$\omega = \frac{L}{R}$$

$$f = \frac{R}{2\pi L}$$

**Ans**

Fig.15.9

# Experiment

## Problem 1

Given:

-AC signal generator of frequency 0.20 kHz
-double channeled oscilloscope
-diode transistor
-capcitor 0.1 $\mu$F
-resistor of unknown value
-connecting board
-connecting wires

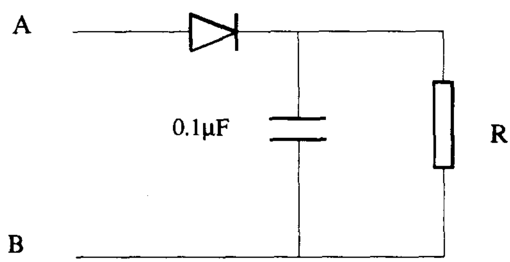



Fig.15.11

Connecting a circuit as shown in Fig.15.11.

Connect AC signal at frequency 0.20 kHz and peak to peak voltage of 4 V across AB.

Calculate power comsummed by resistor R from values measured experimentally.

## Solution

Connect a circuit in accordance with the instruction given in the problem and shown in Fig. 15.11.

Connect voltage across R to Y -input of the oscilloscope.

The curve observed on the screen of the oscilloscope is shown in Fig.15.12.

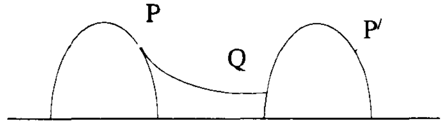



Fig.15.12

Only half of the sinusoidal wave gets through the semi-conductor diode.

The capacitor is charged during the interval of current flow through the diode and dis-harged through resistor R when there is no current in the circuit.

During the charging interval, output voltage follows input voltage.
During the discharging interval, output voltage follows the formula,

$$V = V_0 e^{-\frac{1}{RC}}$$

$$\frac{1}{RC} = \ln(\frac{V_0}{V})$$

At $t = \frac{T}{2}$ $\qquad$ $RC = \frac{T}{2}\ln(\frac{V}{V_0})$

Take $V_0$ reading at P, and V reading at Q.

From the problem, $\quad T = \frac{1}{0.20 \times 10^3}$ $s^{-1}$

Calculate R by substituting R, T, $V_0$ and V in the formula above.

Average power consummed by R , $\quad P = \frac{1}{T}\int_0^T V^2 R dt$

Measure the area under curve from 0 to T divided by period T to obtain the average power required. **Ans**

# Problem 2

Given:

- neon lamp connected to 220 V
- laser of unknown wavelength
- Diffraaction grating of unknown spacing
- transparent scale ( 1mm divided into 100 spacings)
- 1 meter ruler
- a retort stand with clamps

The spectrum of the neon lamp consists a number of spectral lines in the yellow-orange- and red range. One of the yellow lines is conspicuously strong. Determine its wavelength. Give an estimate for the accuracy of the measurement.

# Solution

Direct laser beam through the transparent scale and onto the screen or the wall located at distance D from the scale.

Observe diffraction pattern on the screen or wall as the case mgiht be.
Calculate wavelength of the laser beam, from the formula

$$n\lambda_0 = d_0 \sin \alpha_n$$

where n is an integer 0,1,2,3......representing the order of the observed lines
$d_0$ distance between two adjacent lines on the scale
$\alpha_n$ angle which the diffracted beam makes with the principal axis of the diffraction grating $\alpha_n = x/D$
x distance the spectral line measured from the principal axis

Having calculated $\lambda_0$, conduct a similar experimen t by directing laser beam through the grating of spacing d.

Substitute measured values of θ, associated n and $\lambda_0$ in the formula

$$d \sin\alpha_n = n\lambda_0$$

to obtain the value of d.

Conduct a similar experiment, using neon light instead of the laser source.

Substitute the values of d, $\alpha_n$ and n in the formula $d \sin\alpha_n = n\lambda_0$ to calculate wavelength λ of the pronounced line.

The chief source of error is due to error in x. It is therefore advisable to use the spectral line of 3rd or 4th order to reduce error in x and the calculated value of λ.

# International Physics Olympiad XVI

# 1983

# Portoroz, Yugoslavia

## Theory

### Problem 1

A young male radio amateur maintains a radio contact with two girl friends living in two different towns. He positions an aerial array such that when the first girl living in town A recieves a maximum signal, the other girl in town B recieves minimum or no signal at all, and vice versa. The array is built from two vertical aerial rods each of which transmits signal with the same intensity and uniformly in all directions in the horizontal plane.

1.1 Find the parameters of the array, i.e. the distance between the rods, its oreintation and the phase shift between the electrical signals supplied to the rods, such that the distance between them is minimum.

1.2 Find the numerical solution if the man has a radio station transmitting at 27 MHz and build up the aerial array at Portoroz. Using the map he has found that the angle between the north and the direction of A (Koper) and of B (Buje) small town in Istria are 72° and 157° respectively.

### Solution

1.1 Let the signals sent out at rods 1 and 2 be

$$E_1 = E_0\cos(kx-\omega t) \quad \text{or} \quad E_0\cos[\frac{2\pi}{\lambda}(x-vt)]$$

$$E_2 = E_0\cos(kx-\omega t-\delta) \quad \text{or} \quad E_0\cos[\frac{2\pi}{\lambda}(x-vt)-\delta]$$

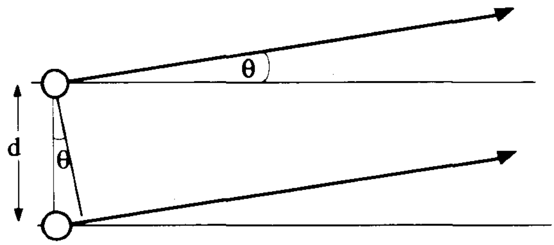



Fig.16.1

Condition for the two signals to reinforce constructively and results in maximum intensty at town A is

$$\left(\frac{2\pi}{\lambda} d \sin \theta_A\right) - \delta = 2n\pi \qquad (1)$$

Condition for the two signals to cancel each other and resutls in minimum intensity at town B is

$$\left(\frac{2\pi}{\lambda} d \sin \theta_B\right) - \delta = (2n'+1)\pi \qquad (2)$$

where $n = 0,1,2,3,4........$
$n' = 0,1,2,3,4........$

$\theta_A$ and $\theta_B$ are angular positions of towns A and B measured from the normal direction dividing distance between the two aerial rods.

Relation between $\theta_A$ and $\theta_B$ is

$$\theta_A - \theta_B = \phi \qquad (3)$$

The solution of this problem is to find $\theta_A$, $\theta_B$ d, , n and n′ which satisfy (1),(2) and (3) and minimum d.

(1)- (2) $\quad d\sin\theta_A - d\sin\theta_B = (n - n' - \frac{1}{2})\lambda$

$$d\sin\theta_A - d\sin(\theta_A - \phi) = (n - n' - \frac{1}{2})\lambda$$

$$2d\cos(\theta_A - \frac{\phi}{2})\sin\frac{\phi}{2} = (n - n' - \frac{1}{2})\lambda$$

d is ~~maximum~~ minimum when $\cos(\theta_A - \frac{\phi}{2})\sin\frac{\phi}{2}$ is maximum and $(n - n' - \frac{1}{2})\lambda$ ) is minimum

$\cos(\theta_A - \frac{\phi}{2})$ is maximum when $\theta_A = \frac{\phi}{2}$

and $\quad (n - n' - \frac{1}{2})\lambda$ is minimum when $n - n' = 1$

Hence
$$d = \frac{\lambda}{4\sin\frac{\phi}{2}}$$

$$\theta_A = \frac{\phi}{2}$$

$$\theta_B = \frac{\phi}{2}$$

(1)+(2) $\frac{2\pi}{\lambda} 2d \sin\frac{\phi}{2} - 2\delta = 2\pi(n+n')$

$\pi - 2\delta = 2\pi(2n'+1)$ or $= 2\pi(2n-1)$

$\delta = -\frac{\pi}{2} - 2n'\pi = \frac{3\pi}{2} - 2n\pi$

1.2 Wavelength $\lambda = \frac{c}{v} = 11.1$ m

Angle $\phi$ between the directions of A and B = 150° - 72° = 85°

Minimum length of the aerial rod is $d_{MIN} = \frac{1.1}{4\sin 42.5^o} = 4.1$ m

And the line drawn perpendicular to both rods makes angle of 72° + 42° =114.5° with north-south direction.

# Problem 2

A long bar in the shape of a rectangular parallelpiped with sides a, b, and c ( a >> b >>c) is made from semi-conductor InSb. Current I flows in the bar along the direction parallel to the edge a. The bar itself is in an external magnetic field B which is parallel to the edge c. The magnetic field produced by current I can be neglected. The current carriers are electrons. The average velocity of electrons in a semi-conductor in the presence of an electric field only is $\mathbf{v} = \mu\mathbf{E}$ where $\mu$ is mobility of the electron.

When there is external magnetic field, the net electric field is no longer parallel to the current. This phenomenon is known as Hall's effect.

2.1 Determine the magnitude and the direction of the electric field in the bar in order to have current I described above in the bar.

2.2 Calculate the difference of the electric potential between two opposing points on the surface of the bar along the direction of the edge b.

2.3 Find the analytic expression for the DC component of the electric potential difference in 2.2, if the current and the magnetic field are AC i.e.

$$I = I_0 \sin \omega t$$
$$B = B_0 \sin(\omega t + \delta)$$

2.4 Using the results from 2.3, design and describe an electric circuit which will enable us to measure the power consumption of an electric apparatus in connection with AC network or AC main.

Data Electron mobility in InSb = 7.8 $mV^{-1}s^{-1}$

Electron concentration in InSb = $2.5 \times 10^{22}$ /m3

I = 1.0 A, B = 0.10 T, b = 1.0 cm, c = 1.0 mm

electron charge $e = 1.6 \times 10^{-10}$ C

# Solution

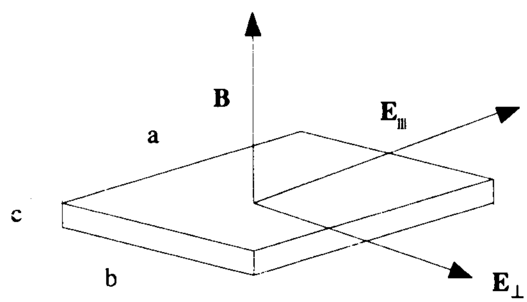



Fig.16.2

(Direction of motion of electron beam is opposite to that of current per unit area j)

2.1 Calculation of electron 's velocity

$$I = \mathbf{j.s}$$

j current per unit area normal to applied current

s area normal to the direction of applied current

$$I = n\, e_0\, v\, b\, c$$

$$v = \frac{I}{n.e_0 b.c} = 25 \text{ m/s}$$

Calculation of electric field

$$v = \mu E_{||}$$

where $E_{||}$ component of the ellectric field parallel to the direction of v.

$$E_{\parallel} = \frac{v}{\mu}$$

$$= 3.2 \quad \text{V/m} \qquad \textbf{Ans}$$

Calculation of electric field in the direction parallel to b from Lorentz force acting on the electron.

$$\mathbf{E}_{\perp} = \mathbf{v} \times \mathbf{B} = 2.5 \quad \text{V/m}$$

Magnitude of the electric field

$$|\mathbf{E}| = \sqrt{E_{\parallel}^2 + E_{\perp}^2}$$

$$= 4.06 \quad \text{V/m} \qquad \textbf{Ans}$$

2.2 Calculation of potential from

$$U_H = E_{\perp}\, b = 25 \quad \text{mV}$$

2.3 Calculation of $U_H$ as a function of time

$$U_H = v.B.b$$

$$= \frac{I.B.b}{n.e_o b.c}$$

$$= \frac{I_o.B_o.b}{n.e_o b.c}[\sin \omega t \sin(\omega t + \delta)]$$

$$= \frac{I_o.B_o.b}{n.e_o b.c}.\frac{1}{2}[\cos(2\omega t + \delta) + \cos\delta]$$

DC component of $U_H$ is $\dfrac{I_o.B_o.b}{n.e_o b.c}$ **Ans**

2.4 The AC wattmeter required by the problem is shown in Fig.16.3 below.

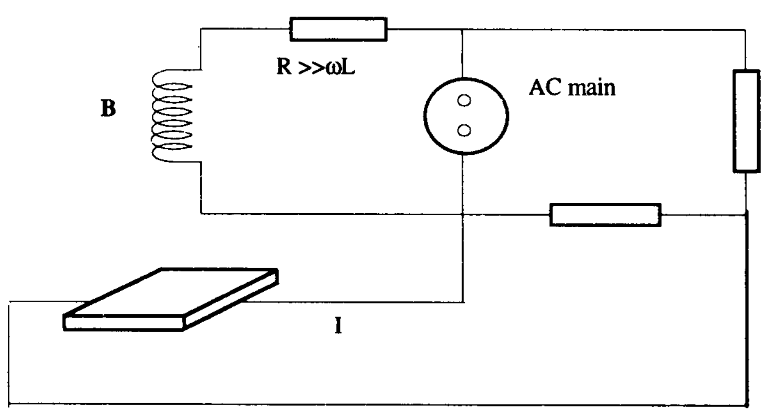



Fig.16.3

## Operation of Wattmeter

1. Connect AC signal to the circuit consisting of a load and an induction coil which produces magnetic field $B_0$. Choose peak current equal to DC component $I_0$ to be used in 2. . Measaure DC Hall'svoltage ($U_{H1}$)

2. Connect DC s ignal to the circuit consisting of a load and an induction coil which produces magnetic field B0. Use DC component equal to peak current $I_0$. Measaure DC Hallûsvoltage ($U_{H2}$).

From (1) $$U_{H1} = \frac{I_0 B_o \cos\delta}{2ne_o c}$$

From (2) $$U_{H2} = \frac{I_0 B_o}{2ne_o c}$$

$$= \cos\delta$$

$$\text{Wattage} = VI\cos\delta$$

The values of V and I are measured by voltmeter and ammeter, while cos is known from the experiment and additional calculation, hence wattage can be calculated **Ans**

# Problem 3

In a space research project, two schemes of launching a space probe out of the solar system are discussed. The first scheme is to launch the probe with a velocity large enough to escape from the gravitational pull of the solar system directly. According to the second scheme, the probe is to be sent approaching one of the outer planets. With the planet's help changes, the probe will change its direction and reach the velocity necessary to escape from the solar system. Assume that the probe moves under the gravitational field of only the sun or the planet, depending whichever field is stronger at that point under consideration.

3.1 Determine the minimum velocity and its direction relative to the earth's motion that should be given to the probe on launching according to the first scheme.

3.2 Suppose that the probe has been launched in the direction determined in 3.1 but with different speed. Find the velocity of the probe when it crosses the orbit of Mars, ie. paralell and perpendicular components with respect ot the orbit of Mars. Note that Mars is not near the point of crossing when the probe is crossing the orbit of Mars.

3.3 If the probe is to enter the gravitational field of Mars, find the minimum launching velocity from the earth necessary for the probe to escape from the solar system.
Hint From the result 3.1 we know the optimum magnitude and direction of the velocity of the probe that is necessary to escape from the solar system after leaving the gravitational field of Mars. (One needs not worry about the precise position of Mars during the encounter.)

Find the relation between this velocity and the velocity components before the probe enter the gravitational field of Mars; ie. the components one determine in 3.2.
Is energy saved by adopting the second scheme?

3.4 Estimate the maximum possible fractional saving of energy in the second scheme with respect to the first scheme.
Note Assume that all the planets revolve around the sun in circular orbits in the same direction and in the same plane. Neglect air resistance, the rotation of the earth around its axis as well as the energy used in escaping the earth's gravitational field.

**Data**: Velocity of the earth around the sun is 30 km/s and the ratio of the distance of the earth and Mars from the sun is 2/3.

# Solution

3.1 The condition for the probe to e scape the soalr system is the sum of its potential energy and kinetic energy in the gravitational field of the solar system must be greater or equal to 0 ,ie

$$\frac{1}{2}m\,v^2_{1S} - G\frac{mM_S}{R_{ES}} \geq 0 \qquad (1)$$

where m is mass of the probe
$v_{1S}$ magnitude of the velocity of the probe relative to the sun
$M_S$ mass of the solar system or approximately the mass of the sun
$R_{ES}$ distance of the earth from the sun

If $v_{ES}$ is the magnitude of the velocity of the earth relative to the sun,
Equation of motion of the earth is

$$\frac{m_E v_{ES}^2}{R_E} = G\frac{m_E M_S}{R_E^2}$$

where $m_E$ is mass of the earth

$$v_{ES} = \sqrt{\frac{GM_S}{R_E}}$$

or

$$\frac{GM_S}{R_E} = v^2_{ES} \qquad (2$$

Substitute $v_{ES}$ from (2) in (3) to obtain,

$$v^2_{1S} = 2v^2_{ES} \qquad (3$$

Let $v_{1E}$ be the velocity of the probe that escapes the solar system measured relative to the earth.
and $v_{1S}$ be the velocity of the probe that escapes the solar system measured relative to the sun.

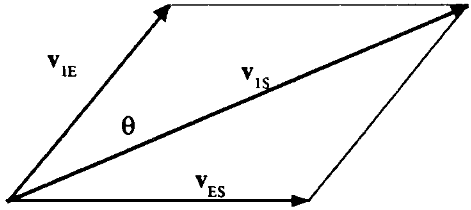



Fig.16.4

Let $\mathbf{v}_{1E}$ make angle θ with the direction of $\mathbf{v}_{ES}$ (velocity of the earth relative to the sun) Hence the resultant velocity between $\mathbf{v}_{1E}$ and $\mathbf{v}_{ES}$ is escaping velocity of the probe relative to the sun $\mathbf{v}_{1S}$.

From the above figure, $\mathbf{v}_{1S} = \mathbf{v}_{1E} + \mathbf{v}_{ES}$

$$v^2_{1S} = v^2_{1E} + v^2_{ES} + 2v_{1E}v_{ES}\cos\theta$$

Substitute $v^2_{1S} = 2v^2_{ES}$ from (3) to obtain,

$$2v^2_{ES} = v^2_{1E} + v^2_{ES} + 2v_{1E}v_{ES}\cos\theta$$

$$v^2_{1E} - 2v_{1E}v_{ES}\cos\theta - v^2_{ES} = 0$$

$$v_{1E} = \frac{-2v_{ES}\cos\theta \pm \sqrt{4v_{ES}^2\cos^2\theta + 4v_{ES}^2}}{2}$$

$$= -v_{ES}(\cos\theta \pm \sqrt{1+\cos^2\theta})$$

$$= -v_{ES}(\cos\theta + \sqrt{1+\cos^2\theta})$$

The minimum value of $v_{1E}$ occurs when $\cos\theta = 1$ or $\theta = \frac{\pi}{2}$

$$(v_{1E})_{MIN} = (\sqrt{2} - 1)\, v_{ES}$$

Substitute $v_{ES} = 30$ km/s

$$(v_{1E})_{MIN} = .414 \times 30 = 12.3 \text{ km/s}$$

Escaping velocity of the probe from the solar system relative to the earth is 12.3 km/s **Ans**

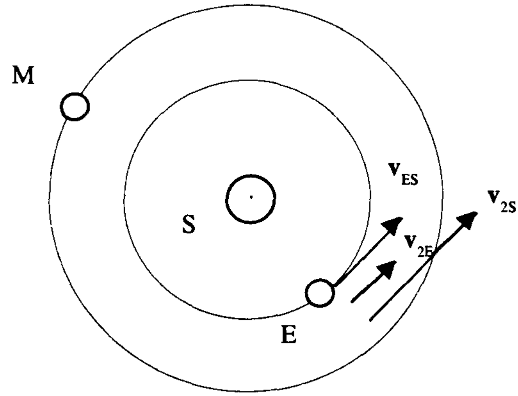



Fig. 16.5

3.2 In the case of the probe being sent from the earth with the magnitude other than $v_{1E}$ = 12.3 km/s but in the same direction with $v_{1E}$,
Let $v_{2E}$ and $v_{2S}$ be velocities of the probe relative to the earth and the sun respectively, when the probe is launched in the same direction with $v_{ES}$.
As in the analysis in 3.1

$$\mathbf{v}_{2S} = \mathbf{v}_{2E} + \mathbf{v}_{ES} \quad (4)$$

Apply the principle of the conservation of energy when the probe crosses the orbit of Mars,

$$\frac{1}{2} m v^2_{2S} - G\frac{mM_S}{R_{ES}} = m[\left(v_{2S}\right)_{\parallel}^2 + \left(v_{2S}\right)_{\perp}^2] - G\frac{mM_S}{R_{MS}} \quad (5)$$

where $(\mathbf{v}_{2S})_{\parallel}$ and $(\mathbf{v}_{2S})_{\perp}$ are components of the velocity of the probe resolved in the directions parallel and normal to the orbital path of Mars.(meassured relative to the sun)

$R_{MS}$ is the distance of Mars from the sun (equal to the orbial radius of Mars around the sun.)

$R_{ES}$ is the distance of the earth from the sun (equal to the orbial radius of the earth around the sun.)

Apply the principle of the conservation of angular momentum to the probe

$$m\,\mathbf{v}_{2E} \times \mathbf{R}_{ES} = m\,\mathbf{v}_{\parallel} \times \mathbf{R}_{MS}$$

where $\mathbf{R}_{ES}$ is position vector of the probe when leaving the earth ( measured from the sun.)
$\mathbf{R}_{MS}$ is position vector of the probe when crossing the orbital path of Mars (measured from the sun.) ( does not contribute to angular momentum as it is parallel to $R_{M}$)

$$mv_{2E} R_E = mv_{\parallel} R_M$$

where $R_{ES} = R_E$, $R_{MS} = R_M$

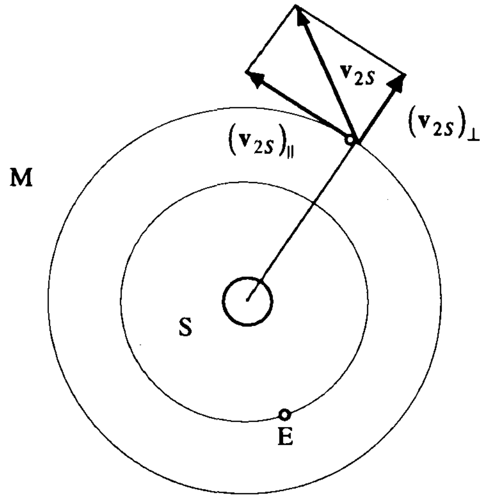



Fig 16.6

$$v_{||}\, R_M = ( \,|\, \mathbf{v}_{2E} + \mathbf{v}_{ES} \,|\, R_E$$

$$v_{||} = ( v_{2E} + v_{ES})$$

$$v_{||} = ( v_{2E} + v_{ES})\, r \qquad (6)$$

Substitute r = 2/3 and note that $\mathbf{v}_{2E}$ , $\mathbf{v}_{ES}$ are in the same direction.

$$v_{||} = \frac{2}{3}\,( v_{2E} + 30) \quad \textbf{Ans}$$

From (5) $$\frac{v_{2S}^2}{2} = \frac{1}{2}(v_{\|}^2 + _{\perp}^2) + GM_S\left(\frac{1}{R_{ES}} - \frac{1}{R_{MS}}\right)$$

Substitute $$GM_S = v_{ES}^2 R_{ES}$$

$$v_{2S}^2 = \frac{1}{2}(v_{\|}^2 + v_{\perp}^2) + 2v_{ES}^2(1-r)$$

$$v_{\perp}^2 = v_{2S}^2 - v_{\|}^2 - 2v_{ES}^2(1-r) \qquad (7)$$

Substitute $v_{||} = ( v_{2E} + v_{ES})\, r$ from (6) and

$v_{2S} = v_{2E} + v_{ES}$ from (4) to obtain

$$v_{\perp}^2 = (\mathbf{v}_{2E} + \mathbf{v}_{ES})^2 + (v_{2E} + v_{ES})^2 r^2 - 2v_{ES}^2(1-r)$$

$$= (v_{2E} + v_{ES})^2(1-r^2) - 2v_{ES}^2(1-r)$$

$$v_{\perp} = \sqrt{(v_{2E} + v_{ES})^2(1-r^2) - 2v_{ES}^2(1-r)} \quad \textbf{Ans} \qquad (8)$$

3.3 The analysis under 3.1 gives the minimum velocity for lthe probe to escape from the gravitational pull of Mars in the form

$$v_{2M} = v_{MS}\left(\sqrt{2}-1\right)$$

having direction tangent to the path of Mars around the sun or in the same direction with $\mathbf{v}_{MS}$ (the velocity of Mars relative to the sun) at the time of launching.

The role of Mars as stated in the problem is to step up the velocity of the probe to the escaping velocity from the solar system as a means of saving energy.

In Mars' frame of reference, the total energy of the probe is constant.However when viewed fromn the the sun's frame of reference, the encounter between the probe and the gravitational field of Mars amounts to a collision between the probe and the gravitational field of Mars.

Since the collission is elastic in nature, (gravitational field of Mars is conservative field), the magnitude of the velocity of the probe relative to Mars prior to moving in the field of Mars is equal to lthe magnitude of the velocity of the probe (also relative to Mars) after leavign the the gravitational field of Mars.

Let the components of the velocity of the probe parallel and normal to the orbital path of Mars be $(\mathbf{v}_{2M})_{\parallel}$ and $(\mathbf{v}_{2M})_{\perp}$ relative to Mars respectively. ( measured in Mars's frame of reference)

Hence $(\mathbf{v}_{2M})_{\parallel} = (\mathbf{v}_{2S})_{\parallel} - \mathbf{v}_{MS}$

$$(\mathbf{v}_{2M})_{\perp} = (\mathbf{v}_{2S})_{\perp}$$

where $(\mathbf{v}_{2S})_{\parallel}$ and $(\mathbf{v}_{2S})_{\perp}$ are components of the velocity of the probe resolved in the directions parallel and normal to the orbital path of Mars (relative to the sun) after the probe leaving the gravitational field of Mars.

Let $\mathbf{v}_{2M}$ be resultant velocity of the probe measured in Mars's frame of reference, prior moving into the gravitational field of Mars.

Hence
$$\mathbf{v}_{2M} = \sqrt{(\mathbf{v}_{2M})_{\parallel}^2 + (\mathbf{v}_{2M})_{\perp}^2}$$

$$\mathbf{v}_{2M} = \sqrt{(\mathbf{v}_{2S} - \mathbf{v}_{MS})^2 + (\mathbf{v}_{2M})_{\perp}^2} \qquad (10)$$

From 3.1 minimum velocity for the probe to escape from the gravitational pull of Mars is

$$(\mathbf{v}_{2M})_{\parallel} = \mathbf{v}_{MS}(\sqrt{2} - 1) \qquad (11)$$

with both velocities in the same direction (11)

Substitute $(v_{2M})_{\parallel}$ from equation (6) .

$(v_{2M})_{\perp}$ from equation (8)

$v_{2M}$ from equation (11) into (10) to obtain,

$$v^2_{MS}(\sqrt{2}-1)^2 = [(v_{2E}+v_{ES})r - v_{MS}]^2 + (v_{2E}+v_{ES})^2(1-r)^2 - 2v^2_{ES}(1-r)$$

To determine $v_{2E}$ from the above equation , we need to know $v_{MS}$
However from the energy equation we have,

$$v_{MS} = \sqrt{\frac{GM_S}{R_{MS}}}$$

Substitute $GM_S$ from (2) $v_{MS} = \sqrt{\frac{R_{ES}}{R_{MS}}}\ v_{ES}$

$$= \sqrt{r}\ v_{ES}$$

$$rv_{ES}^2(3-2\sqrt{2}) =$$
$$(v_{2E}+v_{ES})^2r^2 - 2v_{MS}(v_{2E}+v_{ES})r + v_{MS}^2 + (v_{2E}+v_{ES})^2(1-r^2) - 2v_{ES}^2(1-r)$$

$$= (v_{2E}+v_{ES})^2 - 2\sqrt{r}v_{ES}(v_{2E}+v_{ES})r + rv_{ES}^2 - 2v_{ES}^2(1-r)$$

$$(v_{2E}+v_{ES})^2 - 2r\sqrt{r}v_{ES}(v_{2E}+v_{ES}) + (2\sqrt{2}r-2)v_{ES}^2 = 0$$

$$v_{2E} = v_{ES}[r\sqrt{r} - 1 + \sqrt{r^3+2-2\sqrt{2}.r}]$$

Substitute $v_{ES}$= 30 km/s and r = 2/3

$$v_{2E} = 5.5 \text{ km/s}$$

The probe has to be launched with a velocity of 5.5 km/s (relative to the earth) in the direction along the orbital path of the earth about the sun. **Ans**

3.4 The ratio between the differece in energy used in schemes 1 and 2 and energy used in scheme 1 is

$$\frac{E_1 - E_2}{E_1} = \frac{v_{1E}^2 - v_{2E}^2}{v_{1E}^2}$$

where $E_1$ and $E_2$ are energies used in schemes 1 and 2 respectively.

Substitute $v_{1E}$ = 12.3 km/s

$v_{2E}$ = 5.5 km/s

Ratio of energy saved = 0.80 or 80% **Ans**

# Experiment

## Problem 1

### Investigation of acceleration and deceleration of a brass disc driven by an AC electric motor.

From the measured times of half turns, plot angle $\theta$, angular speed $\dot{\theta}$, and angular acceleration $\ddot{\theta}$ as a function of time. Determine torque and power of the motor as a function of angular velocity.

Instruments provided

- -AC motor with switch and brass disc
- -Induction motor
- -Multi-channel stop-watch (computer)

### Instruction.

The induction sensor senses the iron pegs mounted on the disc, when they are closers than 0.5 mm and sends a signal to the strop-watch. The stop-watch is programmed on a computer so that it registers the time at which the sensor senses the approaching peg and stores it in memory. The stop-watch is run by simple numerical commands, i.e. by pressisng one the following numbers:

Key 5 -Preparation for measurements. The measurement does not start immediately. The stop-watch waits until the number of measurements is specified, that is, the number of successive detection of the pegs:

Key 3 - 30 measurements

Key 6 - 60 measurements

-Either of these commands starts the measurement. When a measurement is completed, the computer displays the results in graphic form. The vertical axis represetns the length of the interval vetween detection of the pegs and horizontal axis is the number of the intervals.

Key 7 - Display results in numerical form.

(The first column is the number of times a peg has passed the detector, the second column is the the time elapsed from the beginning of the measurement and the third column is the length of time interval between the detection of the two pegs.)

-In the case of 60 measurements:

Key 8 -Display the first page of the table.

Key 2 -Display the second page of the table.

Key 4 -Display the results graphically.

A measurment can be interruppted before the prescribed number of measurements by pressing any key and giving the disc another half turn.
The motor running on 25 V DC can be operated by a switch on the mounting base. It may be necessary at times to give a disc a light push or tap the base to start the disc.

The total moment of inertia iof all rotating parts is $(14.0 \pm 0.5) \times 10^{-6}$ km m$^2$

# Solution

Perform the experiment in accordance with the instruction given in the problem, in order to measure time taken by the disc to undergo one half turn and give command to the computer to record and dispaly the experimental results.

Let $\theta$ angle of rotation of the disc
i order of half turns (i.e. first , second, third... half turn )
$t_i+1$ time for the disc to undergo (i +1 )/2 turns
$t_i$ time for the disc to undergo i/2 turns
$\Delta t$ time interval for the disc to rotate each half turn recorded

$t'$ point in time for instantaneous angular velocity and $t_i' = \dfrac{t_{i+1} + t_i}{2}$

or $\dot{\theta}_i = \dot{\theta}_i(t_i') = \dfrac{\pi}{(t_{i+1} - t_i)}$

| i | t(ms) | Δt (ms) | θ(rad) | t′(ms) | $\dot{\theta}$ | $\ddot{\theta}$ |
|---|---|---|---|---|---|---|
| 1 | 0 | | 0 | | | |
| | | | | 272 | 5.78 | |
| 2 | 543.9 | 543.9 | 3.14 | | | |
| | | | | 758.7 | 7.31 | |
| 3 | 973.5 | 429.6 | 6.28 | | | 3.38 |
| | | | | 1,156. | 8.6 | |
| 4 | 1,339 | 365.5 | 9.42 | | | |
| | | | | 1,499.9 | 9.76 | |
| 5 | 1,660.8 | 321.8 | 12.57 | | | 5.04 |
| | | | | 1,798.6 | 11.4 | |
| 6 | 1,936.3 | 275.5 | 15.71 | | | |
| | | | | 2,057.1 | 13.01 | |
| 7 | 2,177.8 | 241.5 | 18.85 | | | 5.96 |
| | | | | 2,287.2 | 14.36 | |
| 8 | 2,396.6 | 218.8 | 21.99 | | | |
| | | | | 2,498.1 | 15.48 | |

| i | t(ms) | Δt (ms) | θ(rad) | t′(ms) | $\dot{\theta}$ | $\ddot{\theta}$ |
|---|---|---|---|---|---|---|
| 9 | 2,599.6 | 203 | 25.13 | | | 9.4 |
| | | | | 2,689.6 | 17.46 | |
| 10 | 2,799.5 | 179.9 | 28.87 | | | |
| | | | | 2,859.4 | 19.66 | |
| 11 | 2,939.3 | 159.8 | 31.42 | | | 18.22 |
| | | | | 3,008.6 | 22.65 | |
| 12 | 3,078 | 138.7 | 34.56 | | | |
| | | | | 3,139.9 | 25.38 | |
| 13 | 3,201.8 | 123.8 | 37.7 | | | 25.46 |
| | | | | 3,256.6 | 28.66 | |
| 14 | 3,311.4 | 109.6 | 40.84 | | | |
| | | | | 3,361.8 | 31.2 | |
| 15 | 3,412.1 | 100.7 | 43.98 | | | 23.4 |
| | | | | 3,458.2 | 34.11 | |
| 16 | 3,504.2 | 92.1 | 47.12 | | | |
| | | | | 3,457.8 | 36.07 | |
| 17 | 3,591.3 | 87.1 | 50.27 | | | 21.72 |
| | | | | 3,632.4 | 38.27 | |
| 18 | 3,673.4 | 82.1 | 53.41 | | | |
| | | | | 3,713.5 | 39.22 | |
| 19 | 3,753.5 | 80.1 | 56.55 | | | 4.76 |
| | | | | 3,792.8 | 39.97 | |
| 20 | 3,832.1 | 78.6 | 59.69 | | | |
| | | | | 3,872.4 | 39.03 | |
| 21 | 3,912.6 | 80.5 | 62.83 | | | -1.69 |
| | | | | 3,952.7 | 39.22 | |
| 22 | 3,992.7 | 80.1 | 56.55 | | | |
| | | | | 3,792.8 | 39.97 | |
| 23 | 4,072.8 | 80.1 | 65.97 | | | - 0.77 |
| | | | | 4,032.8 | 39.22 | |
| 24 | 4,152 | 79.2 | 72.26 | | | |
| | | | | 4,192.3 | 39.03 | |
| 25 | 4,232.5 | 80.5 | 75.4 | | | - 0.15 |
| | | | | 4,272.4 | 39.42 | |
| 26 | 4,312.2 | 79.7 | 78.54 | | | |

( **Note** The results are provided by the host country prior to the problems being approved for the competition.)

From the investigation of time duration for the disc to complete one half turn at different t′ until the disc reaches the constant angular speed, it is concluded that the turning point does not coincide with its centre of mass.The systematic error in the calculation of angular acceleration is removed when the calculation is based on one complete turn i.e. $$\ddot{\theta}(t_i^{//}) = \frac{\Delta\theta_i}{\Delta t_i} = \frac{\Delta\omega_i}{\Delta t_i}$$

where $\Delta t_i = t_{2i+2} - t_{2i}$

$$\omega = \dot{\theta}$$

$$\Delta\omega_i = \frac{2\pi}{t_{2i+3} - t_{2i+1}} - \frac{2\pi}{t_{2i+1} - t_{2i-1}}$$

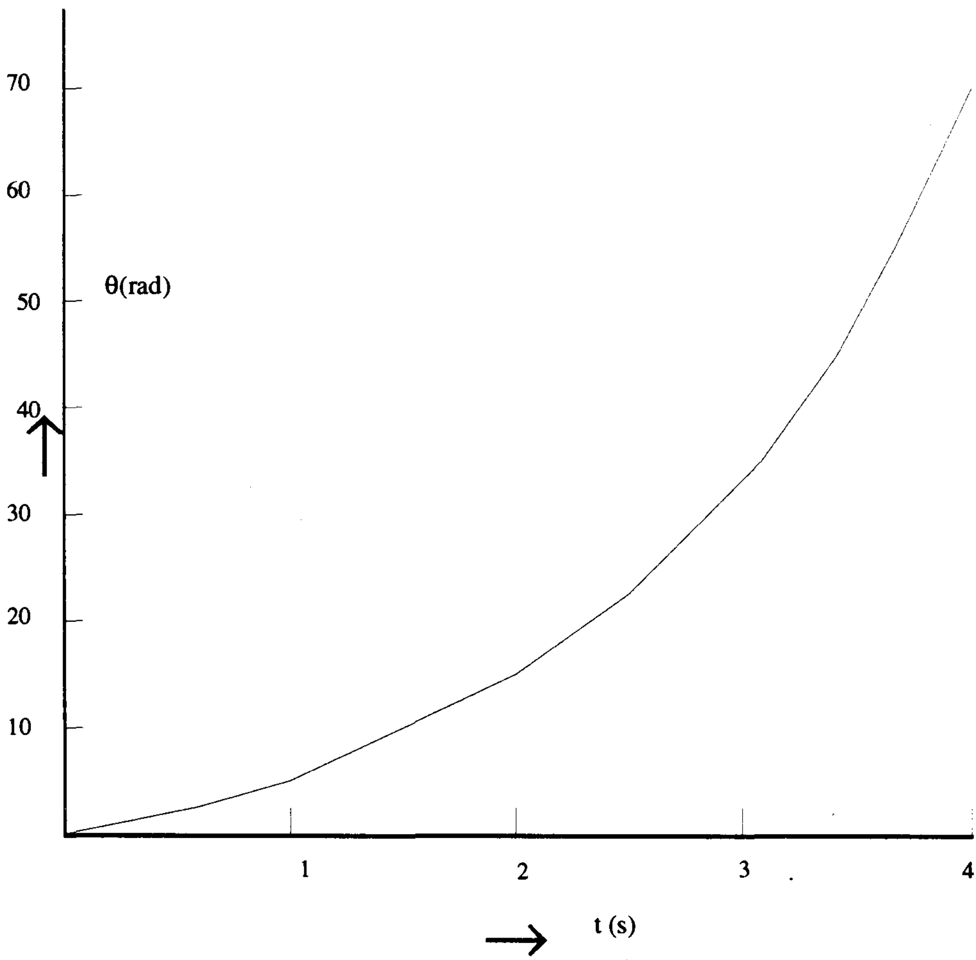



Fig.16.7 Graph of angle θ.as a function of time t .

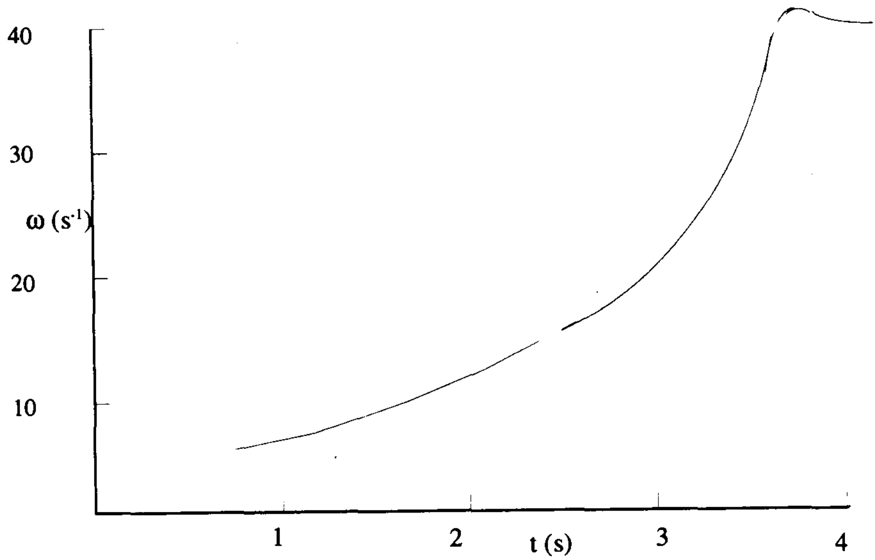



Fig. 16.8 a. Graph of angular speed ( $\omega = \dot{\theta}$ ) as a function of time

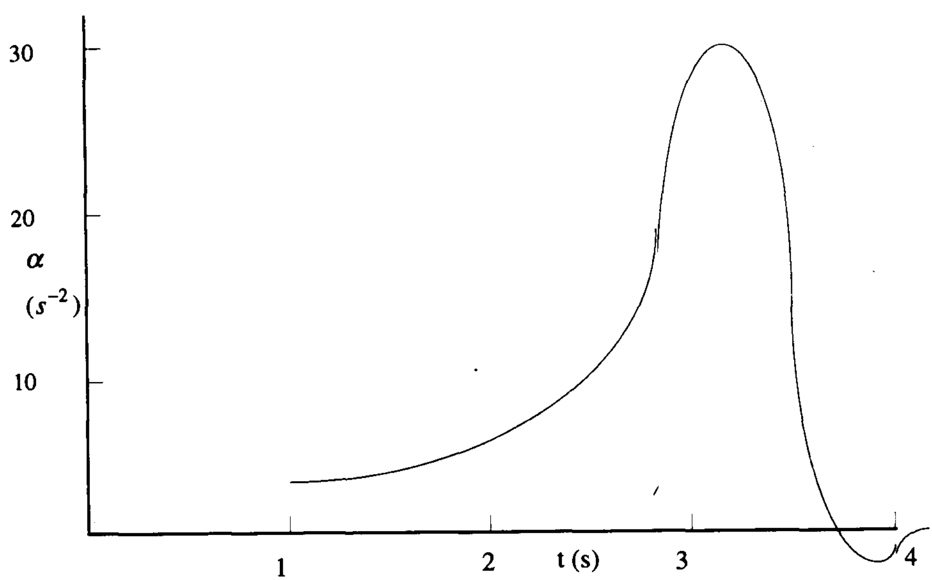



Fig. 16.8 b Graph of angular acceleration ( $\alpha = \dot{\omega} = \ddot{\theta}$ ) against time

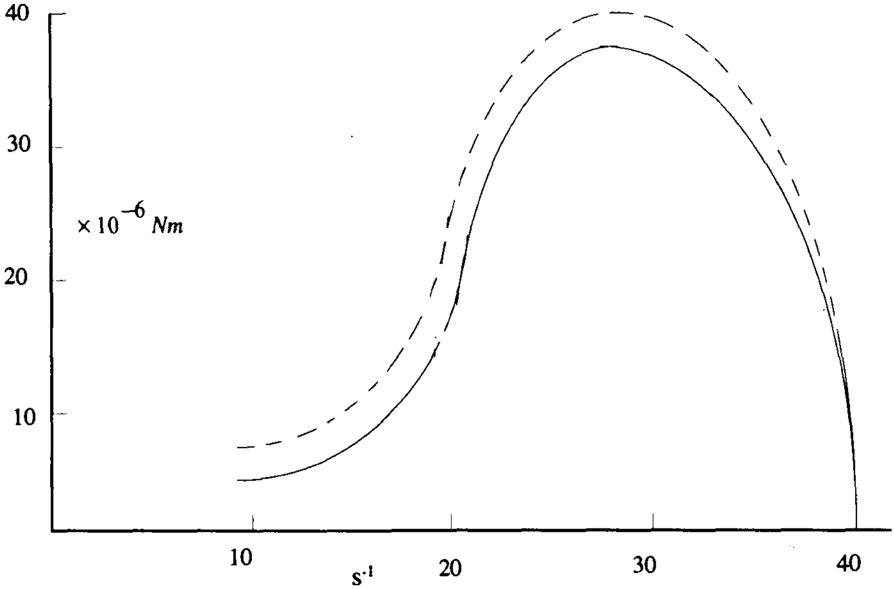



Fig. 16.9 Graph of resultant torque( thick line) and total torque (broken line) as a function of angular acceleration

Torque $\tau$ and power P can be calculated from the formula

$$|\tau| = I\ddot{\theta}$$

and $$P(t) = \tau\dot{\theta}$$

where I is moment of inertia $= (14.10+ 0.5)\times 10^{-6}$ km m$^2$

The values of P(t) at various times can be calculated from the values of $\omega$, $\alpha$ and $\tau$ from graphs in Figs.16.7 and 16.8.

In order to determine torque and total power of the motor as a function of time, we need to know torque due to friction and associated power loss to be added to resultant torque and resultant power. This can be done by switching off the motor and measure angular speed during the time interval of retardation due to friction .(See Fig. 16.10)

Plot a graph of angular speed as a funtion of time.
The slope of the graph at any given point in time is angular deceleration.

The calculated value of torque due to friction is $(3.1 + 0.3)\times 10^{-5}$ Nm

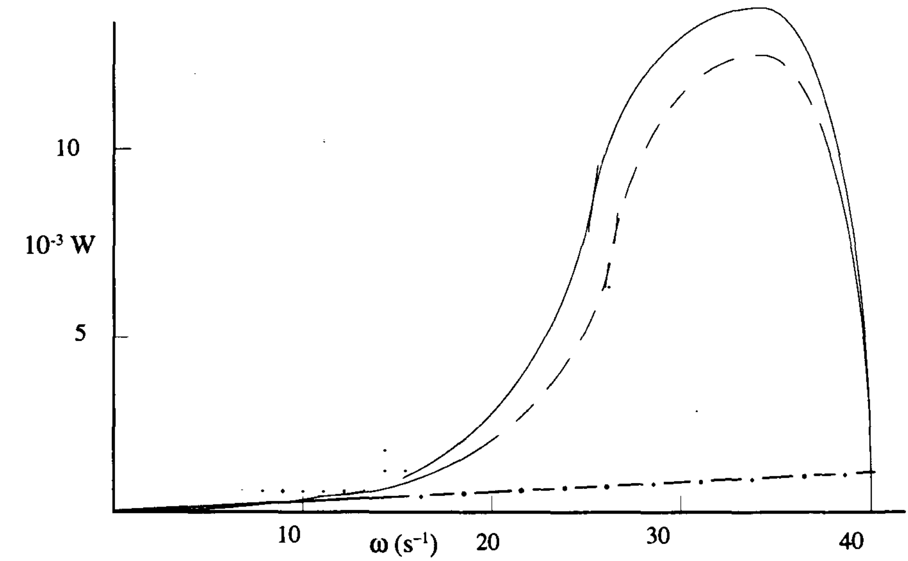



Fig.1610 Graphs of resultant power (broken line), power loss (dotted line) and total power (thick line) as a functionof angular speed

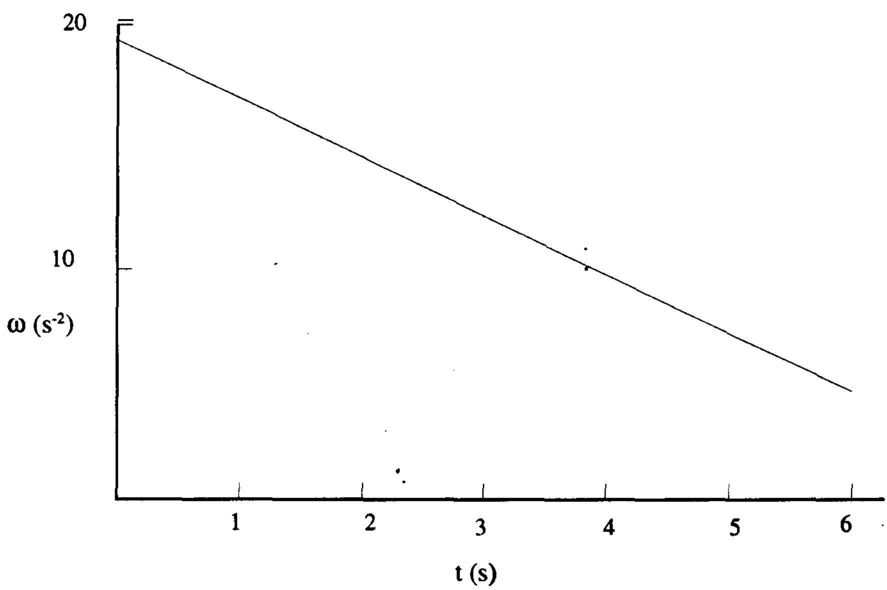



Fig.16.11 Graph of angular speed as a function of time during the time interval of negative values of angular acceleration

# Problem 2

Locate the position of the centres and determine the orientation of a number of identical magnets hidden in the black painted box. A diagram of one such magnet is given in Fig. 16.11. The dimensions of each magnet are 50mm $\times$ 20mm $\times$ 8mm, and of the box, 50cm $\times$ 31cm $\times$ 4.0 cm

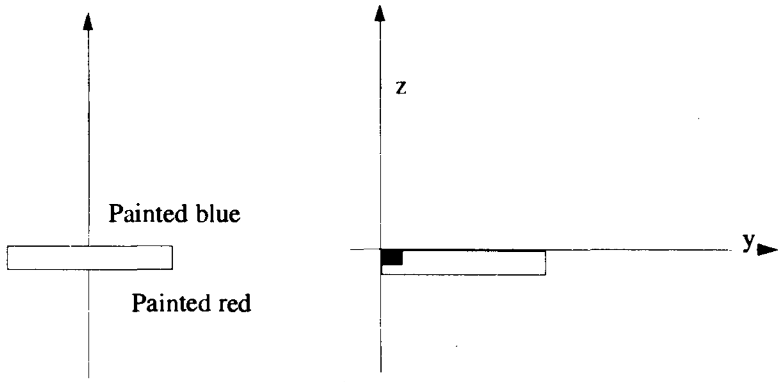



Fig. 16.12

Distances x y and z are measured from the angle marked in black.
Determine the z-component of the magnetic induction vector B in x y plane at z = 0 by calibrating the measuring apparatus beforehand.
Find also the largest magnetic induction vector B obtained from the magnet supplied.

**Apparatus**

1. Permanent magnet given is identical to the hidden magnets in the box.
2. Induction coil, 1400 turns, R = 230
3. Field generating coils, 8800 turns R = 990 ; 2 units
4. Black painted foam box containing hidden magnets
5. Voltmeter ( range 1 V , 3V and 10V recommended)
6. Electronic circuit ( recommended supply voltage 24 V)
7. Ammeter
8. Variable resistor 3.3 k
9. Variable stabilized power supply 0-25 V with current limiter
10. Four connecting wires
11. Supporting plate with fixing holes
12. Rubber bands, multi-purpose (e.g. for coil fixing)
13. Tooth picks
14. Ruler
15. Threads

**Instructions**

Only non-destructive methods for the magnet search are allowed. The final report should include results, formulas, graphs and diagrams. The diagrams should be used instead of comments on the methods used whenever possible.
The proper use of the induced voltage measuring system is explained in reference to Fig. 16.2

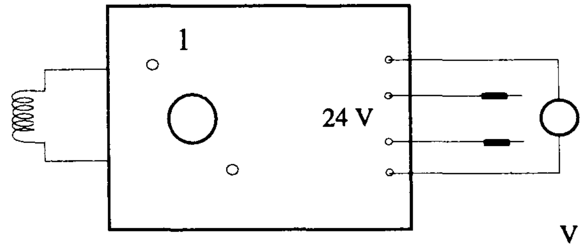



Fig 16.13

This device is capable of responding to the magnetic field. The peak voltage is proportional to the change of the magnetic flux through the coil.

The variable stabilized power supply is switched ON (1) or OFF (0) by the lower left push button. By knob U , the output voltage is increased through the clockwise rotation. The recommended voltage is 24 V . This can be easily accomplished by means of pushing the toggle switch to the 12-25 V position. With this instrument either the output voltage V or output current I is measured, with respect to the position of the corresponding toggle switch(V,A). However, to obtain the output voltage, the upper right switch should be in the ' Vklop' position. By the knob (I), the oputpu t current is limited below the present value. When rotated clockwise the power supply can provide 1.5 A at most.

Given permeabilty of free space $\mu_o = 1.2 \times 10^{-6}$ V.s/( A.m)

## Solution

The directions and orientation of the magnets in the box can be easily determined by suspending a spare magnet (outside the box) and use it as a probe.
Move the suspending magnet over the upper surface of the box to be explored.

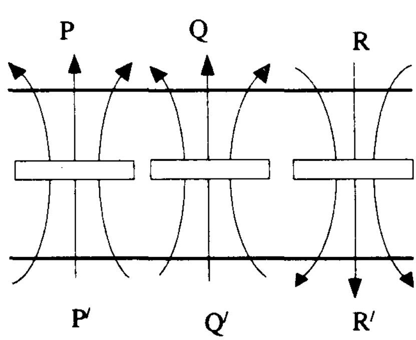



Fig. 16.14

Three areas of strong forces are noted when the probing magnet is in the horizontal position i.e. when B in the box is paralell to z axis of the probing magnet, suggesting that there may be three hidden magnets. P Q · – R as shown in Fig. 16.13 . Aat P and Q The direction of the magnetic induction points out of the surface while at R is directed into the surface. We may hypothesize that there might be three magnets ( identical to the probe) hidden inside the box.

However upon investigating the direction of the induction field on the opposite side of the box, we discover that the direction of the induction field points inward into the box at P/ and points outward at Q/ and R/. The result of the finding contradicts the hypothesis.

The black box is thus likely to consist of two magnets A and B arranged as shown in Fig. 16.4.

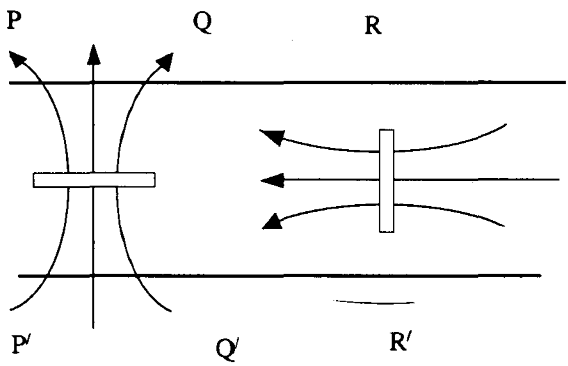



Fig.16.15

In order to identify z axis, we use the field measuring circuit provided and depicted in Fig.16.15
However before we can use the field measuring circiut, it is necessary ot calibrate the instrument first..
Calibration of the field measuring device can be done by measuring known magnetic field expressed in volts or ampere.After the calibraation, we may use the measuring device to measure unknown field in the units of votalge or ampere.

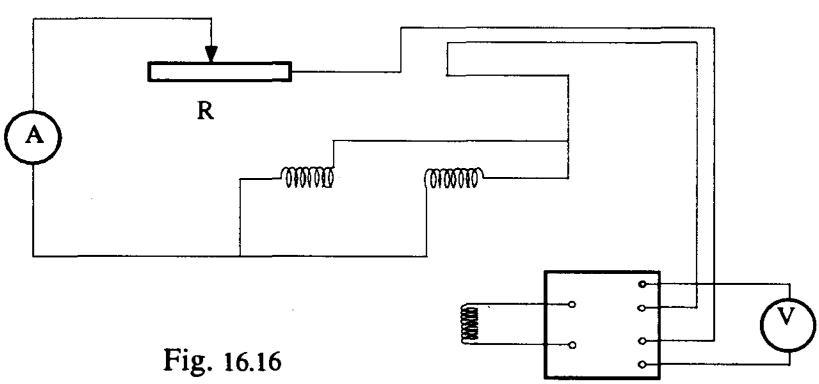



Fig. 16.16

The magnetic induction field between two identical induction coils placed on the same axis is given by

$$B = \frac{\mu_o NI}{(2l + d)}$$

where N number of turns in each induction coil
d distance between the tow coils
l length of each coil
I current through each coil

Let $U_{MAX}$ be maximum voltage recorded by the voltmeter in P (probe) circuit.
Place probe P (searching coil) in the magnetic field between two coils and then remove it to the area where the mganetic induction field is zero. Read maximum votage $U_{MAX}$ from the voltmeter in the probe circuit.

Record the values of B and V in the table below.

| B (tesla) | V (volt) |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |

From the graph, we obtain the relation between B and V in the form

$$B = 0.20 \ V$$

Repeat the experiment, using the sample magnet to find induced voltage which is directly proportional to $B_z$ as a function of z.

Record the experimental values in the table

| V(Induced voltage) | B(Magnetic field) | z (mm) |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |

Plot a graph of B as a function of z.
The graph obtained is similar to that shown in Fig. 16.16

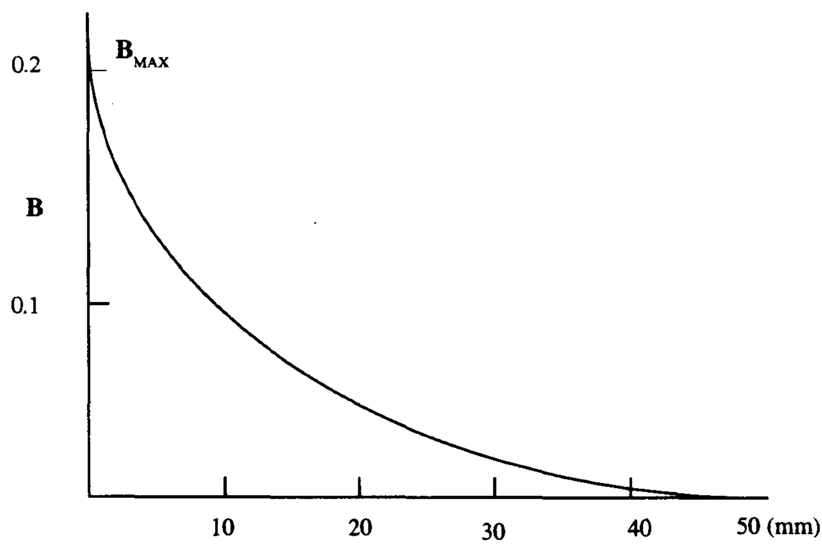



Fig. 16.17 Graph of B as a function of z

From the measurement of magnetic induction field in region P at various distances, we can tell the exact position of magnet A . Likewise, by measuring magnetic induction intensity at vaious distances normal to the axis of B, we can also tell the exact postion of B inside the box.

**Conclusion** There are two magnets inside the black box arranged as shown in Fig. 16.7 and each manet is set from the side of the box at distances determined as described.

# International Physics Olympiad XVII

# 1986

# London, Great Britain

## Theory

### Problem 1

A plane mono-chromatic light wave, with wavelength $\lambda$, and frequency $\nu$, is incident normally on two identical slits, L and M separated by distance d as shown in Fig. 17.1 below. The light wave emerging from each slit in a direction making angle $\theta$ with the principal axis at time t is given by:

$$y = a\cos\frac{2\pi}{\lambda}(x - \nu t)$$

where a is the amplitude for both waves, (assuming x is much larger than d)

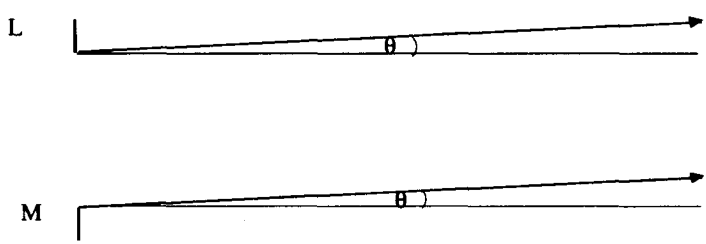



Fig. 17.1

1.1 In the direction making angle $\theta$ with the principal axis, the amplitude of the resultant wave due to the two waves is equal to A, obtained by adding two vectors each of which has magnitude a and direction depending on angle $\theta$. Verify geometrically (using the vector diagram) that

$$A = 2a\cos\beta$$

where $\theta = \frac{2\pi}{\lambda} d \sin\theta$

1.2 The double slits are replaced by a diffraction grating with N equally spaced slits and adjacent slits being separated by a dinstance d. Use the vector method of adding amplitudes to show that the vector amplitudes each of which has magnitude a form part of a regular polygon with vertices on a circle of radius R given by

$$R = \frac{a}{2\sin\beta}$$

Deduce that the resultant amplitude is $\frac{a\sin N\beta}{\sin\beta}$

and determine the resultant phase or phase difference relative to tha t of the light from the slit at the edge of the grating.

1.3 Sketch sin Nβ and $\frac{1}{\sin\beta}$ in the same graph as a function of β

1.4 On a separate graph, show variation of the intensity of the resultant wave as a function of β.

1.5 Show that the number of principal maxima cannot exceed $\frac{2d}{\lambda}+1$.

1.6 Determine maximum a nd minimum intensities that are nearest satellites of the principal maxima of 1st order.

1.7 Show that two wavelengths λ and $\lambda + \Delta\lambda$ where $\Delta\lambda <<< \lambda$, produce principal maxima with an angular separation given by

$$\Delta\theta = \frac{n\Delta\lambda}{d\cos\theta} \qquad \text{where n} = 0, \pm 1, \pm 2, \pm 3....$$

Calculate the angular separation for the sodium D lines for which

$$\begin{aligned} \lambda &= 589.0 \text{ nm} \\ \lambda + \Delta\lambda &= 589.6 \text{ nm} \\ \text{at} \quad n &= 2 \\ \text{and} \quad d &= 1.2 \times 10^{-6} \text{ m} \end{aligned}$$

[Reminder: $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$]

## Solution

If phase of wave from slit M is given by

$$\xi = 2\pi(\frac{x}{\lambda} - vt)$$

then phase from slit L is

$$\xi + \phi = 2\pi\frac{d\sin\theta}{\lambda}$$

Resultant wave is given by

$$a\cos(\xi + \phi) + a\cos\xi \quad = \quad 2a\cos(\xi + \frac{\phi}{2})\cos\xi$$

Let $\beta = \frac{\phi}{2}$

Resultant wave $= 2\,a\cos(\xi + \beta)\ \cos\beta$

The equation above is the equation of light with amplitude 2acos $\beta$ and phase difference of $\beta$ or $\frac{\pi}{\lambda}d\sin\theta$ relative to that of light from slit M.

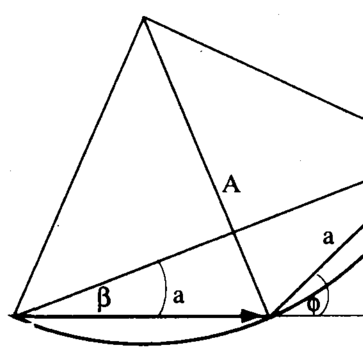



Fig. 17.2

Addition of 2 vectors each of magnitude a and phase difference of $\phi$.

The resultant vector makes angle $\beta$ with the first vector,

where $\beta = \frac{\phi}{2}$ and $\frac{\pi}{\lambda}d\sin\theta$

Resultant amplitude $A = 2a\cos\beta$ **Ans**

1.2 When a grating i s used in place of double slits, each slit in the grating produces wave of amplitude a and phase differecne of ($\phi = 2\beta$) relative to the prceding slit.

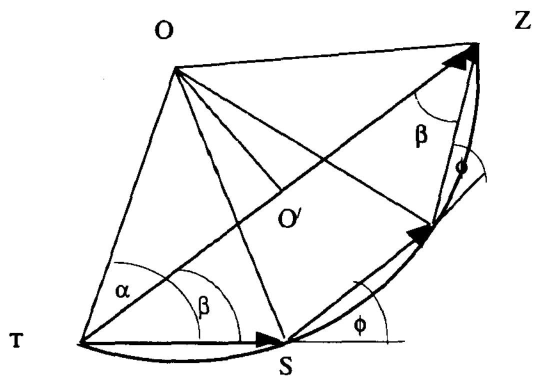



Fig. 17.3

Fig.17.3 above presents the diagram of vectorial addition. Each vector has magnitude a and oreinted along the circumference of the circle. Each vector beginning from the second one makes angle with the preceding vector, and so on. Every vector is a base of the isoceles with the vertice angle at the centre O of the circle.

Let the angle at the base of the isoceles be $\alpha$ (See the first traingle.)
From the geometry, vertice angle $\angle TOS + 2\alpha = \quad = \pi = \phi + 2\alpha$
where $\phi$ is the vertice angle of each isoceles.

From the geometry, N vectors from N -1 isoceles. Hence the angle which is subtended by the resultant vector at O is $(N-1)\phi$.

Consider the first and second isoceles, the angle on thecircumference subtended by chord a is $\frac{\phi}{2}$.
At T on the circumference, the number of angles subtended by N chords is N - 1. Hence the angle which the resultant vector makes with the first vector (phase difference) is

$$(N-1)\,\frac{\phi}{2} \text{ or } (N-1)\beta \quad \textbf{Ans}$$

Let R be radius of the circle,

$$\frac{a}{2} = R\sin\frac{\phi}{2} = R\sin\beta$$

$$R = \frac{a}{2\sin\beta} \qquad \textbf{Ans}$$

Consider triangle TZO,

$$\frac{TZ}{2} = R\sin(N-1)\frac{\phi}{2} = \frac{a}{2\sin\beta}\sin(N-1)\beta$$

Resultant amplitude TZ $= \dfrac{a\sin(N-1)\beta}{\sin\beta}$
1.3

Since N is much larger than 1, N - 1 ≈ N

$$TZ = \frac{a\sin N\beta}{\sin\beta} \qquad \textbf{Ans}$$

1.3

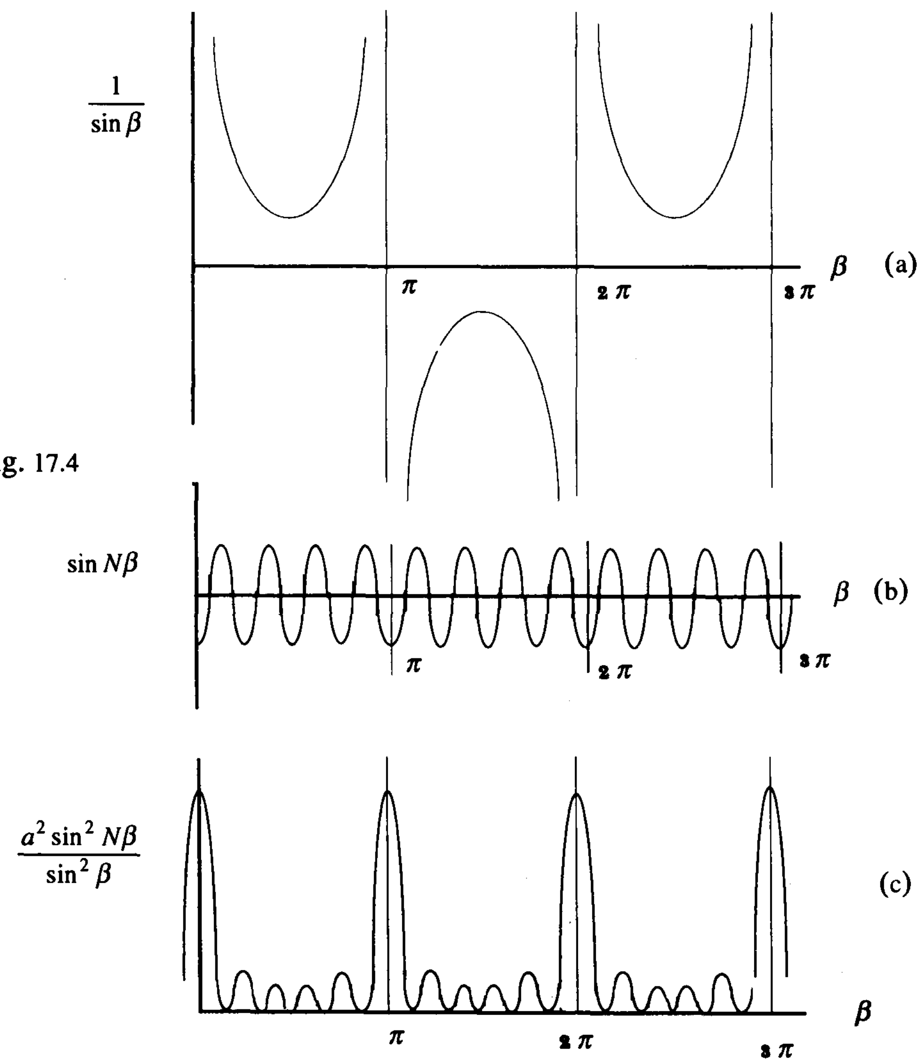



Fig. 17.4

1.4 When $\beta$ approaches $0, \pm\pi, \pm 2\pi, \pm 3\pi.....$

$\frac{1}{\sin\beta}$ approaches $\infty$ while $\sin N\beta$ approaches $0$.

In order to find the value of $\frac{\sin N\beta}{\sin\beta}$ when $\beta = p\pi$, where $p = 0, \pm 1, \pm 2, \pm 3......$

We evaluate $$\lim_{\beta\to 0}\frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to 0}\frac{N\beta}{\beta} = N$$

Principal maxima $I_{MAX} = a^2N^2$

which occurs when $\beta = p\pi$ ( p is integer) **Ans**

1.5 From 1.4 $\beta = p\pi$

p is maximum when $\beta = \frac{\pi}{\lambda} d\sin\theta$

which takes place when $\theta = \frac{\pi}{2}$

Maximum value of p is $\frac{d}{\lambda}$

The number of principal maxima( zeroth order and those on thetwo sides) may not exceed

$\frac{2d}{\lambda} + 1$ **Ans**

1.6 From Fig. 17.4 b, minimum intensity on either side of principal maxima occurs when

$$\sin^2 N\beta = 0$$

$$N\beta = \pm\pi$$

$$\beta = \pm\frac{\pi}{N}$$

$$I_{MIN} = \left(\frac{\sin\pi}{\sin\frac{\pi}{N}}\right)^2$$

$$= 0$$ **Ans**

For maximum intensity which is the satellite of the first order principal maxima to occur the condition is $\sin^2 N\beta = 1$

$$N\beta = \frac{3\pi}{2}$$

$$\beta = \frac{3\pi}{2N}.$$

For maximum intensity which is the satellite of the first order principal maxima is given by

$$I_{MAX} = a^2 \left( \frac{\sin\frac{3\pi}{2}}{\sin\frac{3\pi}{2N}} \right)^2$$

$$\approx a^2 \left( \frac{2N}{3\pi} \right)^2$$

$$= 0.045\ a^2 N^2$$ **Ans**

1.7 Condition for principal maxima

$$\beta = n\pi \qquad \text{where} \qquad n = 0, \pm 1, \pm 2, \pm 3....$$

$$\frac{\pi}{\lambda} d \sin\theta = n\pi$$

Take differentiation with respect to λ

$$d\cos\theta\Delta\theta = n\Delta\lambda$$

$$\Delta\theta = \frac{n\Delta\lambda}{d\cos\theta}$$

$$= \frac{n\Delta\lambda}{d(1-\sin^2\theta)}$$

**Substitute** $\lambda = 589.0$ nm $\quad \lambda + \Delta\lambda = 589.6$ nm

$n = 2 \quad d = 1.2 \times 10^{-6}$ m

$$\Delta\theta = \frac{2 \times 0.6 \times 10^{-9}}{1.2 \times 10^{-6} \sqrt{1 - (\frac{n\lambda}{d})^2}}$$

$$= 5.2 \times 10^{-3} \text{ rad}$$

or $= 0.30^{\circ}$ **Ans**

# Problem 2

Early in the 20th century, the earth was thought to be a sphere of radius R consisting of a homogeneous isotropic solid mantle down to radius $R_C$, while the core region whithin radius $R_C$ contained a liquid .

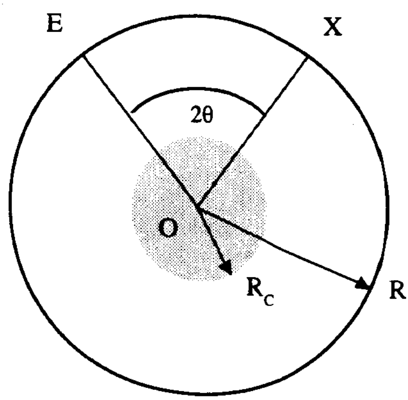



Fig. 17.5

This model can be used to describe seismic waves due to earthquake. ( seismic wave ) The velocities of longitudinal and transversal seismic waves known as P and S waves respectively, are constant.

Let $V_P$ and $V_S$ be velocities of P and S waves in the mantle respectively.

In the mantle $V_P$ and $V_S$ are constant.
In the core region, the velocity of longitudinal wave $V_{CP}$ is constant ,while transversal wave cannot be propagated in the core region.

An earthquake at E on the surface of the earth generates seismic waves which travel through the earth and are recorded by an observer on the earth surface by means of a string of seismometers around the earth. At point X on the surface of the earth the seismic waves or wave are detected. The angular separation between E and X is given by 2θ, (See Fig. 12.6)

$$2\theta = \angle EOX$$

and O is the centre of the earth.

2.1 Show that the seismic waves that travel through the mantle in a straight line will arrive at X at time t (travel time after the earthquake), is given by

$$t = \frac{2R\sin\theta}{v}$$

$$\theta \le \cos^{-1}(\frac{R_C}{R})$$

and $v = v_P$ for P wave
$v = v_S$ for S wave

2.2 For some positions of X giving $\theta > \cos^{-1}\frac{R_C}{R}$ seismic P waves arrive at the observing station after undergoing two refractions a t the mantle-core interface. Draw the path of such seismic P wave.

Also derive a relation between θ and angle i representing the angle of incidence at the mantle-core interface for P waves.

2.3 Given the following data:

$R = 6.370$ km
$R_C = 3.470$ km
$V_P = 10.85$ km/s
$V_S = 6.31$ km/s
$V_{CP} = 9.032$ km/s

and the results obtained in 2.2

2.3.1 Plot a graph of against i, and comment on the consequence of the form of this graph for observers stationed at different points on the durface of the earth.

2.3.2 Sketch a graph showing variation of the travelling time taken by P and S waves as a function of θ in the range of 0 and 90°.

2.4 After an earthquake, an observer measures the time delay between the arrival of S wave, followed by P wave, as 2 minutes and 11 seconds. Deduce angular separation of the center of the earthquake from the observer using the data provided in 2.3.

2.5 The observer in the previous measurement notices that some time after the arrival of the P and S waves, there are two further recordings on the seismometer separated by a time interval of 6 minutes and 37 seconds. Explain this result and verify that it is indeed associated with the angular separation determined in the previous section.

# Solution

2.1

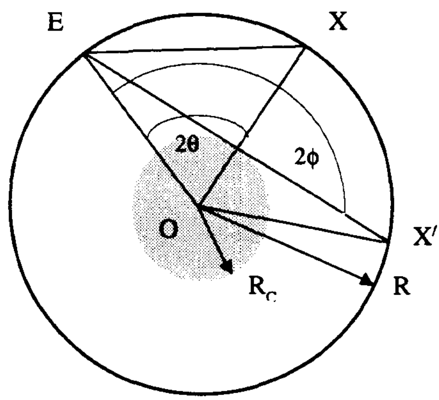



Fig. 17.6

From the above figure, distance EX $= 2R\sin\theta$

minimum travelling time $t = \dfrac{2R\sin\theta}{V}$

where $v = v_P$ for P waves

$v = v_S$ for S waves.

Time calculated applies to only s ome certain range of θ only.

If $\theta > \angle EOX$ P wave travels through the liquid core.

Let $\angle EOX' = 2\phi$

$$\phi = \cos^{-1}(\frac{R_C}{R})$$

Hence $t = \dfrac{2R\sin\theta}{V}$ is valid for $\theta \le \cos^{-1}(\dfrac{R_C}{R})$ only. **Ans**

( $v_P$ for P waves , $v_S$ for S waves)

2.2 $\dfrac{R_C}{R} = 0.5477$

and $\dfrac{V_{CP}}{V_P} = 0.8313$

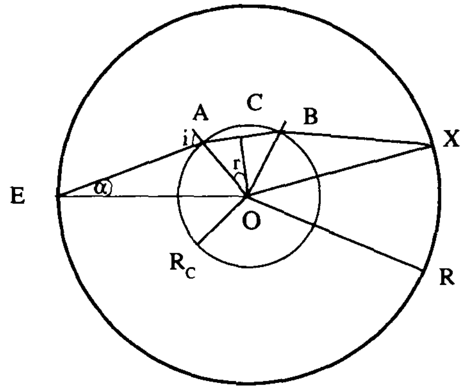



Fig. 17.7

From the figure above, θ = AC + EA

$$= 90° - r + i - \alpha$$

$$\sin i = n \sin r$$

From the principle of refraction,

$$\frac{\sin i}{\sin r} = \frac{V_P}{V_{CP}}$$

Apply sine law to Δ EOA to obtain,

$$\frac{R_C}{\sin \alpha} = \frac{R}{\sin i}$$

$$\alpha = \sin^{-1}\left[ (\frac{R_C}{R}) \sin i \right]$$

$$r = \sin^{-1}\left[ (\frac{V_{CP}}{V_P}) \sin i \right]$$

Hence $$\theta = 90° - \sin^{-1}\left[ (\frac{V_{CP}}{V_P}) \sin i \right]$$

2.3 If i = 0 then θ = 90°

If i = 90° then θ = 75.8°

2.3.1 The most convenient way of finding the minimum value of $\theta$ is to calculate $\theta$ for various i's, and plot a graph of $\theta$ against i. From such a graph, $\theta_{MIN}$ can be easily read from the graph.

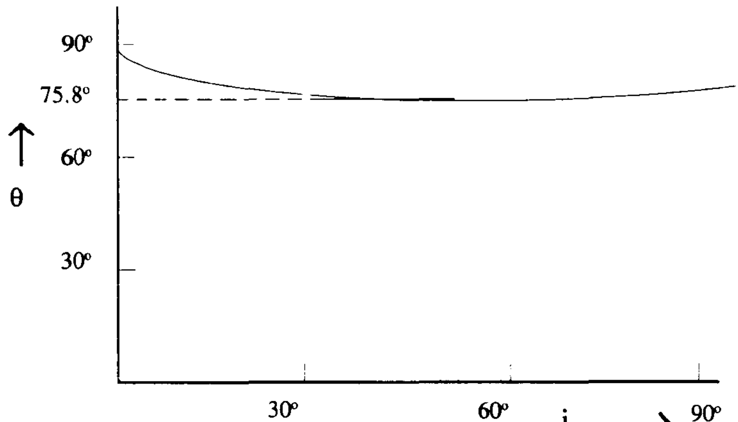



Fig. 17.8

From the graph $\theta_{MIN} = 75.8°$

and occurs when i = 55° **Ans**

Investigation of the consequences

If $\theta$ has a minimum value of 75.8° an observer at position $2\theta$,

and $$151.6^o \geq 2\theta \geq \cos^{-1}(\frac{R_C}{R})$$

will not detect the earthquake for two major reasons:

1. The observer cannot record direct waves because he is blocked by the liquid core.

2. Neither can the observer detect the refracted waves, because the refracted waves cannot deviate by more than $2\theta = 151.6°$.

2.3.2 The graph of travelling time of the P and S waves against angular distance is shown in the following figure.

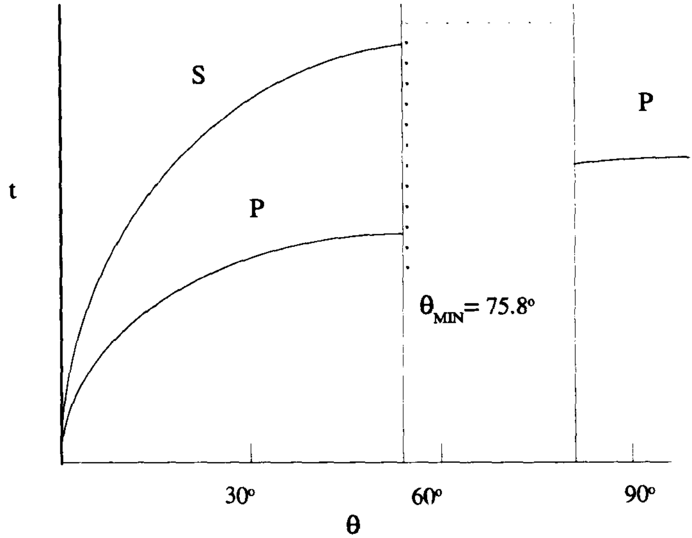



Fig. 17.9

2.4 From $t = \dfrac{2R\sin\theta}{V}$

Let interval between the arrival of P and S waves be Δt.

$$\Delta t = 2R\sin\theta\left[\frac{1}{V_S} - \frac{1}{V_P}\right]$$

Substitute $131 = 2 \times 6370 \times \sin\theta\left(\dfrac{1}{6.31} - \dfrac{1}{10.85}\right)$

$$\sin\theta = \frac{1.31 \times 10^{-2}}{2 \times 6.37 \times 10^{3}(0.158 - 0.092)}$$

$$= 1.54 \times 10^{-2}$$

$$= 8.85^{\circ}$$

$$2\theta = 17.70^{\circ}$$

This value of 2θ is then $2\cos^{-1}\dfrac{R_C}{R}$ or 114° **Ans**

2.5

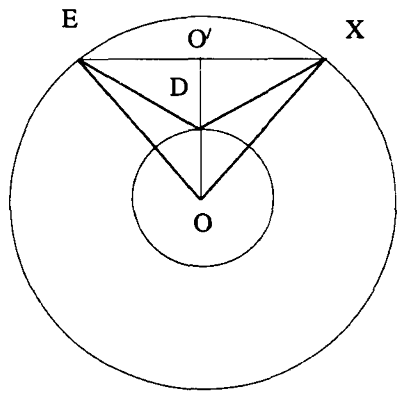



Fig. 17.10

From the data given, it is most probable that S and P waves arrive by direct route as well as are reflected at the liquid core, hence the seismometer records two sets of results within small time interval.

Using the same set of symbols in the diagram, time delay t' between the first and second recordings, is given by

$$\Delta t' = (ED + DX)\left[\frac{1}{v_S} - \frac{1}{v_P}\right]$$

$$= 2ED\left[\frac{1}{v_S} - \frac{1}{v_P}\right]$$

From $\Delta$ EO'D

$$ED^2 = (R\sin\theta)^2 + (R\cos\theta - R_C)^2$$

$$= R^2 + R_C^2 - 2RR_C\cos\theta$$

$$\Delta t' = \sqrt{2R^2 + R_C^2 - 2RR_C\cos\theta}\left[\frac{1}{v_S} - \frac{1}{v_P}\right]$$

$$= 131\frac{\sqrt{(6.370\times10^3)^2 + (3.470\times10^2)^2 - 2\times6.370\times10^3\times3.470\times10^3\cos8.85^o}}{6.370\times10^3\sin8.85^o}$$

= 396.7 s

= 6 minutes 37 seconds

The assumption that two sets of recordings of S and P waves within some time delay is due to the later set of S and P waves are refracted at the core is consistent with the fact that the angular separation between the source of earthquake and the observer is 17.70° **Ans** are solutions of the equation of motion,

## Problem 3

Three particles each of which of mass m , are in equilibrium and linked by unstretched and uncompressed light springs as shown in Fig 17.11. Hookes' law spring constant of each string is k. The whole assembly is constrained to move in a circular path as in indicated in the figure.

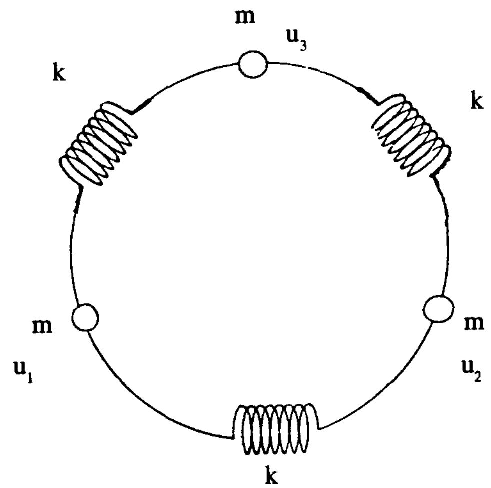



Fig. 17.11

3.1 If each mass is displaced from equilibrium by small displacement $u_1$ , $u_2$, $u_3$ respectively, write down the equation of motion for each mass.

3.2 Show that the system has simple harmonic solutions in the form

$$u_n = a_n \cos \omega t$$

with acceleartion $\qquad a_n = -\omega^2 u_n$

where ω is angular frequency.

Show that ω has possible values of 0 and $\sqrt{3}\omega_0$ where $\omega_o^2 = \dfrac{k}{m}$

3.3 The system of three mass points under discussion is enlarged to a system of N particles each of which of mass m joined by springs to its neighbouring particles. Initially the springs are neither stretched nor compressed and all mass points are at equilibrium. If all of them are slightly displaced from their equilibrium positions, write down the equation of motion for nth mass point ( n = 0,1,2,3,4....) in terms of its displacement and those of adjacent masses.

Given: $$u_n = a_n \sin(\frac{2ns\pi}{N} + \phi)\cos \omega_s t$$

for s = 1,2,3,4........... and n =1,2,3,4,...

ϕ = arbitray phase

$$\left(2\omega_0^2-\omega^2\right)(3\omega_0^2-\omega^2)\left(\omega_0^2-\omega^2\right)-6\omega_0^6+2\omega_0^4\omega^2=0$$

$$(3\omega_0^2-\omega^2)[\left(2\omega_0^2-\omega^2\right)\left(\omega_0^2-\omega^2\right)-2\omega_0^4]=0$$

If $[\left(2\omega_0^2-\omega^2\right)\left(\omega_0^2-\omega^2\right)-2\omega_0^4]\neq 0$ then $\left(\omega^2-3\omega_0^2\right)=0$

$$\omega=\sqrt{3}\omega_0$$

If $(3\omega_0^2-\omega^2)\neq 0$ then $[\left(2\omega_0^2-\omega^2\right)\left(\omega_0^2-\omega^2\right)-2\omega_0^4]=0$

$$2\omega_0^4-\omega^2\omega_0^2-2\omega^2\omega_0^2+\omega^4-2\omega_0^4=0$$

$$\omega^2(\omega^2-3\omega_0^2)=0$$

$$\omega=0 \text{ or } \omega=\sqrt{3}\omega_0$$

The system oscillates in three modes characterized by angularfrequencies 0 or $\sqrt{3}\omega_0$

where $\omega_0=\sqrt{\dfrac{k}{m}}$ **Ans**

**Second Method**

From the symmetrical property of the system, three modes of motion are possible ie.:

In the first mode the three mass points oscillate in the same direction along the cuircular track .$u_1=u_2=u_3$

(1) gives $m\ddot{u}_1=0\ \omega=0$

In the second mode, one of the mass points, in this case le t it be the mass point on the top, remain s at rest while the two mass points below oscillate towards and away from each other (breathing mode).

In this case $u_1=0$, $u_2=-u_3$

Substituion in (2) gives $m\ddot{u}_2=-3ku_2$.

$$\omega=\sqrt{\frac{3k}{m}}=\sqrt{3}\omega_0 \textbf{ Ans}$$

3.3 With N mass points linked by N springs a system of mass points alternating with springs is created. Equation of motion of nth mass point is:

$$m\ddot{u}_n=-k(u_n-u_{n-1})-k(u_n-u_{n+1})$$

Substitute $u_n(t)=u_n(0)\sin(\dfrac{2ns\pi}{N}+\phi)\cos\omega_s t$

to obtain $-\omega_s^2\cos\omega_s t=-\omega_0^2\left[\sin\left(\dfrac{2(n-1)s\pi}{N}\right)-2\sin\dfrac{2ns\pi}{N}+\sin\left(\dfrac{2(n+1)s\pi}{N}\right)\right]\cos\omega_s t$

$$\omega_s^2=\omega_0^2\left[2\sin\frac{2ns\pi}{N}\cos\frac{2s\pi}{N}-2\sin\frac{2ns\pi}{N}\right]$$

$$\omega_s^2 = 2\omega_0^2\left(\cos\frac{2s\pi}{N} - 1\right)$$

$$\omega_s^2 = 4\omega_0^2 \sin^2\frac{s\pi}{N}$$

$$\omega_s = 2\omega_0 \sin\frac{s\pi}{N} \qquad \text{where } s = 0,1,2,3,\ldots\ldots.$$

This impies that $\omega_s$ may have value from 0 to $2\omega_0$ or $2\sqrt{\frac{k}{m}}$, when N approaches $\infty$ - corresponding to the value of s from 1 to N/2 **Ans**

3.4 $$\frac{u_n}{u_{n+1}} = \frac{\sin\frac{2ns\pi}{N}}{\sin\frac{2(n+1)s\pi}{N}}$$

$$= \frac{\sin\frac{2ns\pi}{N}}{\left[\sin\frac{2ns\pi}{N}\cos\frac{2s\pi}{N} + \cos\frac{2ns\pi}{N}\sin\frac{2s\pi}{N}\right]}$$

a.For large N and small ω, $\frac{s}{N} \to 0$

$$\cos\frac{2s\pi}{N} \approx 1$$

and $\sin\frac{2s\pi}{N} \approx 0$

Hence $\frac{u_n}{u_{n+1}} \approx 1$ iedispcement changes slowly with n.

b. For large N and ω, $\omega_{MAX} = 2\omega_0$ corresponding to $s = \frac{N}{2}$

$$\cos\frac{2s\pi}{N} = \cos\pi = -1$$

$$\sin\frac{2s\pi}{N} = \sin\pi = 0$$

$$\frac{u_n}{u_{n+1}} = \frac{\sin n\pi}{\sin n\pi \times (-1) + o}$$

$\frac{u_n}{u_{n+1}} = -1$ displacement changes abruptly with increse in n by 1 **Ans**

# Experiment

## Problem 1

### Experiment 1

Apparatus provided

-Spectrometer equipped with a collimator and a telescope.

-Three syringes; one for water, one for liquid A and one for liquid B.

-A beaker and two tubes; one containing sample of liquid A and other sample of liquid B.

-Three retort stands with clamps.

-Shielded source of white light powered by 12 VAC.

-Black cardboard, plasticine and black tape.

-Two plastic squares with holes as field stops, to be placed over the ends of the telescope using rubber bands provided.

-Sheets of graph paper.

-Three dishes to be used for collecting water, liquids A and B lost from syringes.

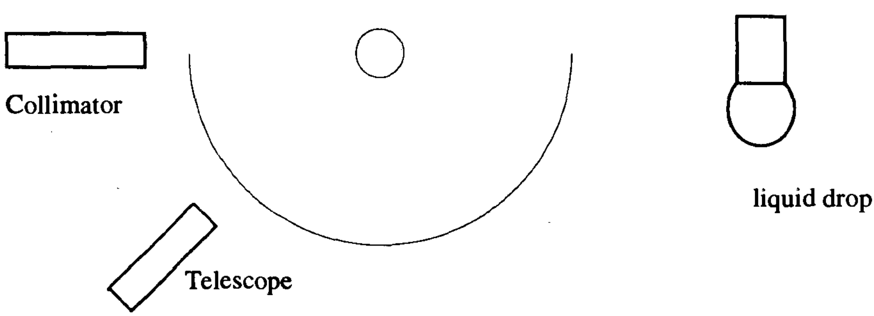



Fig. 17.15 (Diagram of apparatus setup)

**Instructions and Information**

1.Adjust the collimator to obtain parallel beam of light. This may be done by the following sequence of operations:

-focus the telescope on a distant object, using adjusting knob on the telescope, so that the cross hairs and the object are both in focus.

-Position the telscope so that it is directly opposite to the collimator with its slit illuminated in such a way that the slit can be viewed through the telescope.

-Adjust the position of the collimator lens by means of adjusting the knob on the collimator, so that the image of the slit is in focus on the cross hairs of the telescope's eyepiece.

-Lock the spectrometer table, selecting a suitable "zero" on the vernier scale, so that any angular measurements of the telescope position can be conveniently made.

2. Remove the eyepiece from the telescope and place plastic stops symmetrically over both ends of the telescope, using the elastic rubber bands provided so that the angle field of view is reduced.

3. Open up the slit of the collimator.

4. Use the syringe to generate a vertically suspending pendant drop symmetrically above the centre of the spectrometer table so that it is fully illuminated by the light from the collimator and can be viewed through the telescope.

5. The central horizontal region of the suspended liquid drop will produce rainbows as a result of two refractions and k internal reflections. ( k =1,2,3,4......) The first order of rainbow corresponds to one internal reflection. (k =1.) The second order of rainbow corresponds to two internal reflections. (k =1.) The kth order of rainbow corresponds to k internal reflections and so on.

Each rainbow contains all the colours of the spectrum. These colours can be directly observed by eye and their angular positions can be accurately measured using the telescope. The form ation of rainbow is due to beam of white light incident on the drop at an appropriate value of angle of incidence and is different for each rainbow or each order of rainbow.

6. The first order rainbow can be recognized as it has the greatest intensity and appears on the right-hand sideof the drop. (if the collimator is to the left of the telescope.) The second order rainbow appears with the greatest intensity on the lefthand side of the drop(also if the collimator is to the left of the telescope.) These two rainbows are within an angular separa tion of 20° of each other for water droplets. The weak intensity of fifth order rainbow can be observed on the right-hand side of the drop at position between other two"blue" extreme ends of the first and second order rainbows.

7. Light reflected directly from the extreme surface of the drop an that refracted twice but not internally reflected, will produce bright white glare spots that will hinder observation.

8. The refractive indices , n, of the liquids are:

Water $n_W$ = 1.333
Liquid A $n_A$ = 1.467
Liquid B $n_B$ = 1.534

In addition to the experimental report, complete the summary sheet.

# Problem 2

## Experiment 2

Apparatus provided
- RML Nimbus computer
- 10 sheets of graph paper

## General Information

The microcomputer has been programmed to solve the Newtonian equations of motion for a two-dimensional system of 25 interacting particles in the xy plane. It is able to generate the positions and velocities of all particles at discrete, equally spaced time intervals. By pressing appropriate keys (which will be described), access to dynamic information about the system can be obtained.

The system of particles is confined to a box which is initially(at time t = 0) arranged in two-dimensional square lattice. A picture of the system is displayed on the screen together with the numerical data requested. All particles are identical; the colours enable the particles to be disdinguished. As the system evolves in time,the positions and the velocities of the particles will change. If a particle is seen to leave the box, the programme automatically generates a new particle which enters the box at the opposite face with the same velocity, thus conserving the number of particles in the box.

Any two particles i and j, separated by a distance $r_{ij}$ interact with a well-defined potential $U_{ij}$. It is convenient to use dimensionless quantities throughout the computation. The quantities given below are used throughout the calculations.

| NAME OF QUANTITIES | SYMBOLS |
|---|---|
| Distance | r* |
| Velocity | v* |
| Time | t* |
| Energy | E* |
| Mass of particle | m*= 48 |
| Potential | $U_{ij}^*$ |
| Temperature | T* |
| Kinetic energy | E* = (1/2)m*v*2 |

## Instruction

The computer programme allows access to three different sets of numerical information and display them on the screen. Access is controlled by the grey function keys on the left-hand side of the keyboard, labelled F1, F2,F3,F4, and F10. These keys should be pressed and released - do not hold down a key, nor press it repeatedly. The programme may take up to 1 second to respond.

## First Information Set; Problems 1-5

Pressing Fl allows the screen to display the three quantities <vx,n> <vy,n> and <U> defined by

$$\langle vx,n\rangle = \frac{1}{25}\sum_{i=1}^{35} (v^*_{ix})^2$$

$$\langle vy,n\rangle = \frac{1}{25}\sum_{i=1}^{35} (v^*_{iy})^2$$

and

$$\langle U\rangle = \frac{1}{25}\sum_{i=1}^{35} U_{ij}^*$$

where $v^*_{ix}$ = the dimensionless x-component of velocity for the ith particle.
$v^*_{iy}$ = the dimensionless y-component of velocity for the ith particle.
n is an integer and $\geq$ 1

Note: The summation over $U_{ij}^*$ excludes the case in which i = j

After pressing F1, it is necessary to input the integer n ( n $\geq$ 1 ) by pressing one of the white keys in the top row of the keyboard,before the information appears on the screen.

The information is displayed in dimensionless time intervals $\Delta t^*$ ie.

$s\,\Delta t^* \quad s = 0,1,2,3,4.........$

$\Delta t^*$ is set by the computer programme to the value of 0.100000 .

The value of s is displayed at the bottom right-hand side of the screen. Initially it has the value s = 0.
The word "waiting" on the screen indicates that the calculation has halted and information concerning the value of s is displayed.
Pressing the long bar ( the"space"bar ) at the bottom of the keyboard will allow the calculation o f the evolution of the system to proceed in time steps t* . The current value of s is always displayed on the screen. Whilst th e calculation is proceeding the

4. Obtain the dimensionless total energy of the system at times indicated under 2. Does the system conserve energy? State the accuracy of the total energy calculation.

5. The system is initially ( at S = 0 ) and not in the thermodynamic equilibrium. After a period of time the system reaches thermodynamic equilibrium in which the total dimensionless kinetic energy fluctuates about a mean value of $E_k$*.

Determine the value of $E_k$* and indicate the time, SD after which the system is in thermo dynamic equilibrium.

6. Using the accumulated velocity data, during thermodynamic equilibrium, draw up a histogram giving the number Δ N of velocity components against dimensionless velocity component, using the constant velocity components interval $\Delta v^*_c = 0.05$, specified in the table available from the Second Information Set. Data accumulated from approximately 200 time steps should be used and the starting time integer S should be recorded.

Verify that $$\Delta N = A\, e^{-\frac{24(v_c^*)^2}{\alpha}}$$

where A and α are constants
Determine α.

7. For the system of particles in thermodynamic equilibrium, evaluate the everage value of R2 , <R2> where R is the straight line distanc e between the position of a particle at a fixedinitial time number SR and time number S.

The time number difference SZ = (S-SR) takes the value SZ = 0,2,4,.........24.

Plot a graph of $\langle R^2 \rangle$ as a function of SZ for any appropriate value of SR. Calculate the gradient of the function in the linear region and specify the time number range for which this gradient is valid.

In order to improve the accuracy of the plot, repeat the previous calculations for three ( additional) different values of SR and determine the average $\langle R^2 \rangle$ for the four sets of results together with the "linear" gradient and time number range.

Deduce, with appropriate reasoning, the thermodynamic equilibrium state of the system, either solid or liquid.

## Solution

1.Average momentum of system at requested steps (S)

| <PX,1> | s | <VX,1> | <VY,1> | <PY,1> |
|---|---|---|---|---|
| 0000000, | 0 | .000000, | .000000,. | 0000000, |
| .0000048, | 40 | .000010, | .000016,. | 0000077, |
| 0000086, | 80 | .000018,. | 000001, | .0000005, |
| .0000067, | 120 | .000014, | .000007, | .0000034, |
| .0000077, | 160 | .000016, | .000010,. | 0000048, |

The above table shows that momentum fluctuate around a constant value suggesting that the momentum of the system is constant.

Accuracy is calculated from the acuuracy of the velocity $= 100 \times \frac{\Delta v}{\sqrt{\overline{v^2}}}$

where $\Delta$ v is maximum error of the velocity

$\sqrt{\overline{v^2}}$ is root mean square velocity ( RMS )

Range of accuracy $= 100 \times \frac{.000018}{0.1}$

$= 0.002 \%$

2.and 3. Graphs of potential and kinetic energy as a function of time are given in the following pages.

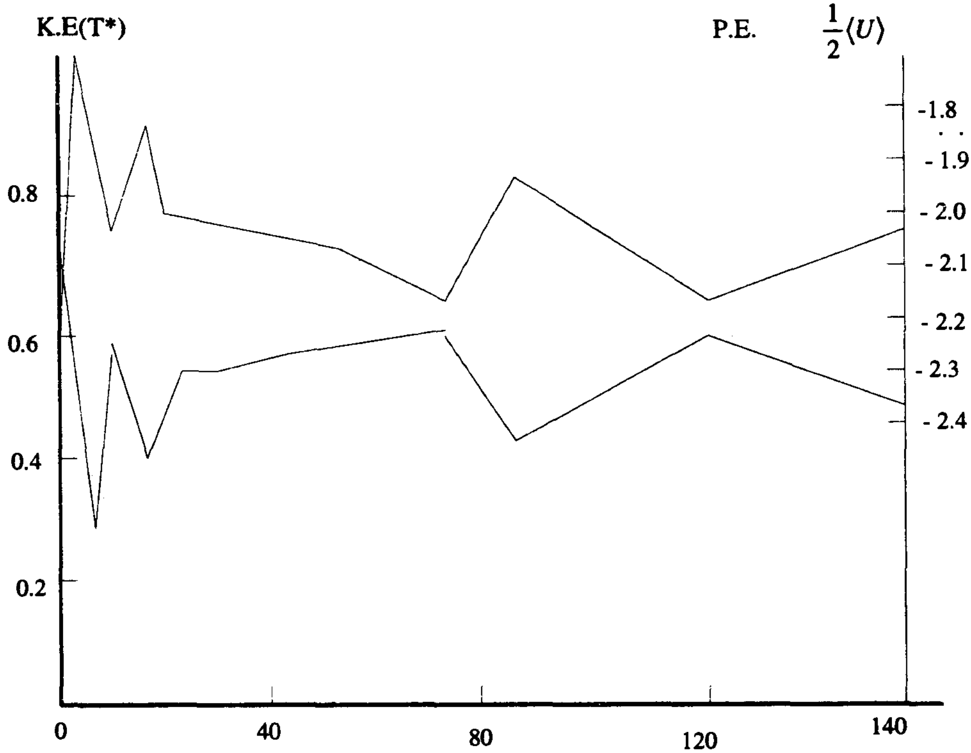



Fig. 17.7 Graph of potential and kintetic energyas a function of time

4. Total energy as calculated by the computer

| Time (S) | <E> Total Energy |
|---|---|
| 0 | -1.61499, |
| 2 | -1.62886, |
| 4 | -1.62878, |
| 6 | -1.62301, |
| 12 | -1.62882, |
| 18 | -1.62599, |
| 24 | -1.62796, |
| 30 | -1.6703, |

| | |
|---|---|
| 50 | -1.62753, |
| 70 | -1.62676, |
| 90 | -1.62580, |
| 130 | -1.62713, |
| 180 | -1.62409, |

From the table and the graph, it may be inferred that total energy of the system is conserved.

5. $E^*_k$ at equilibrium(average 24 to 180 ) = $0.534 \pm 0.05$

Equilibrium time(See Fig. 17.17) SD ~ $(10 \text{ to } 20) \times 0.1$

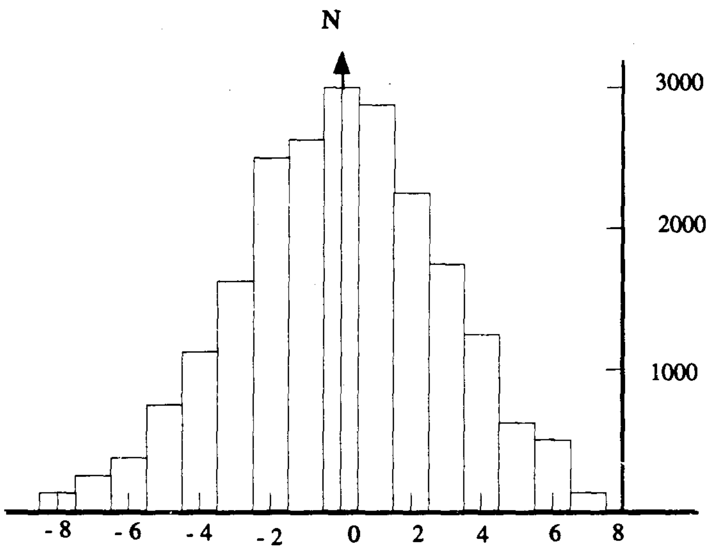



Fig. 17.18 Block graph of N against V

The curve is bell-shaped suggesting that the relation follows the equation

$$N = A.\, e^{-\frac{24(v_c^*)^2}{\alpha}}$$

Value of S recorded > 20 e.g. 60

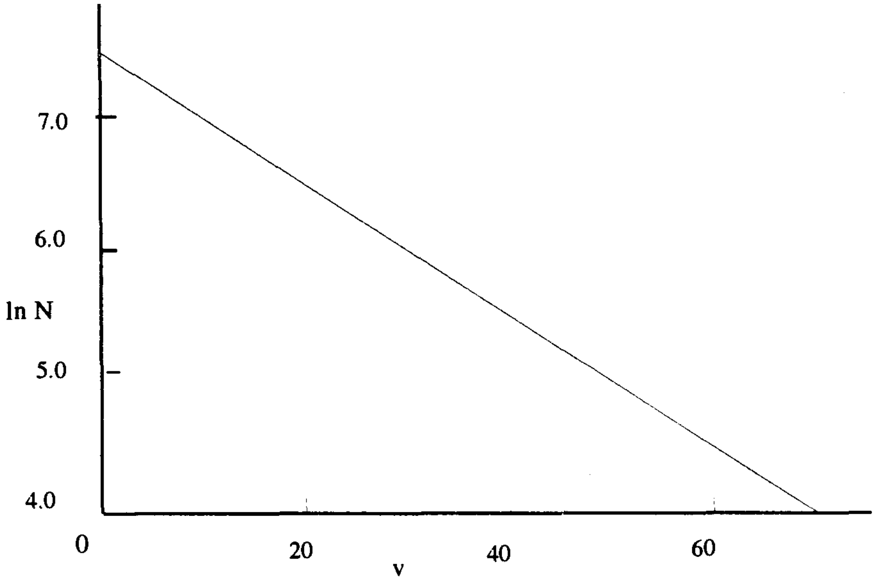



Fig. 17.19 Graph of ln N against V.

From the graph

The value of α ( at SD = 60 ) = 0.503

Accuracy of α ( for SD = 60) = ± 0.02

7. The values of $\langle R^2 \rangle$ calculated at different times

| SZ = S-SR | $\langle R^2 \rangle$ | | | | Averaged Value |
|---|---|---|---|---|---|
| | SR= 281 | SR = 301 | SR = 334 | SR = 370 | |
| 0 | 0, | 0, | 0, | 0 | 0, |
| 2 | .00088, | . 00067, | .00091, | .00079, | .00081, |
| 4 | .00287, | .00276, | .00382, | .00298, | .00311, |
| 6 | .00523, | .00628, | .00858, | .00623, | .00658, |
| 8 | 00797, | .01101, | .01449, | .01039, | .01097, |

| SZ = S-SR | $<R^2>$ | | | | Averaged Value |
|---|---|---|---|---|---|
| | SR= 281 | SR = 301 | SR = 334 | SR = 370 | |
| 10 | .01143, | .01656,. | 02095, | .01523, . | 01640, |
| 12 | .01528, | .00235, | .02768, | .02022, . | 02138, |
| 14 | .01874, | .02845, | .03453, | .02564, . | 02864, |
| 16 | .02184, | 03539, | .04157, | .03160, | .03260, |
| 18 . | 02526, | .04293, | .04902, | .03833, | .03889, |
| 20 | .02979, | .05080, | .05718, | 04532, . | 04577, |
| 22 . | 03538, | .05918,. | 06605, | .05150, | .05303, |
| 24 | .04063, | . 06784, | .07533, | .05569, . | 05987, |

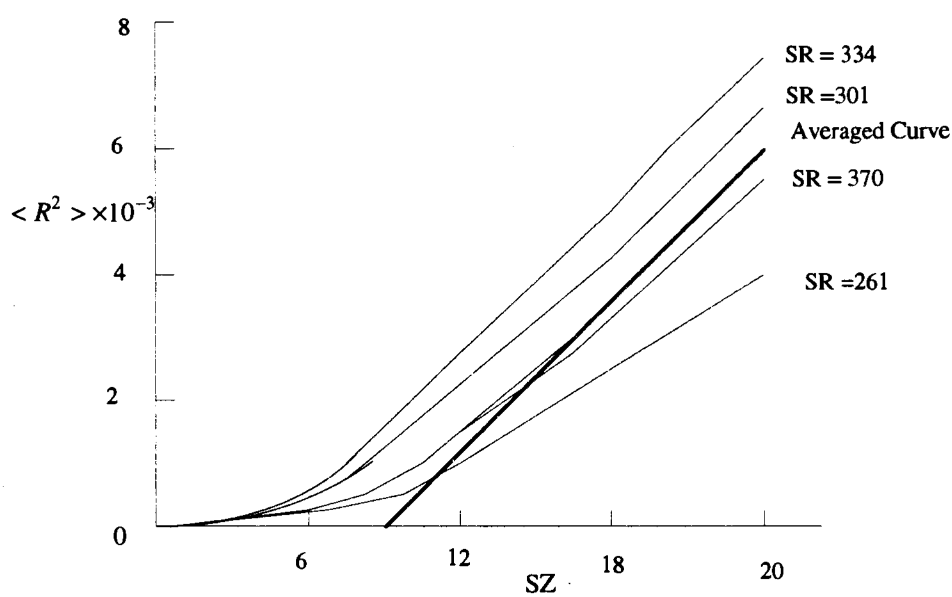



Fig. 17.20 Graph of $<R^2>$ against time SZ

**Question:** For what time number range is graph ,obtained using the first value of SR is linear?

**Answer:** Using the first value of SR, the curve is linear from time interval SZ = 18 to 24,

Slope of the curve in this region ≈0.27 to 0,047

Accuracy of the slope = ±0.002

Slope of averaged <$R^2$> in linear region ≈ 0.035

Error= ±0.01

**Question:** Is the system liquid or solid?

**Amswer:** Liquid

# International Physics Olympiad XVIII

# 1987

# Jena, German Democratic Republic

## Theory

### Problem 1

Moist air flows adiabatically up the slope of the mountain as shown in Fig. 18.1

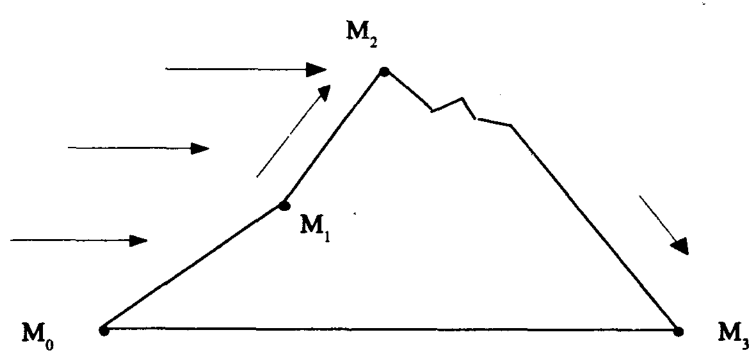



Fig. 18.1

Meteorological stations at $M_0$ and $M_3$ measure atmospheric pressure and obtain the same value of 100 kPa, while meteorological station at $M_2$ records atmospheric preassure reading of 70 kPa. The temperature reading at $M_0$ is 20° C.

As the moist air is moving upwards along the slope of the mountain, it begins to turn into clouds at the pressure of 84.5 kPa.

Consider 200 kg of moist air moving along the slope of themountain and spreading out to cover every square meter of the area on the slope. After a travelling time of 1600 s, the moist air reaches the area on the slope where meteorological station $M_2$ is situated. During this time interval, condensation of every kilogram of moist air yields 2.45 g of rain.

Determine:

1.1 Temperature $T_1$ at $M_1$ where cloud ceiling is formed.

1.2 Altitude $h_1$ ( Location of $M_1$ measured from $M_0$ ) of the cloud ceiling , assuming that the density of the atmosphere decreases linearly with altitude.

1.3 Temperature $T_2$ on the ridge where $M_2$ is situated.

1.4 Height of water column (precipitation level) condensed from an air current over the interval of 3 hours, assuming that rain is a uniform mass of water between locations of $M_1$ and $M_2$.

1.5 Temperature $T_3$ on the the other side of the mountain where station $M_3$ is located. Comment on the state of the atmoshere at location of $M_3$ compared to that at location of $M_1$.

Useful Information:

The air which we are considering is an ideal gas.
Water vapour is assumed to have no effect on specific heat and density of the atmosphere.
Latent heat of vaporization does not depend on temperature.

Calculate:

Temperature to the accuracy of 1 K.
Altitude of clouds to the accuracy of 10 m.
Height of water column(precipitation level) to the accuracy of 1 mm.

Given:

Specific heat of the atmosphere at constant pressure in the temperature range under consideration,

$$c_P = 1005 \text{ J. kg}^{-1}..\text{K}^{-1}$$

Density of the atmosphere at pressure $P_0$ and $T_0$ at station $M_0$ is

$$= 1.189 \text{ kg.m}^{-3}$$

Latent heat of vaporization of water

$$L_v = 2500 \text{ kJ kg-1}$$

$$\frac{c_P}{c_V} = \gamma = 1.4$$

$$g = 9.81 \text{ ms}^{-2}$$

## Solution

1.1 Determination of $T_1$ at the cloud ceiling.
Equation for adiabatic change

$$P_0 V_0^{\gamma} = P_1 V_1^{\gamma} \quad (1)$$

where $P_0$ and $V_0$ are pressure and volume of the air at station $M_0$,

$P_1$ and $V_1$ are pressure and volume of the air at station $M_1$.

Equation of state for an ideal gas

$$\frac{P_0 V_0}{T_0} = \frac{P_1 V_1}{T_1} \quad (2)$$

Volume is the variable which cannot be directly measured under the situation of this problem and may be eliminated by combining equations (1) and (2) as follows,

(2) $\gamma$
$$\left(\frac{P_0 V_0}{T_0}\right)^{\gamma} = \left(\frac{P_1 V_1}{T_1}\right)^{\gamma} \quad (3)$$

(1) + (3)
$$P_0^{(1-\gamma)} T_0^{\gamma} = P_1^{(1-\gamma)} T_1^{\gamma}$$

$$\left(\frac{T_1}{T_0}\right)^{\gamma} = \left(\frac{P_0}{P_1}\right)^{(1-\gamma)}$$

$$\frac{T_1}{T_0} = \left(\frac{P_0}{P_1}\right)^{(\frac{1}{\gamma}-1)}$$

$$\frac{T_1}{T_0} = \left(\frac{P_1}{P_0}\right)^{(1-\frac{1}{\gamma})}$$

where $P_0 = 100$ kPa $\qquad P_1 = 84.5$ kPa

$T_0 = ?$ $\qquad T_1 = 273+20 = 293$ K

Substitution gives
$$\frac{293}{T_0} = \left(\frac{84.5}{100}\right)^{(1-\frac{1}{1.4})}$$

$$T_0 = 293 \times \left(\frac{100}{84.5}\right)^{\frac{2}{7}}$$

$$= 293 \times 0.953$$

$$= 279.2 \quad \text{K} \quad \textbf{Ans}$$

1.2 Determination of altitude of cloud ceiling h.

Atmospheric pressure is equal to weight of the air column of unit cross-sectional area on the spot where the atmosphereic pressure is to be measured. In reverse we can determine the length of the air column h from the difference in atmosperic pressure taken at the bottom and the top of the air column.

Here $$P_0 - P_1 = g\, h\, \rho_{av}$$

where $$\rho_{AV} = \frac{\rho_0 + \rho_1}{2}$$

$\rho_1$ can be calculated from the equation of state for an ideal gas ie.

$$\frac{P_0 V_0}{T_0} = \frac{P_1 V_1}{T_1}$$

$$\frac{P_0 M}{\rho_0 T_0} = \frac{P_1 M}{\rho_1 T_1}$$

where **M** is mass of the atmospheric air of volume $V_0$ at temperature $T_0$ ( 1 g mol ) and also the mass of the atmospheric air of volume $V_1$ at temperature $T_1$ ( 1 g mol )

$$\rho_1 = \frac{P_1 T_0}{P_0 T_1} \rho_0$$

Substitute $\rho_0$ = 1.189 kg $m^{-3}$ in the equation above to obtain

$$\rho_1 = \frac{84.5 \times 293}{100 \times 279} \times 1.189$$

$$= 1.055 \quad \text{kg m}^{-3}$$

$$\rho_{AV} = \frac{1.189 + 1.055}{2}$$

$$= 1.122$$

$$h = \frac{(100 - 84.5) \times 10^{-3}}{9.81 \times 1.122}$$

$$= 1{,}408 \quad m$$

Altitude of the cloud ceiling is 1,408 m **Ans**

1.3 Determination of temperature $T_2$ at the ridge of the mountain

Change in temperature of the air that flows from the cloud ceiling to the ridge is governed by two phenomena ie.

- Cooling to temperature $T_x$ by an adiabatic process
- Heat intake from condensation of water vapour into rain causing temperature change of $\Delta T$

Let $T_2 = T_x + \Delta T$

From 1.1 $$\frac{T_X}{T_1} = \left(\frac{P_2}{P_1}\right)^{(1-\frac{1}{\gamma})}$$

where $T_x = ?$ $\quad P_2 = 70$ kPa

$T_1 = 279$ K $\quad P_1 = 84.5$ kPa $\quad \gamma = 1.4$

Substitution gives $$T_x = 279 \times \left(\frac{70}{84.5}\right)^{(1-\frac{1}{1.4})}$$

$$= 279 \times (0.828)^{0.826}$$

$$= 279 \times 0.947$$

$$= 264.2 \quad K$$

Heat released in 1 kg of air is $L_v m = c_p \Delta T$

where $L_v$ = latent heat of vaporization = 2500 × 102 J/kg

$m$ = mass of vapour undergoing condensation $2.45 \times 10^{-3}$ kg

$c_p$ = specific heat of air at constant pressure = 1.005 J $kg_{-1}$ $K^{-1}$

$$\Delta T = \frac{2500 \times 10^2 \times 2.45 \times 10^{-3}}{1005}$$

$$= 6.125 \quad K$$

$$T_2 = 264.2 + 6.125$$

$$= 273 \quad K \quad \textbf{Ans}$$

1.4 Determination of the height of water column (precipitation level)

The quantity of rain that is taken out of the air column during its journey upwards per 1 square kilometer per second is

$$= \frac{2000 \times 2.45 \times 10^{-3}}{1500} \quad kg\ s^{-1}$$

$$= 3.27 \times 10^{-3} \quad kg\ s^{-1}$$

In the interval of 3 hours or $3 \times 60 \times 60$ seconds, the quantity of rain condensated in the air column considered is

$$= 3.27 \times 10^{-3} \times 3 \times 60 \times 60 \quad kg$$

$$= 35.3 \quad kg$$

1 kg of water over the area of 1square meter corresponds to the column length of 1 mm

Hence 35.3 kg of water over 1 square meter give the water column of length 35.3 mm **Ans**

1.5 Determination of $T_3$ at station $M_3$

The air descends downward adiabatically

From (1) $$\frac{T_3}{T_1} = \left(\frac{P_3}{P_2}\right)^{(1-\frac{1}{\gamma})}$$

where $T_2 = 270$ K $\qquad P_2 = 70 \times 10^3$ N/m$^2$

$T_3 = ?$ $\qquad P_3 = 100 \times 10^3$ N/m$^2$

$$T_3 = 270 \times \left(\frac{100}{70}\right)^{0.826}$$

$$= 270 \times (1.429)^{0.826}$$

$$= 270 \times 1.11$$

$$= 299.7 \quad K \quad \textbf{Ans}$$

**NB.** The air current upon reaching $M_3$ from $M_1$ becomes drier and warmer. It is drier because of precipitation of rain, and warmer because it takes in heat released by the prcipitation. Under normal conditio n ie. a flat land without a mountain, the conditions of the atmosphere at $M_1$ and $M_3$ would have been the same.

# Problem 2

Electron beam from electron source P enters magnetic field $\mathbf{B}_C$ of an electric toroidal coil along the direction of the magnetic field. If the angle subtended by the aperture which regulates the electron beam is small ( $2\alpha << 1$ ) and the electron beam travels along the cicumference of radius R measured from the center of the toroid to the midpoint of the crossection of the toroidal coil. (See Fig.18.12) .Voltage $V_0$ is applied to accelerate the electron beam to the velocity required.

Ignore interaction between electrons in the beam due to their negligible effect and assume that the magnitude of $\mathbf{B}_C$ i.e. $\mathbf{B}_C$ is constant.

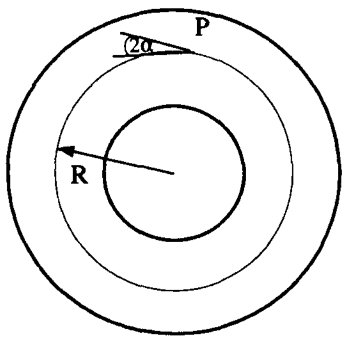



Fig. 18.2

2.1 In order to confine the motion of the electron beam along the circumference of the circle with radius R described earlier, it is necessary to use $\mathbf{B}_E$ to constrain the direction of motion of the electrons. Determine the magnitude of magnetic field $\mathbf{B}_E$ which keeps the elecron beam along the circumference of the circle of radius R in the elctric toroidal coil.

2.2 Determine magnetic field $\mathbf{B}_C$ of an electric coil which focuses the elctron beam travelling along the circumference of the elctric toroidal coil at the spots away from P by and . (When calculating the path of the electron, the radius of curvature of magnetic field $\mathbf{B}_C$ needs not be brought into the analysis.)

2.3 If magnetic field $\mathbf{B}_E$ cannot keep the elctron beam along the circumferene of the circle of radius R all the time, the elctron beam will gradually drift along the direction normal to the plane of the toroidal coil.

2.3.1 Show that the radial deviation of the electron beam from the position of equilibrium is finite.

2.3.2 Show that the direction of drifting motion of the electron beam is perpendicular to the sagital plane of the toroid.

**Necessary Data**

$$\frac{e}{m} = 1.75 \times 10^{11} C.kg^{-1}$$

$V_0 = 3$ kV

$R = 50 \times 10^{-3}$ m

# Solution

2.1 Determination of $\mathbf{B}_E$

$$\mathbf{F} = -e\,\mathbf{u}_0 \times \mathbf{B}_E$$

$$\frac{mu_0^2}{R} = eu_0B_E$$

$$\frac{mu_0}{R} = eB_E \qquad (1)$$

The electron beam is accelerated by potential $V_0$

Hence

$$\frac{1}{2}mu_0^2 = eV_0$$

$$mu_0 = \sqrt{2emV_0} \qquad (2)$$

Substitute $mu_0$ from (2) in (1) to obtain

$$\frac{\sqrt{2emV_0}}{R} = eB_E$$

$$B_E = \frac{1}{R}\sqrt{\frac{2mV_0}{E}}$$

Substitute values of R, $\frac{e}{m}$ and $V_0$

$$= \frac{1}{50 \times 10^{-3}}\sqrt{\frac{2 \times 3 \times 10^3}{1.76 \times 10^{11}}}$$

$$= \quad 0.32 \times 10^{-2} \text{ Tesla} \quad \textbf{Ans}$$

2.2 Determination of $\mathbf{B}_C$ of the toroidal coil.

In case of the velocity of the electron beam **v** being different from $\mathbf{u}_0$, velocity **v** is resolved into two components: the first is along the circumference of the circle of radius R, and other normal to the plane defining the circle of radius R.

$$\mathbf{v} = \mathbf{v}_{\parallel} + \mathbf{v}_{\perp}$$

Lorents force due to field $\mathbf{B}_C$ acts on the electron having velocity normal to the circular plane .

$$\frac{mv_{\perp}^2}{r} = ev_{\perp}B_C$$

where r is radius of the circular path of the electron beam measured from the axis of the toroidal coil, i.e. the circumference of the circle which defines the axis of the toroid.

$$r = \frac{mv_{\perp}}{eB_C}$$

$$T = \frac{2\pi r}{v_{\perp}}$$

$$= \frac{2\pi m}{e.B_C}$$

In order for the electron to be focused at positions corresponding to $\frac{\pi}{2}, \pi, \frac{3\pi}{2}$,

$$v_{\parallel}T = \frac{2\pi R}{4}$$

$v_{\parallel}$ does not change because $2\alpha <<< 1$

$$v_{\parallel} = u_0 \cos\alpha \approx u_0$$

Hence $$u_0 \frac{2\pi m}{eB_C} = \frac{\pi R}{2}$$

$$B_C = \frac{4}{R}\frac{u_0 m}{e}$$

Substituting $B_C$ to obtain $$u_0 = \sqrt{\frac{2eV_0}{m}}$$

$$B_C = \frac{4}{R}\sqrt{\frac{2mV_0}{e}}$$

$$= \frac{4}{50\times10^{-3}}\sqrt{\frac{2\times3\times10^3}{1.76\times10^{11}}}$$

$$= 1.48\times10^{-2} \text{ Tesla}$$ **Ans**

2.3 Consider motion of the electron beam when there is deviation from the circumference of radius R.

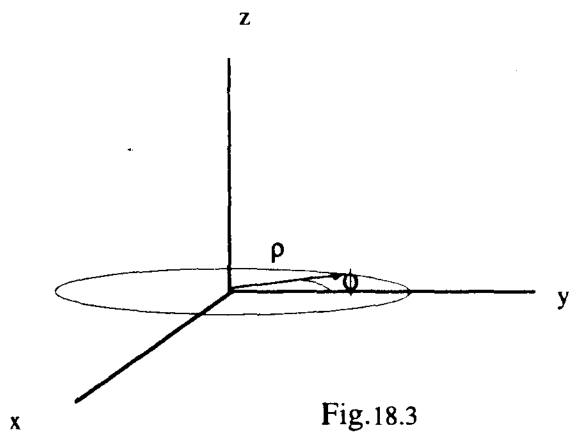



Fig.18.3

In the cylindrical coordinates characterized by variables ρ, ϕ and z ,kinetic energy of the electron in the toroid is given by

$$K.E. = \frac{m}{2}(v_\rho^2 + v_\phi^2 + v_z^2)$$

when $v_\rho = \dot{\rho}$

$$v_\phi = \rho\dot{\phi}$$

$$v_z = \dot{z}$$

$\dot{\rho}$ is small compared to $v_\phi$ and $v_z$

Hence $$K.E. = \frac{m}{2}(v_\phi^2 + v_z^2)$$

Let $$u_0^2 = (v_\phi^2 + v_z^2)$$

As there is no torque in the direction of $\mathbf{1}_z$ angular momentum about z remains constant.

$$mv_\phi\rho = mu_0R$$

$$v_\phi = \frac{u_0\rho}{R}$$

Lorentz's force due to $\mathbf{B}_C$ acting along z axis is

$$|\mathbf{F}_z| = -e|\mathbf{B}_C|$$

$$m|\mathbf{a}_z| = -e|\mathbf{B}_C|\rho$$

where $\mathbf{a}_z$ is acceleration of the electron along z axis.

$$\frac{\Delta v_z}{\Delta t} = -eB_C\frac{\Delta \rho}{\Delta t}$$

$$\Delta v_z = \frac{-eB_C}{m}\Delta\rho$$

$v_z$ starts from 0 when $\rho = R$ hence

$$v_z = \frac{-eB_C}{m}(\rho - R)$$

Substitute $v_\phi$ from (2) and $v_z$ from (3) in (1) to obtain,

$$u_0^2 = \left(\frac{u_0 R}{\rho}\right)^2 + \left(\frac{eB_C}{m}\right)^2(\rho - R)^2$$

$$1 = \left(\frac{R}{\rho}\right)^2 + \left(\frac{eB_C}{mu_0}\right)^2(\rho - R)^2 \qquad (3)$$

Sketch a graph of $f(\rho) = \left(\frac{R}{\rho}\right)^2 + \left(\frac{eB_C R}{mu_0}\right)^2\left(\frac{\rho}{R} - 1\right)^2$ as a function of $\rho$ is shown in Fig.18.4

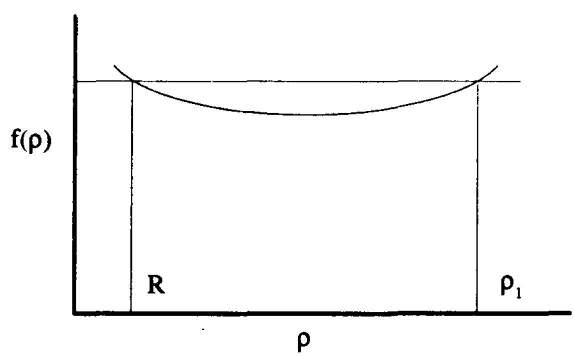


Fig. 18.4

Since $f(\rho) \leq 1$ the value of $\rho$ is between R and $\rho_1$ only, and can be deteermined from (3) The drifting of the elctron beam is along -z axis **Ans**

# Problem 3.

A sinusoidal voatage signal travels along an infinite LC grid depicted in Fig. 18.5. The phases of AC voltage signal across two successive capacitors differ by $\phi$.

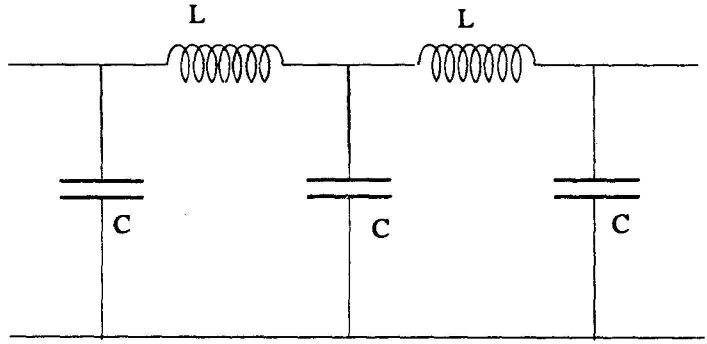



Fig. 18.5

Determine:

3.1 $\phi$ as a function of $\omega$ L and C

3.2 The velocity of sine wave along the network if the length of each cell consisting of L and C is l.

3.3 The condition for the vlocity to independenbt on $\omega$ (approximately)

3.4 A mechanical model which is the analog or equivalent to the circuit above and also the equation which is the analog of the equation in the elctrical model.

Formulas and Necessary Data:

$$\cos\alpha - \cos\beta = -2\sin(\frac{\alpha+\beta}{2})\sin(\frac{\alpha-\beta}{2})$$

$$\sin\alpha - \sin\beta = 2\cos(\frac{\alpha+\beta}{2})\sin(\frac{\alpha+\beta}{2})$$

## Solution

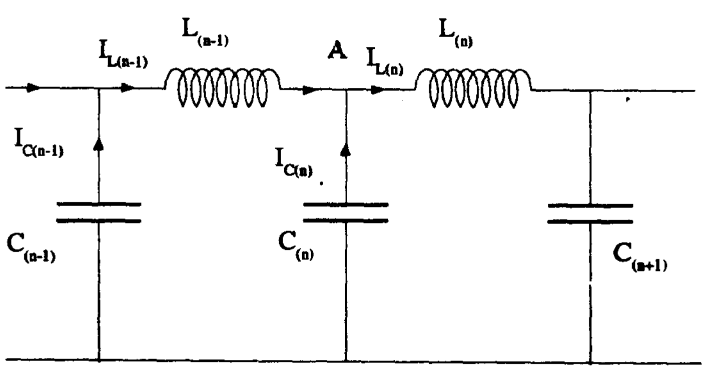



Fig 18.6

3.1 Referring to Fig 18.6 ,let
current through $C_{(n-1)}$ be $I_{C(n-1)}$
through $C_{(n)}$ be $I_{C(n)}$
through $L_{(n)}$ be $I_{L(n)}$
through $L_{(n-1)}$ be $I_{L(n-1)}$

**First Method**

Apply Kirchhoff's law at A (current)

$$I_{L(n-1)} + I_{C(n)} - I_{L(n)} = 0 \tag{1}$$

Apply Kirchhoff's law for circuit 1(voltage)

$$V_{C(n-1)} + V_{L(n-1)} - V_{C(n)} = 0 \tag{2}$$

From data given

$$V_{C(n)} = V_0 \sin(\omega t + n\phi) \tag{3}$$

$$I_{C(n)} = \frac{d}{dt}(Q_{C(n)}) = C_{(n)}\frac{d}{dt}(V_{C(n)})$$

$$I_{C(n)} = \omega C V_0 \cos(\omega t + n\phi) \tag{4}$$

$$V_{C(n-1)} = V_0 \sin[\omega t + (n-1)\phi] \tag{5}$$

Substitute $V_{c(n-1)}$ and $V_{C(n)}$ from (3) and (5) in (2) to obtain

$$V_{L(n-1)} = V_0 \sin(\omega t + n\phi) - V_0 \sin[\omega t + (n-1)\phi]$$

$$= 2V_0 \cos\left[\omega t + (n - \frac{1}{2})\phi\right]\sin\frac{\phi}{2}$$

$$L\frac{d}{dt}(I_{L(n-1)}) = V_{L(n-1)}$$

$$= 2V_0 \cos\left[\omega t + (n - \frac{1}{2})\phi\right]\sin\frac{\phi}{2}$$

$$I_{L(n-1)} = \left(\frac{2V_0}{\omega L}\right)\sin\left[\omega t + (n - \frac{1}{2})\phi\right]\sin\frac{\phi}{2} + k$$

Here $I_{L(n-1)} = 0$ when $\omega t + (n - \frac{1}{2})\phi = 0$

k = 0 current leads voltage by factor of $\frac{\pi}{2}$

$$I_{L(n-1)} = \left(\frac{2V_0}{\omega L}\right)\sin\left[\omega t + (n - \frac{1}{2})\phi\right]\sin\frac{\phi}{2} \tag{6}$$

$$I_{L(n)} = \left(\frac{2V_0}{\omega L}\right)\sin\left[\omega t + (n + \frac{1}{2})\phi\right]\sin\frac{\phi}{2} \tag{7}$$

(Capacitor impedes voltage while inductor impedes current)

Substitute $I_{C(n)}$ from (4) and $I_{L(n-1)}$ from (6) into (7) to obtain

$$\left(\frac{2V_0}{\omega L}\right)\sin\left[\omega t + (n-\frac{1}{2})\phi\right]\sin\frac{\phi}{2} - \left(\frac{2V_0}{\omega L}\right)\sin\left[\omega t + (n+\frac{1}{2})\phi\right]\sin\frac{\phi}{2} + \omega C V_0\cos(\omega t + n\phi) = 0$$

$$\left(\frac{2V_0}{\omega L}\right)\left[\sin\left[\omega t + (n-\frac{1}{2})\phi\right] - \left(\frac{2V_0}{\omega L}\right)\sin\left[\omega t + (n+\frac{1}{2})\phi\right]\right]\sin\frac{\phi}{2} + \omega C V_0\cos(\omega t + n\phi) = 0$$

$$\left(\frac{2V_0}{\omega L}\right)\left[2\cos(\omega t + n\phi)\sin(-\frac{\phi}{2})\right]\sin\frac{\phi}{2} + \omega C V_0\cos(\omega t + n\phi) = 0$$

$$\frac{4\sin^2\frac{\phi}{2}}{\omega L} + \omega C = 0$$

$$\omega^2 LC = 4\sin^2\frac{\phi}{2}$$

The value of $\sin^2\frac{\phi}{2}$ varies between 0 and 1, hence

$$0 \quad \le \quad \omega^2 LC \quad \le \quad 4$$

$$0 \quad \le \quad \omega \quad \le \quad \frac{2}{\sqrt{LC}}$$ **Ans**

**Second Method**

Apply Kirchhoff's law for circuit 1(voltage)

$$V_{C(n-1)} + V_{L(n-1)} - V_{C(n)} \quad = \quad 0$$

From data given

$$V_{C(n)} = V_0\sin(\omega t + n\phi)$$

$$V_{C(n-1)} = V_0\sin[\omega t + (n-1)\phi]$$

$$V_{L(n-1)} = V_0\sin(\omega t + n\phi) - V_0\sin[\omega t + (n-1)\phi]$$

$$= 2V_0\cos\left[\omega t + (n-\frac{1}{2})\phi\right]\sin\frac{\phi}{2}$$

In the same spirit, if we apply Kirchhoff's law for circuit 2(voltage), we will obtain,

$$V_{Ln} = 2V_0\cos\left[\omega t + \left(n+\frac{1}{2}\right)\phi\right]\sin\frac{\phi}{2}$$

Apply the principle that current through a capacitor leads voltage across the capacitor under consideration by 90° ,while current through an inductor lags behind voltage across the respective induction coil by 90°.

Hence $$I_{L(n-1)} = I_{L0}\cos\left[\omega t + \left(n-\frac{1}{2}\right)\phi + \frac{\pi}{2}\right]$$

$$I_{Ln} = I_{L0}\cos\left[\omega t + \left(n+\frac{1}{2}\right)\phi + \frac{\pi}{2}\right]$$

$$I_{Cn} = I_{C0}\sin\left(\omega t + n\phi - \frac{\pi}{2}\right)$$

where $I_{L0}$ and $I_{C0}$ are maximum current in the induction coil and capacitor respectively.

$$I_{L0} = \frac{2V_0\sin\frac{\phi}{2}}{\omega L}$$

$$I_{C0} = \omega C V_0$$

Apply Kirchhoff's law at A (current)

$$\frac{2V_0\sin\frac{\phi}{2}}{\omega L}\cos\left[\omega t + \left(n-\frac{1}{2}\right)\phi + \frac{\pi}{2}\right] + \omega C V_0\sin\left(\omega t + n\phi - \frac{\pi}{2}\right)$$

$$= \frac{2V_0\sin\frac{\phi}{2}}{\omega L}\cos\left[\omega t + \left(n+\frac{1}{2}\right)\phi + \frac{\pi}{2}\right]$$

$$\omega C V_0\sin\left(\omega t + n\phi - \frac{\pi}{2}\right)$$

$$= \frac{2V_0\sin\frac{\phi}{2}}{\omega L}\left\{\cos\left[\omega t + \left(n+\frac{1}{2}\right)\phi + \frac{\pi}{2}\right] - \cos\left[\omega t + \left(n-\frac{1}{2}\right)\phi + \frac{\pi}{2}\right]\right\}$$

$$-\omega C V_0\cos(\omega t + n\phi) = -\frac{2V_0\sin\frac{\phi}{2}}{\omega L}2\sin\left(\omega t + n\phi + \frac{\pi}{2}\right)\sin\frac{\phi}{2}$$

$$\omega^2 CL\cos(\omega t + n\phi) = 4\sin^2\frac{\phi}{2}\cos(\omega t + n\phi)$$

$$\omega^2 CL = 4\sin^2\frac{\phi}{2}$$

$\sin^2\frac{\phi}{2}$ varies between 0 and 1

$$0 \le \varpi^2 LC \le 4$$

$$0 \le \omega \le \frac{2}{LC}$$

3.2 Time for sine wave to cross one cell is $\Delta t$ and $\omega\Delta t = \phi$

This can be easily verified using

voltage $V_{C(n)}$ at time t = voltage $V_{C(n+1)}$ at time t - $\Delta t$

$$V_0 \sin(\omega t + n\phi) = V_0 \sin[\omega(t-\Delta t) + (n+1)\phi]$$

$$\omega t + n\phi = \omega(t-\Delta t) + (n+1)\phi$$

$$\omega\Delta t = \phi$$

Velocity of wave signal $v = \dfrac{l}{\Delta t}$

$$= \frac{\omega l}{\phi}$$

$$= \frac{\omega l}{2\sin^{-1}\left(\dfrac{\omega\sqrt{LC}}{2}\right)} \quad \textbf{Ans}$$

If the velocity does not depend on ω or even varies slowly with ω,then

$$v \approx \frac{\omega l}{\omega\sqrt{LC}}$$

$$\approx \frac{l}{\sqrt{LC}} \quad \textbf{Ans}$$

3.4 Total energy of the network consisting of N cells

$$W = \frac{1}{2}\sum_{n=1}^{N}(CV_{C(n)}^2 + LI_{L(n)}^2)$$

The mechanical analog of this electrical system is a string of mass m's joined together by springs each of which has spring constant k as depicted in Fig 18.6

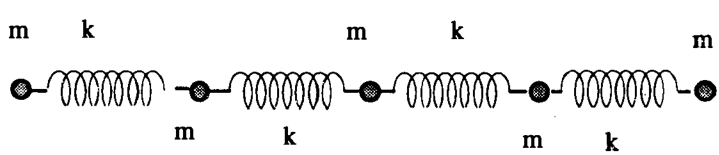



Fig 18.6

In the case of the mechanical model

$$W = \frac{1}{2}\sum_{n=1}^{N}[m\dot{x}_n^2 + k(x_n - x_{n-1})^2] \tag{9}$$

(NB Mass m on the extreme right corresponds to n = 0 )

In order to write (8) as the strict analog of (2) we start from

$$V_{C(n)} = \frac{Q_{C(n)}}{C} \text{ and} \tag{10}$$

$$I_{L(n-1)} + I_{C(n)} - I_{L(n)} = 0$$

Hence $\dot{Q}_{L(n-1)} + \dot{Q}_{C(n)} - \dot{Q}_{L(n)} = 0$

$$Q_{L(n-1)} + Q_{C(n)} - Q_{L(n)} = 0 \quad \text{(Conservation of charge)}$$

$$Q_{C(n)} = Q_{L(n-1)} - Q_{L(n)} \tag{11}$$

Substitute (10) and (11) in (8) to get

$$W = \frac{1}{2}\sum_{n=1}^{N}[\frac{1}{C}(Q_{L(n-1)} - Q_{L(n)})^2 + L\dot{Q}_{L(n)}^2]$$

$$W = \frac{1}{2}\sum_{n=1}^{N}[L\dot{Q}_{L(n)}^2 + \frac{1}{C}(Q_{L(n)} - Q_{L(n-1)})^2]$$

| | Mechanical Model | Electrical Model |
|---|---|---|
| Variable | $x_n$ | $Q_n$ |
| | $\dot{x}_n$ | $\dot{Q}_{L(n)}$ |
| | m | L |
| | k | $\frac{1}{C}$ **Ans** |

# Experiment

## Problem1

**Determination of refractive indices, $n_p$ of glass prism and $n_l$ of liquid**

1.1 Find refractive index of a given prism $n_p$ using two different methods. Draw accurate diagrams to illustrate experimental procedures and also to provide the basis on which the formula for calculating the value of the refractive index is derived. (Only one prism is to be used.)

**Apparatus**

- Two identical prisms each of which has the angles of 30° 60° and 90° respectively, and the side subtending the right angle is frosted.
- A set square
- A round table
- A sample of liquid
- A sheet of graph paper
- A set of drawing paper and a pencil

**Useful formulas**

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

**Additional Remarks**

The opaque side of the prism may be marked with a pencil. Use of the lamp in the experiment is upon the discretion of the contestant.

## Solution

### First Method

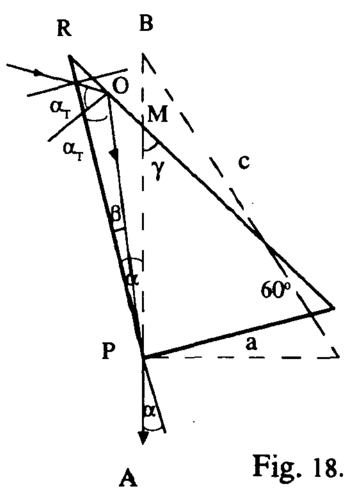



Fig. 18.7

- Draw line AB along the vertical direction as shown in Fig 18.7
- Set the prism on the paper so that the longer side of the prism adjacent to the right angle is along AB and the shorter side of the prism is on the right of line AB.
- Call the vertex of the right-angled triangle P.
- Rotate the prism about P in the anti - clockwise direction while sighting along line AB through the prism.
- At the beginning, the filed of view is totally dark until total reflection occurs and the field of view becomes very bright.
- At this position markthe position of the prism by line MO

Let c represent distance MQ

and a distance PQ

i) $$n_P \sin \alpha_T = 1.\sin 90^o$$

$$\sin \alpha_T = \frac{1}{n_P} \qquad (1)$$

ii) $$\frac{\sin \beta}{\sin \alpha} = {}_P\mu_a = \frac{1}{{}_a\mu_P} = \frac{1}{n_P} \qquad (2)$$

iii) $$\angle BPQ = 90^o - \alpha_T = 30^o + \beta$$

$$\beta = 60^o - \alpha_T \qquad (3)$$

iv) $$\angle MPQ = 90^o - \alpha$$

$$90^o - \alpha + \gamma + 60^o = 180^o$$

$$\gamma - \alpha = 30^o \qquad (4)$$

v) Apply sine law at ΔPQM

$$\frac{a}{\sin \gamma} = \frac{c}{\sin(90^o - \alpha)} \qquad (5)$$

$$\frac{a}{\sin(30^o + \alpha)} = \frac{c}{\cos \alpha}$$

$$a \cos \alpha = c[\sin 30^o \cos \alpha + \cos 30^o \sin \alpha]$$

$$\frac{a \cos \alpha}{c} = \frac{\cos \alpha}{2} + \frac{\sqrt{3} \sin \alpha}{2}$$

$$\left(\frac{2a - c}{c}\right)^2 (1 - \sin^2 \alpha) = 3 \sin^2 \alpha$$

$$\sin \alpha = \frac{2a - c}{2\sqrt{a^2 - a.c + c^2}} \qquad (6)$$

From (2) $$\sin \alpha = n_P \sin(60^o - \alpha_T)$$

$$= n_P[\sin 60^o \cos \alpha_T - \cos 60^o \sin \alpha_T]$$

$$= n_P\left[\frac{\sqrt{3}}{2}\cos\alpha_T - \frac{1}{2}\sin\alpha_T\right]$$

$$= n_P\left[\frac{\sqrt{3}}{2}\left(\sqrt{1-\frac{1}{n_P^2}} - \frac{1}{2n_P}\right)\right]$$

$$= \frac{\sqrt{3}}{2}\left(\sqrt{n_P^2-1}\right) - \frac{1}{2}$$

$$1 + 2\sin\alpha = \sqrt{3}\left(\sqrt{n_P^2-1}\right)$$

$$n_P^2 - 1 = \frac{(1+2\sin\alpha)^2}{3}$$

$$n_P = \left[1 + \frac{(1+2\sin\alpha)^2}{3}\right]^{\frac{1}{2}} \qquad (7)$$

The value of sinα can be calculated from (6) using experimentally measured values of a and c. With known value of sinα, $n_p$ can be calculated **Ans**

## Second Method

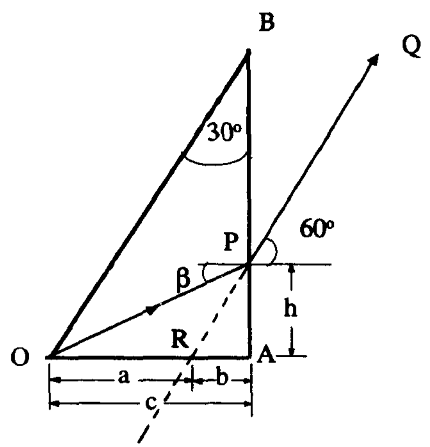



Fig 18.8

- Draw line AB along the vertical line
- At P on AB draw line RPQ making 60° angle with the horizontal line as shown in Fig 18.8
- Place the prism on the paper so that the side subtending 60° angle coincides with line AB. Direct the line of sight along PQ,while slide the prism along line AB until point O or the vertex angle is in view.
- At this position, draw lines along the edges of the prism to mark its position.

From the figure, $\tan\beta = \frac{h}{c}$ (1)

$$\tan 60^o = \frac{h}{b} \qquad (2)$$

$$h = \sqrt{3}b$$

$$\frac{\sin 60^o}{\sin\beta} = \quad {}_a\mu_P = n_P$$

From (1) $\tan^2\beta = \frac{\sin^2\beta}{\cos^2\beta} = \left(\frac{h}{c}\right)^2$

$$\frac{\sin^2\beta}{1-\sin^2\beta} = \left(\frac{h}{c}\right)^2$$

Substitute sinβ and h to obtain

$$\frac{3}{4n_P^2}.\frac{1}{1-\frac{3}{4n_P^2}} = \left(\frac{h}{c}\right)^2 = 3\left(\frac{b}{c}\right)^2$$

$$\frac{3}{4n_P^2-3} = 3\left(\frac{b}{c}\right)^2$$

$$4n_P^2 - 3 = \left(\frac{c}{b}\right)^2$$

$$n_P^2 = \frac{3+\left(\frac{c}{b}\right)^2}{4}$$

$$n_P = \frac{1}{2}\left[3+\left(\frac{c}{b}\right)^2\right]^{\frac{1}{2}}$$

c and b are mearued and known, hence $n_P$ can be calculated. **Ans**

## 1.2 Determination of refractive index of the liquid using two given prisms

- Place two prisms together with the frosted sides facing each other. and then lay the two prisms on the dish annointed with the liquid in an appropriate quantity.The liquid gradually diffuses through the contact area between the two prisms to form a thin film of liquid.
- The 60° vertices of the two prisms plus the thin film of liquid between them form a point where light is incident and then reflects or refracts as the case may be.
- Place the dish on the rotatable table, so that point K where the 60° vertices of the two prisms meet is at the centre of the rotating table.
- From the opposite side of the table, look at K against the lighten background along the

line of symmetry separating the two prisms.

- Slowly rotate the table in the counter-clockwise direction, while keeping the line of sight at K against the lighting background ( Look at K through one prism.)
- Keep rotating the table until the lighteng background disappears from view due to total reflection in side the prism.

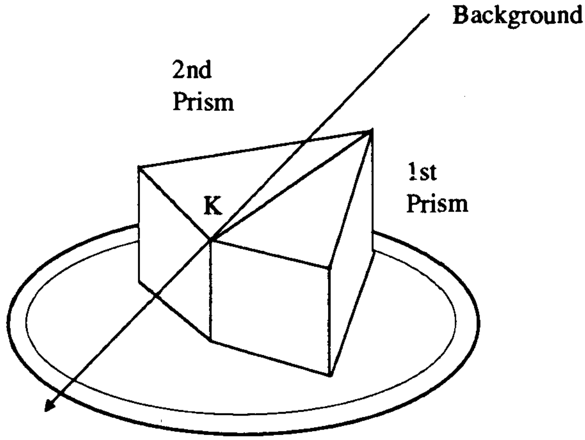



Fig.18.9

- Mark position of point B where the line of symmetry intersects side b of the second prism.

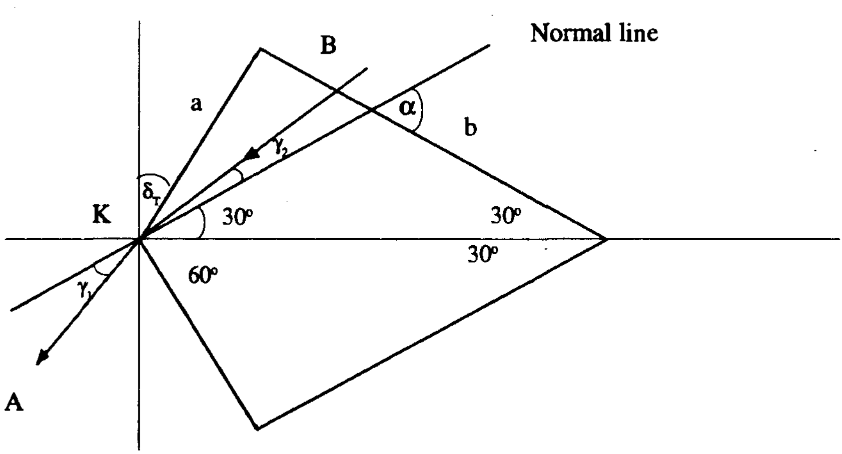



Fig 18.10

Note that the film is very thin such that light beam travelling through the film does not alter the direction.

Consider the path of light beam along AK.

$$\frac{\sin\gamma_1}{\sin\gamma_2} = n_P \qquad (1)$$

From the diagram $$\sin\alpha = \frac{a}{\sqrt{a^2+b^2}} \qquad (2)$$

Consider $\Delta$BKK′ $$\alpha = \beta + 30^o \qquad (3)$$

$$\gamma_1 = 30^o - \beta = 60^o - \alpha \qquad (4)$$

Let $\delta_T$ be the angle of incident of light beam from B to K which causes total reflection (between the second prism and the liquid)

$$90^o - \delta_T = \angle BKK' = 30^o - \gamma_2$$

$$\delta_T = 60^o + \gamma_2 \qquad (5)$$

The refractive index of liquid $$n_l = \frac{n_P \sin\delta_T}{\sin 90^o}$$

Substitute $\delta_T$ from (4) to obtain $$n_l = n_P \sin(60^o + \gamma_2) \qquad (6)$$

Express all angles in radians $$n_l = \sin(\frac{\pi}{3} + \gamma_2) \qquad (7)$$

From (1) $$\gamma_2 \approx \frac{\gamma_1}{n_P} \approx \frac{\frac{\pi}{3} - \alpha}{n_P}$$

The value of $\alpha$ is known from (2) hence $n_l$ can be calculated **Ans**

# International Physics Olympiad XIX

# 1988

# Bad Ischala, Austria

## Theory

### Problem 1 Spectroscopy of Particle Velocities

### Basic Data

The absorption and emission of photon is a reversible process. A good example is to be found in the excitation of an atom from the ground state to a higher energy state and the atom's subsequent return to the ground state. In such a case we may detect the absorption of photon from the phenomenon of spontaneous emission or fluorescence. Some of the more modern instrumentation make uses of this principle to identify atoms, and also to measure or calculate the value of the velocity in the velocity spectrum of the electron beam.

In one ideal experiment ( See Fig. 19.1 ), a singly- charged ion travels in the opposite direction to light from a laser source with velocity v. The wavelength of light from the laser source is adjustabale. An ion with velocity 0 can be excited to a higher energy state by the application of laser light having wavelength for example $\lambda = 600$ nm. If we excite a moving ion, our knowledge on Doppler's effect tells us that we need to apply laser light of wavelength other than the value given above.
Given a velocity spectrum embracing volcity magnitude from $v_1 = 0$ m/s to $v_2 = 6{,}000$ m/s (See Fig.19.1)

1.1

1.1.1 What range of wavelength of laser beam must be used to excite ions of all velocities in the velocity spectrum given above?
(N.B. The problem must be analyzed using classical Doppler's effect, ie Doppler's effect based on Theory of Special Relativity may not be used in this part of the problem. )

1.1.2 A rigorous analysis of the problem calls for application of the principle from Theory of Special Relativity ie.

$$\nu' = \nu\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{x}}}$$

Determine the error when the clssical formula for Doppler's effect is used to solve the problem.

1.2 Assuming the ions are accelerated by potential U before being excited by the laser beam, determine relatioship between the width of velocity spectrum of the ion beam and accelerating potential. Does the accelerating voltage increase or decrease velocity spectrum width ?

1.3 Each ion has the value $e/m = 4\times10^6$ C/ kg, two energy levels corresponding to wavelength $\lambda^{(1)} = 600$ nm and wavelength $\lambda^{(2)} = \lambda^{(1)} + 10^{-3}$ nm. Show that lights of the two wavelengths used to excite ions overlap when no accelerating potential is applied. Can accelerating voltage be used to separate the two spectra of laser light used to excite ions so that they no longer overlap? If the answer is positive, calculate the minimum value of the voltage required.

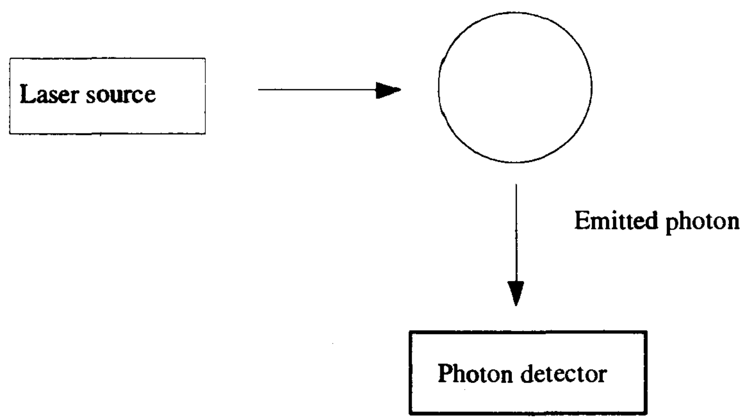



Fig. 19.1

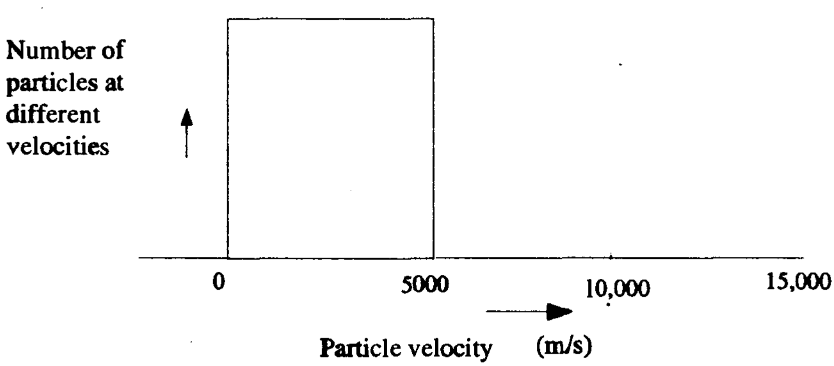



Fig. 19.2

## Solution

1.1

1.1.1 Let v be the velocity of the ion towards the laser source relative to the laser source.

$\nu'$ frequency of laser light as observed by the observer moving with the ion(ie.in the frame in which the velocity of the ion is 0.)

$\nu$ frequency of laser light as observed by the observer at rest with respect to the laser source

.Classical formula for Doppler's effect is given as

$$\nu' = \nu\left(1+\frac{v}{c}\right) \quad (1)$$

Let $\nu^*$ be the frequency absorbed by ion( characteristic of individual ions)

$\nu_L$ frequency of laser light used to excite ion at rest

Hence $\nu^* = \nu_L$

For a moving ion, the frequency used to excite ions must be lower than $\nu^*$.

Let $\nu_H$ be frequency used to excite the moving ion .

**When no accelerating voltage is applied**

| Frequency of laser light used to excite ions | Magnitude of velocity of ions | Frequency of laser light absorbed by ions | Wavelength of laser light used to excite ions |
|---|---|---|---|
| $\nu_H$ | 0 | $\nu^*$ | $\lambda_1$ |
| $\nu_L$ | $v = 6\times10^3 m/s$ | $\nu^*$ | $\lambda_2$ |

$\nu_L < \nu_H$ $\nu_L = \nu^*$

Calculation of frequency $\nu_H$ absorbed by moving ions.

$$v^* = v_L(1+\frac{v}{c}) \tag{2}$$

where $v^* = v_H = 5\times10^{14}$ Hz

$v = 6\times10^3$ m/s

Difference in the values of the frequencies absorbed by stationary ion and ion moving with velocity **v** $= v_H - v_L$

Difference in the values of the wavelengthes absorbed bystationary ion and ion moving with velocity **v** $= \lambda_L - \lambda_H$

(Higher frequency implies shorter wavelength)

$$\lambda_L - \lambda_H = \frac{c}{v_L} - \frac{c}{v_H}$$

From (2) $$\lambda_L - \lambda_H = \frac{c}{v^*}(1+\frac{v}{c}) - \frac{c}{v^*}$$

$$= \frac{v}{v^*}$$

In this case $$\lambda_L - \lambda_H = \frac{6\times10^3}{5\times10^{14}} \quad \text{m}$$

$$= \quad 12\times10^{-3} \text{ nm} \qquad \textbf{Ans}$$

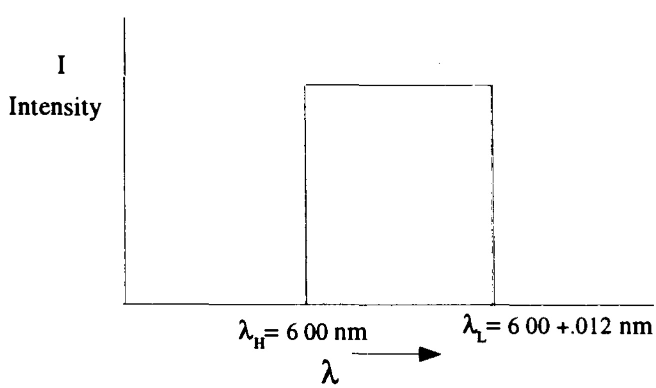



Fig. 19.3 Spectrum of laser light used to excite ions

1.1.2 The formula for calculation of $\nu'$ as observed by the observer moving towards light source based on the Principle of Theory of Special Relativity,

$$\nu' = \nu\sqrt{\frac{\left(1+\frac{v}{c}\right)}{\left(1-\frac{v}{c}\right)}}$$

where v is the manitude of the velocity of the observer towards the light source

$\nu'$ frequency absorbed by the ion moving with velocity v towards the light source(also observed by the observer moving with velocity v towards the laser source.

$\nu$ frequency of laser light as observed by an observer at rest.

(To put in a metaphoric way, the moving ion sees the laser light of frequency $\nu'$ even though the scientist who operates the laser source insists that he is sending a laser beam of frequency $\nu$. )

$$\nu' = \nu\sqrt{\left(1+\frac{v}{c}\right)\left(1+\frac{v}{c}+\frac{v^2}{c^2}+\ldots\ldots\right)}$$

$$= \nu\sqrt{\left(1+\frac{v}{c}\right)^2+\left(1+\frac{v}{c}\right)\frac{v^2}{c^2}+\ldots}$$

$$= \nu\left(1+\frac{v}{c}\right)\left[1+\frac{v^2}{c^2}\cdot\frac{1}{\left(1+\frac{v}{c}\right)}+\ldots\ldots\right]^{\frac{1}{2}}$$

$$= \nu\left(1+\frac{v}{c}\right)\left[1+\frac{v^2}{2c^2}\frac{1}{\left(1+\frac{v}{c}\right)}+\ldots\ldots\right]$$

The second term in the bracket represents the error if the classical formula for Doppler's effect is employed.

$$\frac{v}{c} = 2\times10^{-5}$$

$$\frac{v^2}{2c^2}\frac{1}{\left(1+\frac{v}{c}\right)}=\frac{1}{2}.\frac{4\times10^{-10}}{1+2\times10^{-5}}$$

$$\approx 2\times10^{-10}$$

The error in the application of classical formula for Doppler's effect however is of the order of the factor $2\times10^{-10}$. This means that classical formula for Doppler's effect can be used to analyze the problem without losing accuracy.

**1.2 When accelerating voltage is used.**

| Frequency of laser light used to excite ions | Magnitude of velocity of ions | Frequency of laser light absorbed by ions | Wavelength of laser light used to excite ions |
|---|---|---|---|
| $\nu_H^{/}$ | $v_H^{/}$ | $\nu^* = 5\times10^{14}$ | $\lambda_H^{/}$ |
| $\nu_L^{/}$ | $v_L^{/}$ | $\nu^* = 5\times10^{14}$ | $\lambda_L^{/}$ |

Lowest limit of the kinetic energy of ions $\frac{1}{2}m(v_L^{/})^2 = eU$ and $v_L^{/}=\sqrt{\frac{2eU}{m}}$

Highest limit of the kinetic energy of ions $\frac{1}{2}m(v_H^{/})^2=\frac{1}{2}mv^2+eU$ and $v_H^{/}=\sqrt{v^2+\frac{2eU}{m}}$

Spectrum width of velocity spectrum $v_H^{/}-v_L^{/}=\sqrt{v^2+\frac{2eU}{m}}-\sqrt{\frac{2eU}{m}}$ (3)

(Note that the final velocity of accelerated ion is not the sum of v and $\sqrt{\frac{2eU}{m}}$ as velocity changes with time .)

In equation (3) if $\sqrt{\frac{2eU}{m}}$ is negligibly small, the change in the width of the spectrum is negligible, by the same token of argument if $\sqrt{\frac{2eU}{m}}$ is large or approaches $\infty$ , the width of the spectrum of the light used in exciting the ions becomes increasingly narrow and approaches 0 **Ans**

1.3 Given two energy levels of the ion,
corresponding to wavelength $\lambda^{(1)} = 600$ nm
and $\lambda^{(2)} = 600 + 10^{-2}$ nm

For the sake of simlicity, the following sign notations will be adopted:

The superscript in the bracket indicates energy level 1 or 2 as the case may be. The sign $'$ above denotes the case when accelerating voltage is applied. And also the subscripts H and L appl y to absorbed frequencies (and also wavelengths) correspond to the high velocity and low velocity ends of the velocity spectrum of the ion beam respectively.

The subscript following $\lambda$ (or $v$) can be either 1 or 2, with number 1 corresponding to lowest velocity of the ion and number 2, the highest velocity of the ion. When no accelerating voltage is not applied, the subscript 1 implies that minimum velocity of the ion is 0, and the highestvelocity of ion, 6000 m/s. If accelerating voltage U is applied, number 1 indicates that the wavelength of laser light pertains to the ion of lowest velocity and 2 the ion of highest velocity ie..

Finally the sign * indicates the value of the wavelength ($\lambda^*$) or frequency($v^*$) absorbed by the ion.(characteristic absorbed frequency)

**When no accelerating voltage is applied**

For the First Energy Level

| Frequency of laser light used to excite ions | Magnitude of velocity of ions | Frequency of laser light absorbed by ions | Wavelength of laser light used to excite ions |
|---|---|---|---|
| $v_H^{(1)}$ | 0 | $v^{(1)*} = 5\times10^{14}$ Hz | $\lambda_1^{(1)}$ |
| $v_L^{(1)}$ | $v = 6\times10^3$ m/s | $v^{(1)*} = 5\times10^{14}$ Hz | $\lambda_2^{(1)}$ |

$$v_H^{(1)*} = v_L^{(1)*} = v^{(1)*} = 5\times10^{14}\ \text{Hz}$$

Difference in frequencies of laser light used to excite ions $= v_H^{(1)} - v_L^{(1)}$

Difference in wavelengths of laser light used to excite ions $= \lambda_L^{(1)} - \lambda_H^{(1)}$

$$= \frac{6\times10^3}{5\times10^{14}}$$
$$= .012 \text{ nm} \quad \textbf{Ans}$$

For the second energy level

| Frequency of laser light used to excite ions | Magnitude of velocity of ions | Frequency of laser light absorbed by ions | Wavelength of laser light used to excite ions |
|---|---|---|---|
| $\nu_H^{(2)}$ | 0 | $\nu^{(2)*}=5\times10^{14}$ Hz | $\lambda_H^{(2)}$ |
| $\nu_L^{(2)}$ | $v = 6\times10^3$ m/s | $\nu^{(2)*}=5\times10^{14}$ Hz | $\lambda_L^{(2)}$ |

$$\nu_H^{(2)*} = \nu_L^{(2)*} = \nu^{(2)*} = 5\times10^{14} \text{ Hz}$$

Difference in frequencies of laser light used to excite ions $= \nu_H^{(2)} - \nu_L^{(2)}$
Difference in wavelengths of laser light used to excite ions $= \lambda_L^{(2)} - \lambda_H^{(2)}$

$$= \frac{6\times10^3}{5\times10^{14}} = .012 \text{ nm}$$

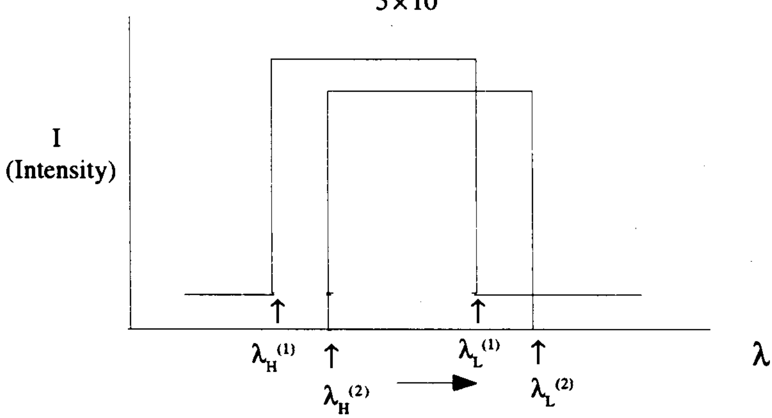



Fig. 19.4 Spectrum of laser light used to excite ions when no accelerating voltage is applied(Absorption Spectrum)

nformation given in the problem $\lambda_H^{(1)} = 600$ nm
$\lambda_L^{(2)} = 600 + .001$ nm

Hence the spectra of of laser light (absorption spectrum) usedto excite ion at two eneergy levels overlap as shown in Fig. 19.4 above.

**When accelerating voltage is applied**

Let $\lambda_H^{(1)/}$ and $\lambda_L^{(1)/}$be range of wavelengths used to excite ions in the first energy level, when accelerating voltage is applied.( Note the prime sign to denote the situation in which accelerating voltage is used.)
and $\lambda_H^{(2)/}$ and $\lambda_L^{(2)/}$ represent range of wavelengths used to excite ions in the second energy level also when accelerating voltage is applied.

Condition for the two spectra not to overlap..

$$\lambda_H^{(2)/} \geq \lambda_L^{(1)/} \qquad \text{(See Fig.19.4)} \qquad (4)$$

(Keep in mind that lower energy means longer wavelength and vice versa.)

From condition in (3) $$\lambda_L - \lambda_H = \frac{v}{v^*} \qquad (5)$$

The meanings of this equation is if the velocity of the ion is v, the wavelength which the ion "sees" is $\lambda_L$.
When $\lambda_H$ is wavelength which the ion of 0 velocity "sees".

Equation (5) may be rewritten in the context of the application of accelerating voltage in order for the the two spectra of laser light will not overlap as follows:

$$\lambda_L^{(N)/} - \lambda_H^{(N)} = \frac{v^/}{v^*} \qquad (6)$$

where N is the order of the energy level.

The subscript L relates λ to lowest velocity of the ion which "sees" frequency ν* .that lowest velocity in this case is $\sqrt{\frac{2eU}{m}}$

and the subscript H relates λ to highest velocity of the ion -in this case is $\sqrt{v^2 + \frac{2eU}{m}}$

Equation (6) will be used to calculate:

-Width of velocity spectrum of the ion accelerated by voltage U.
-potential U which results in condition given by (4).

Let us take up the second energy level (lower eneregy level of the two) of the ion first.

$$\lambda_L^{(2)\,\prime} - \lambda_H^{(2)} = \frac{v'}{v^*} \qquad (7)$$

Substitute $v' = \sqrt{\dfrac{2eU}{m}}$

$\lambda_H^{(1)} = 600 + 10^{-3}$ nm

$\nu^* = 5 \times 10^{14}$ Hz

$v = 0$ m/s

$$\lambda_H^{(2)\,\prime} = (600{+}.001) \times 10^{-9} + \frac{\sqrt{\dfrac{2eU}{m}}}{5 \times 10^{14}} \quad \text{m} \qquad (8)$$

Considering the first energy level of the ion

$$\lambda_L^{(1)\prime} - \lambda_H^{(1)} = \frac{v'}{v^*} \qquad (9)$$

In this case $v' = \sqrt{v^2 + \dfrac{2eU}{m}}$

$\nu^* = 5 \times 10^{14}$ Hz

$v = 6000$ m/s

$\lambda_H^{(1)} = 600 \times 10^{-9}$ m

$$\lambda_L^{(1)\,\prime} = 600 \times 10^{-9} + \frac{\sqrt{v^2 + \dfrac{2eU}{m}}}{5 \times 10^{14}} \quad \text{m} \qquad (10)$$

Substitute $\lambda_H^{(2)\,\prime}$ from (8) $\lambda_L^{(1)\prime}$ from (10) in (4) to get.

$$(600{+}.001) \times 10^{-9} + \frac{\sqrt{\dfrac{2eU}{m}}}{5 \times 10^{14}} \geq 600 \times 10^{-9} + \frac{\sqrt{v^2 + \dfrac{2eU}{m}}}{5 \times 10^{14}}$$

$$500 \geq \frac{\sqrt{v^2 + \dfrac{2eU}{m}}}{5 \times 10^{14}} - \frac{\sqrt{\dfrac{2eU}{m}}}{5 \times 10^{14}}$$

$$\sqrt{36 \times 10^6 + 2 \times 4 \times 10^6 U} - \sqrt{2 \times 4 \times 10^6 U} \leq 500$$

Assume that U is of the order of $10^2$ and over,

then $\sqrt{8 \times 10^6 U}\left(1 + \dfrac{9}{4U}\right) - \sqrt{8 \times 10^6 U} \leq 500$

$$\frac{1}{\sqrt{2U}} \times 9 \times 10^3 \leq 500$$

$$\sqrt{2U} \geq 324$$

$$U \geq 162$$

as the minimum value of voltage to avoid overlapping of asorption spectra **Ans**

The minimum value of accelerating voltage to avoid overlapping of absorption spectra is appoximately 162 V **Ans**

## Problem 2 Maxwell's Wheel

**Introduction.** A cylindrical wheel of uniform density, having mass M = 0.40 kg, radius R = 0.060 m, thickness d = 0.010 m is suspended by means of two light strings of the same length from the ceiling. Each string is wound around the axle of the wheel. Like the strings, the mass of the axle is negligible. When the wheel is turned manually, the strings are wound up until the centre of mass is raised 1.0 m above the floor. If the wheel is allowed to move downward vertically under the pulling force of the gravity, the strings are unwound to the full length of the strings and the wheel reaches its lowest point. The strings then begin to be wound in the opposite sense resulting in the wheel being raised upwards.

Analyze and answer the following questions; assuming that the strings are in vertical position and the points where the strings touch the axle are directly below their respective suspending points. (See Figs 19.5 a and b.)

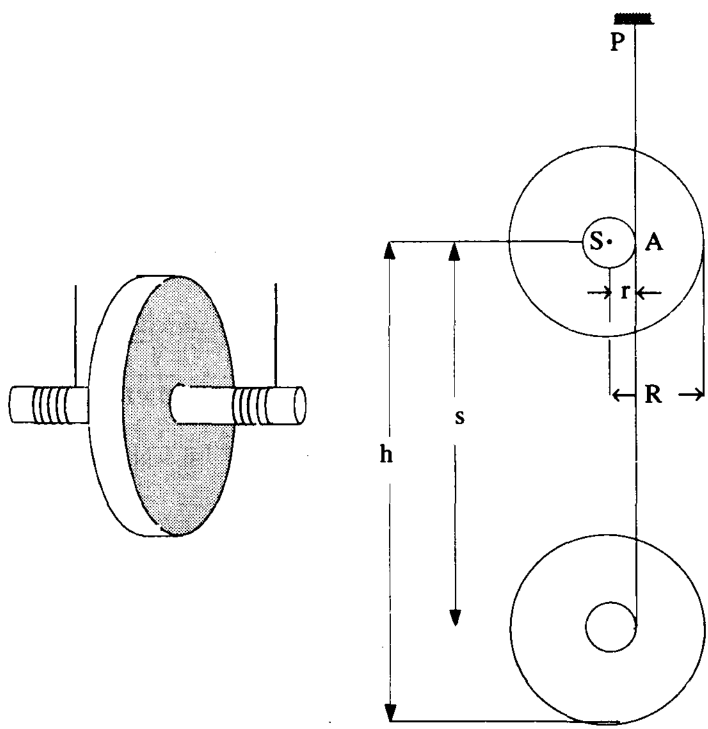



Fig. 19.5

2.1 Determine angular speed of the wheel when the centre of mass of the wheel covers vertical distance s.

2.2 Determine kinetic energy of linear motion of the centre of mass $E_T$ after the wheel travels distance $s = 0.50$ m, and calculate ratio between $E_T$ and energy in any other form in this problem upto this point.

Given the radius of the axle = 0.0030 m.

2.3 Determine tension in the string while the wheel is moving downward.

2.4 . Calculate angular speed $\omega'$ as a function of angle $\phi$ when the strings begin to unwind themselves in opposite sense as depicted in Fig. 19.6. Sketch a graph of variables which describe motion (In the cartesian system which suits the problem) and also speed of the centre of mass as a function of $\phi$.

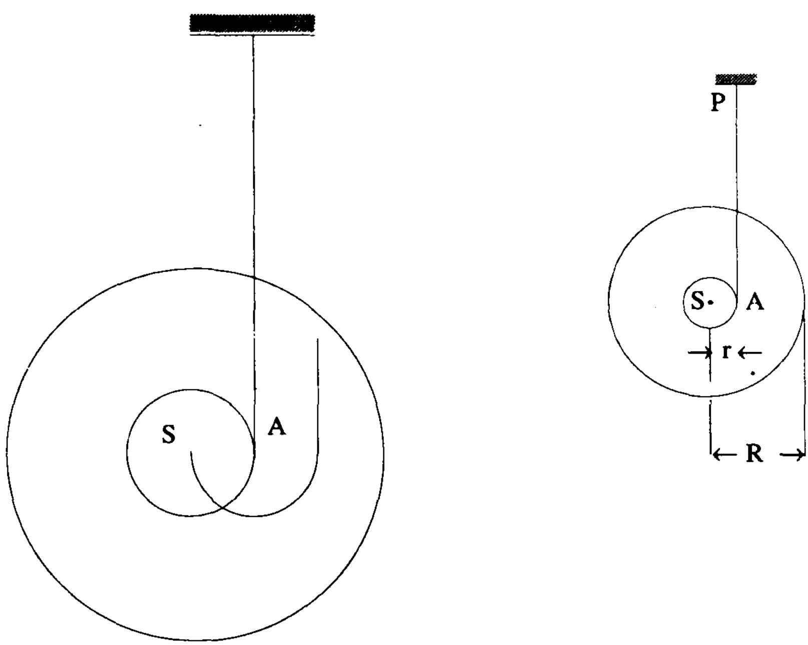



Fig 16.6

2.5 If the string can withstand maximum tenstion $T_m$ = 10 N, find the maximum length of the string which may be unwound without breaking by the wheel.

## Solution

2.1 From the princile of the conservation of energy

$$Mgs = \frac{1}{2} I_A \omega^2$$

where $\omega$ is angular speed of the wheel

$I_A$ moment of inertia about the axis through A

Note: If we take moment of inertia about S instead of (1) we would have,

$$Mgs = \frac{1}{2} I_S \omega^2 + \frac{1}{2} mv^2$$

where v is the speed of the centre of mass along the vertical.
This equation is the same as (1) in meanings since

$$I_A = I_S + Mr^2 \text{ and } I_S = MR^2$$

From (1) $\omega = \sqrt{\frac{2Mgs}{I_A}}$

Substitute $I_A = \frac{1}{2} Mr^2 + MR^2$

$$\omega = \sqrt{\frac{2gs}{r + \frac{R^2}{2}}}$$

Substitute
r = $3.00 \times 10^{-3}$ m
R = $6.0 \times 10^{-2}$ m
s = 0.50 m

$$\omega = \sqrt{\frac{2 \times 9.81 \times 0.50}{9 \times 10^{-6} + \frac{1}{2} \times 36 \times 10^{-4}}}$$

$= \sqrt{5248}$

= 72.4 rad/s **Ans**

2.2 Kinetic energy of linear motion of the centre of mass of the wheel is

$$E_T = \frac{1}{2} Mv^2 = \frac{1}{2} M\omega^2 r^2$$

$$= \frac{1}{2} \times 0.40 \times 5248 \times 9 \times 10^{-6}$$

$$= 9.76 \times 10^{-3} \quad \text{J}$$

(1) Potential energy of the wheel $E_P$ = Mgs

$$= 0.40 \times 9.81 \times 0.50 \quad \text{J}$$

$$= 1.962 \quad \text{J}$$

$$= 72.4 \quad \text{rad/s}$$

Rotational kinetic energy of the wheel

$$E_R = \frac{1}{2} I_S \omega^2$$

$$= \frac{1}{2} \times 0.40 \times 1.81 \times 10^{-3} \times 5248$$

$$= 1.899 \quad \text{J}$$

$$\frac{E_T}{E_P} = \frac{9.76 \times 10^{-3}}{1.962}$$

$$= 4.97 \times 10^{-3}$$

$$\frac{E_T}{E_R} = \frac{9.76 \times 10^{-3}}{1.899}$$

$$= 5.13 \times 10^{-3} \quad \text{J} \quad \textbf{Ans}$$

2.3 Let $\frac{T}{2}$ be tension in each string.

(2) Torque $\tau$ which causes rotation is given by

$$\tau = mgr = I_A \alpha$$

where $\alpha$ is angular acceleration

$$\alpha = \frac{Mgr}{I_A}$$

Equation of motion of the wheel is,

$$Mg - T = Ma$$

Substitute $a = \alpha r$ from (3) and $I_A = \frac{1}{2} Mr^2 + MR^2$ to get

$$T = Mg + \frac{Mgr^2}{\frac{1}{2} MR^2 + Mr^2}$$

$$= \left[ 1 + \frac{2r^2}{R^2 + 2r^2} \right] Mg$$

Tension in each string is given by $\frac{T}{2}$

$$\frac{T}{2}=\left[1+\frac{2.9\times10^{-6}}{3.6\times10^{-3}+1.8\times10^{-5}}\right]\left[\frac{0.40\times9.81}{2}\right]$$

$$= \quad 1.96 \quad \text{N} \quad \textbf{Ans}$$

2.4

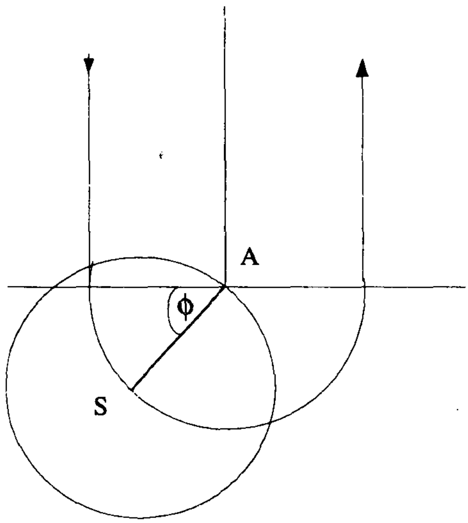



Fig 19.7

After the whole length of the strings is competely unwound,the wheel continues to rotate about A (which is at rest for some interval to be discussed.)

Let $\dot{\phi}$ be angular speed of the centre of mass about the axis through A.

Equation of rotational motion of the wheel about A may bewitten as

$$|\tau| = I_A\ddot{\phi}$$

where $\tau$ is torque about A

$I_A$ is moment of inertia about the axis through A.

$\ddot{\phi}$ angular acceleration about the axis through A

$$Mgr\cos\phi = I_A\ddot{\phi}$$

$$\ddot{\phi} = \frac{Mgr\cos\phi}{I_A}$$

$\times \dot{\phi}$ $$\dot{\phi}\ddot{\phi} = \frac{Mgr\cos\phi\dot{\phi}}{I_A}$$

$$\frac{1}{2}\frac{d(\dot{\phi})^2}{dt} = \frac{Mgr\cos\phi}{I_A}\frac{d\phi}{dt}$$

$$(\dot{\phi})^2 = \frac{2Mgr\sin\phi}{I_A} + c \qquad \text{( c = arbitrary constant )}$$

$\dot{\phi} = \omega$ when $\phi = 0$ (s = H)

From (3) $$\omega = \frac{2MgH}{I_A}$$

$$c = \frac{2MgH}{I_A}$$

$$\dot{\phi} = \omega = \left[\frac{2MgH\sin\phi}{I_A}\left(1+\frac{r}{H}\right)\right]^{\frac{1}{2}}$$

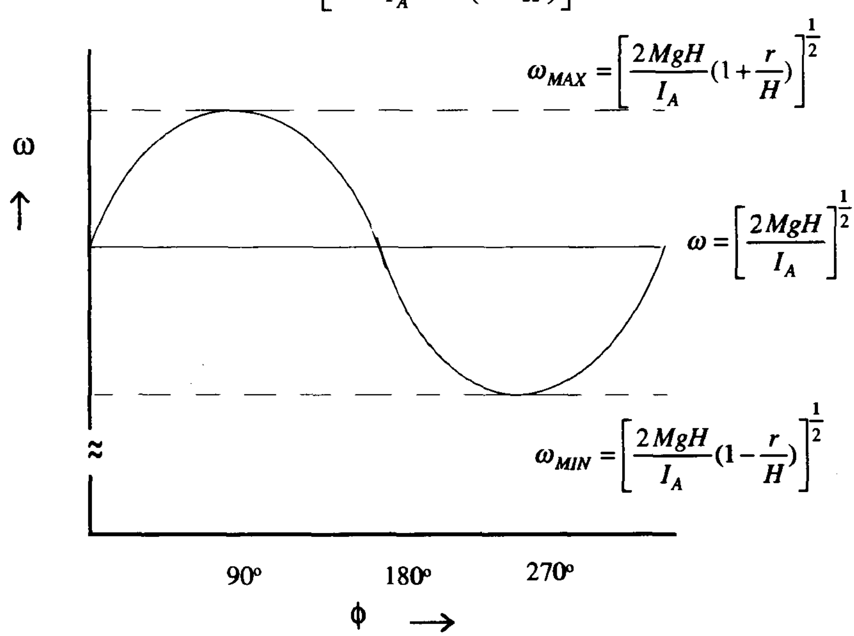



Fig.19.8

For $\frac{r}{H} <<< 1$ $$\omega = \omega'_{MAX} = \left[\frac{2MgH}{I_A}\right]^{\frac{1}{2}}$$

$$v = r\omega'_{MAX} = r\left[\frac{2MgH}{I_A}\right]^{\frac{1}{2}}$$

Component of the displacement

along x axis is, $x = r\sin\phi - r$

and along y axis is $y = r\cos\phi - r$

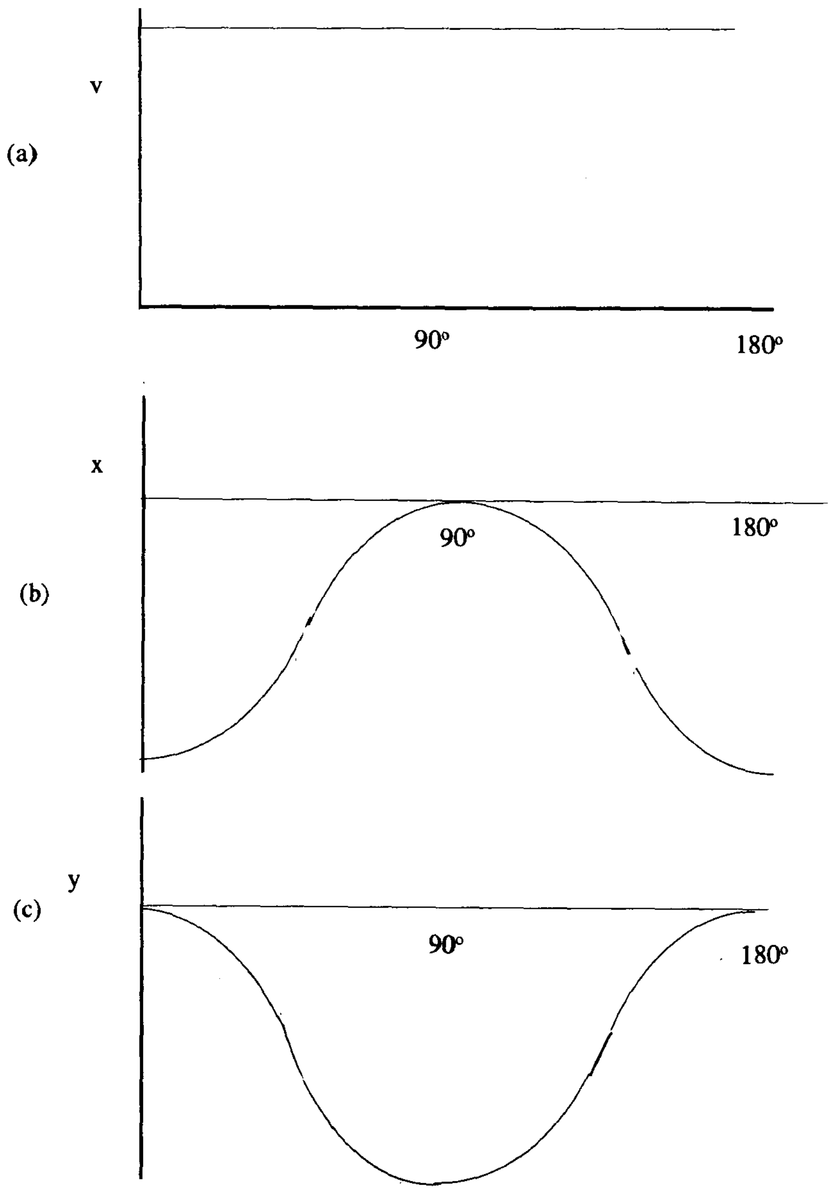



Fig.19.9

2.5 Maximum tension in each string occurs

$$\dot{\phi} = \omega'_{MAX}$$

Equation of motion is

$$T_{MAX} - Mg = M\left(\omega'_{MAX}\right)^2 r$$

Substitute T = 20 N

$$20 = M\left[g + \left(\omega'_{MAX}\right)^2 r\right]$$

Substitute $\omega'_{MAX} = \left[\dfrac{2Mgs}{I_A}\right]^{\frac{1}{2}}$

where s is the maximum length of the strings supporting the wheel without breaking ,

and $I_A = M\left[\dfrac{R^2}{2} + r^2\right]$

$$20 = M\left[g + \frac{4gsr}{R^2 + 2r^2}\right]$$

Subtitute $r = 3 \times 10^{-3}$ m

$$R = 6 \times 10^{-2} \text{ m}$$

$$20 = Mg[\ 1 + \frac{4 \times 3 \times 10^{-3} s}{36 \times 10^{-4} + 2 \times 9 \times 10^{-6}}\ ]$$

$$20 = Mg\ [\ 1 + 3.33\ s]$$

$$1 + 3.33\ s = \frac{20}{0.40 \times 9.81}$$

$$1 + 3.33\ s = 5.1$$

$$3.33\ s = 4.1$$

$$s = 1.24 \text{ m}$$

Maximum length of the strings which support maximum tension without breaking is 1.24 m **Ans**

## Problem 3 Recombination of positive and negative ions in ionized gas

**Introduction** A gas consists of positive ions of some elemen t (at high temperature) and electrons. The positive ion belongs to the atom of unknown mass number Z. What is known about this ion is it has only one electron in the shell or orbit.

Let this ion be represented by symbol $A^{(Z-1)+}$ and given

$$\varepsilon_0 = 8.85 \times 10^{-12} \quad \text{A.s/V.m or C/V.m}$$

Electron charge $\quad e = 1.602 \times 10^{-10}$ A.s

$$q^2 = \frac{e^2}{4\pi\varepsilon_0} = 2.037 \times 10^{-28} \text{ J.m}$$

Planck's constant $\quad \bar{h} = 1.054 \times 10^{-10}$ J..s

Mass of electron $\quad m_p = 9.108 \times 10^{-31}$ kg

Bohr's atomic radius $\quad r_B = \dfrac{\bar{h}}{m_0 q^2} = 5.92 \times 10^{-11}$ m

Rydberg's energy $\quad E_R = \dfrac{q^2}{2r_B} = 2.180 \times 10^{-18}$ J

Rest mass of proton $\quad m_p c^2 = 1.503 \times 10^{-10}$ J

Answer the following five questions:

3.1 Assume that the ion which has one electron left in the shell ie. $A^{(Z-1)+}$ is in the ground s tate.

In this lowest energy state, the square of the average distance of the electron from the nucleus or $r^2$ with components along x y and z axes being $(\Delta x)^2$ $(\Delta y)^2$ and $(\Delta z)^2$ respectively and,

$$r^2_0 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$

also the square of the average momentum by,

$$p^2_0 = (\Delta p_X)^2 + (\Delta p_Y)^2 + (\Delta p_Z)^2.$$

Given $\Delta p_X \geq \dfrac{\bar{h}}{2\Delta x}$ , $\Delta p_Y \geq \dfrac{\bar{h}}{2\Delta y}$ and $\Delta p_Z \geq \dfrac{\bar{h}}{2\Delta z}$

Write inequality involving $(p_0)^2(r_0)^2$ in a complete form.

3.2 The ion represented by $A^{(Z-1)+}$ may capture an additional electron and consequently emits a photon, write down an equation which is to be used for calculating the frequency of emitted photon .

3.3 Calculate energy of ion $A^{(Z-1)+}$ using the value of the lowest energy. The calcualtion should be approximated based on the following principles:

a. Potential energy of the ion should be expressed in terms of the average value of $\frac{1}{r}$ ( ie. $\frac{1}{r_0}$ , $r_0$ is given in the problem )

b. In calculating kinetic energy of the ion, use the average value of the square of the momentum given in 3.1 after being simpified by $(p_0)^2(r_0)^2 \approx (\bar{h})^2$

3.4 Calculate energy of ion $A^{(Z-2)+}$ taken to be in the ground state, using the same pricipie as the calculation of the energy of ion $A^{(Z-1)+}$. Given the average distance of each of the two electrons in the outermost shell (same as $r_0$ given in 3.3) denoted by $r_1$ and $r_2$, assume the average distance between the two electrons is given by $r_1+r_2$ and the average value of the square of the momentum of each electron obeys the principle of uncertainty ie.

$$(p_1)^2(r_1)^2 \approx (\bar{h})^2 \text{ and } (p_2)^2(r_2)^2 \approx (\bar{h})^2$$

**Hints**: Make use of the information that in the ground state $r_1 = r_2$.

3.5 Consider in particular the ion $A^{(Z-1)+}$ is at rest in the ground state when capturing an additional electron and the captured electron is also at rest prior to the capturing. Determine the numerical value of Z, if the frequency of emitted photon accompaning electron capturing is $= 2.057 \times 10^{17}$ rad/s. Identify the element which gives rise to this ion.

## Solution

3.1
$$r_0^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$

$$p_0^2 = (\Delta p_X)^2 + (\Delta p_Y)^2 + (\Delta p_Z)^2$$

Since
$$\Delta p_X \geq \frac{\bar{h}}{2\Delta x} \qquad (\Delta p_X)^2 \geq (\frac{\bar{h}}{2\Delta x})^2$$

$$\Delta p_Y \geq \frac{\bar{h}}{2\Delta y} \qquad (\Delta p_Y)^2 \geq (\frac{\bar{h}}{2\Delta y})^2$$

$$\Delta p_Z \geq \frac{\bar{h}}{2\Delta z} \qquad (\Delta p_Z)^2 \geq (\frac{\bar{h}}{2\Delta z})^2$$

$$p_0^2 \geq \frac{(\bar{h})^2}{4}\left[\frac{1}{(\Delta x)^2}+\frac{1}{(\Delta y)^2}+\frac{1}{(\Delta z)^2}\right]$$

and $$(\Delta x)^2 = (\Delta y)^2 = (\Delta z)^2 = \frac{r_0^2}{3}$$

$$p_0^2 r_0^2 \geq \frac{9}{4}(\bar{h})^2 \quad \textbf{Ans}$$

3.2 Given $|v_e|$ is the speed of the external electron before the capture

$|V_i|$ speed of $A^{(Z-1)+}$ before capturing

$|V_f|$ speed of $A^{(Z-1)+}$ after capturing

$E_n$ energy of the emitted photon = $h\nu$

From the principle of conservation of energy

$$\frac{1}{2}mev_e^2 + \frac{1}{2}(M+m_e)V_i^2 + E[A^{(Z-1)+}] = \frac{1}{2}(M+2m_e)V_f + E[A^{(Z-2)+}]$$

where $E[A^{(Z-1)+}$ and $E[A^{(Z-2)+}]$ are energy of electron in the outermost shell of ions $A^{(Z-1)+}$ and $A^{(Z-2)+}$ respectively.

From the principle of conservation of momentum

$$m_e\mathbf{v}_e + (M+m)\mathbf{V}_i = (M+2m_e)\mathbf{V}_f + \left(\frac{h\nu}{c}\right)\mathbf{1}_r$$

where $\mathbf{1}_r$ is unit vector pointing in the direction of motion of the emitted photon. **Ans**

3.3 Determination of energy of $A^{(Z-1)+}$

Potential energy $= -\dfrac{Ze^2}{4\pi\varepsilon_0 r_0} = -\dfrac{Zq^2}{r_0}$

Kinetic energy $= \dfrac{p^2}{2m}$

If the motion of the electrons is confined within xy plane, principles of uncertainty in 3.1 can be written as,

$$r_0^2 = (\Delta x)^2 + (\Delta y)^2$$
$$p_0^2 = (\Delta p_X)^2 + (\Delta p_Y)^2$$

$$= \frac{1}{4}(\bar{h})^2 \left[ \frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} \right]$$

$$= \frac{1}{4}(\bar{h})^2 \left[ \frac{2}{r_0^2} + \frac{2}{r_0^2} \right]$$

$$p_0^2 r_0^2 = (\bar{h})^2$$

$$E[A^{(Z-1)+}] = \frac{p_0^2}{2m_e} - \frac{Zq^2}{r_0}$$

$$= \frac{(\bar{h})^2}{2m_e r_e^2} - \frac{Zq^2}{r_0}$$

Energy is minimum when $\frac{dE}{dr_0}$ is 0.

$$\frac{dE}{dr_0} = -\frac{(\bar{h})^2}{m_e r_e^3} + \frac{Zq^2}{r_0^2} = 0$$

$$\frac{1}{r_0} = \frac{Zq^2 m_e}{(\bar{h})^2}$$

$$E[A^{(Z-1)+}] = \frac{(\bar{h})^2}{2m_e}\left(\frac{Zq^2 m_e}{\bar{h}}\right)^2 - Zq^2 \frac{Zq^2 m_e}{(\bar{h})^2}$$

$$= -\frac{m_e}{2}\left(\frac{Zq^2}{\bar{h}}\right)^2$$

$$= -\frac{q^2 Z^2}{2r_B} = -E_R Z^2 \textbf{ Ans}$$

3.4 In the case of $A^{(z-1)+}$ captures second electron

Potential energy of both electrons $= -2\frac{Zq^2}{r_0}$

Kinetic energy of the two electrons $= 2\frac{p^2}{2m} = \frac{(\bar{h})^2}{m_e r_0^2}$

Potential energy due to interaction between the two electrons $= \frac{q^2}{|\mathbf{r}_1 - \mathbf{r}_2|}$

From data given $|\mathbf{r}_1 - \mathbf{r}_2| = 2r_0$

$$E[A^{(Z-2)+}] = \frac{(\bar{h})^2}{m_e r_0^2} - \frac{2Zq^2}{r_0^2} + \frac{q^2}{2r_0}$$

Total energy is lowest when $\frac{dE}{dr_0} = 0$

$$0 = -\frac{2(\bar{h})^2}{m_e r_0^3} + \frac{2Zq^2}{r_0^3} - \frac{q^2}{2r_0^2}$$

$$\frac{2(\bar{h})^2}{m_e r_0^2} = \frac{q^2(2Z - \frac{1}{2})}{r_0^2}$$

$$\frac{1}{r_0} = \frac{q^2 m_e}{2(\bar{h})^2}(2Z - \frac{1}{2})$$

$$\frac{1}{r_0} = \frac{1}{r_B}(Z - \frac{1}{4})$$

$$E[A^{(Z-2)+}] = \frac{(\bar{h})^2}{m_e}[\frac{q^2 m_e}{2(\bar{h})^2}]^2 - \frac{q^2(2Z - \frac{1}{2})}{\bar{h}} \cdot \frac{q^2 m_e (2Z - \frac{1}{2})}{2\bar{h}}$$

$$= \frac{m_e}{4}[\frac{q^2(2Z - \frac{1}{2})}{\bar{h}}]^2 - \frac{m_e}{2}[\frac{q^2(2Z - \frac{1}{2})}{\bar{h}}]^2$$

$$= -\frac{m_e}{4}[\frac{q^2(2Z - \frac{1}{2})}{\bar{h}}]^2$$

$$= -\frac{m_e[q^2(Z - \frac{1}{4})]^2}{(\bar{h})^2}$$

$$= -\frac{q^2(Z - \frac{1}{4})^2}{r_B^2}$$

$$= -2E_R(Z - \frac{1}{4})^2 \qquad \textbf{Ans}$$

3.5 In the case of ion $A^{(Z-1)+}$ is at rest when it captures the second electron also at rest before the capturing. From the information provided in the problem, the frequency of photon emitted is given by,

$$\nu = \frac{\omega}{2\pi} = \frac{2.057 \times 10^{17}}{2\pi} \quad \text{Hz}$$

Energy equation is simplified to

$$E[A^{(Z-1)+}] - E[A^{(Z-2)+}] = \bar{h}\omega = \bar{h} \times 2.507 \times 10^{17}$$

$$-E_R Z^2 - \left[-2E_R\left(Z-\frac{1}{4}\right)^2\right] = \bar{h}\omega$$

Substitute $\bar{h} = 1.05 \times 10^{-34}$ J.s

$E_R = 2.180 \times 10^{-18}$

$$2.180 \times 10^{-18}[-Z^2 + 2(Z-\frac{1}{4})^2] = 1.05 \times 10^{-34} \times 2.607 \times 10^{17}$$

$$Z^2 - Z + \frac{1}{8} = \frac{1.05 \times 2.607 \times 10^{-17}}{2.108 \times 10^{-18}} \approx 12.9$$

$$Z^2 - Z - 12.7 = 0$$

$$Z = \frac{1 \pm \sqrt{1+51}}{2} = \frac{8.21}{2} = 4.1$$

$Z = 4$ implies that the ion belongs to beryllium **Ans**

# Experiment

## Problem 1 Polarized Light Experiment

**General Information**

In the experimental part, the following apparatus and eqipment are provided:

- An electric tungsten bulb made of frosedt-surface glass complete with mount ing stand, 1 set.
- Three wooden clamps, each of which contains a slit for light experiment.
- Two glass plates one of which is rectangular and other square-shaped.
- One polaroid sheet (cicular-shaped)
- One red film or filter.
- Self adhesive tape tape (one roll).
- Self-adhesive labelling tape 6 pieces
- One cellophane sheet
- Black paper 1 sheet.
- One graph paper
- Drawing triangle with a handle.
- Unerazable luminocolour pen 312, exra fine and black colour.
- Lead pencils F and H types.
- Pencil sharpener
- Erazer
- A pair of scissors

**Important Instruction to be Followed**

- There are 4 pieces of labelling tape coded for each contestant. Stick the tape one each on the instrument marked with the sign # . Having done this, the contestant may proceed to perform the experiment to answer the questions.

- Cutting, etching, scraping, folding the polaroid is strictly forbidden.

- If marking is lto be made on the polaroid, use the lumino-colour pen provided and put the cap back in place after finishing.

- When marking is to be made on white paper sheet, use the white tape.

- Use lead pencil to draw or sketch a graph.

- Black paper may be cut into pieces for use in the experiment, but the best way of using the black paper is to roll it into a cylinder as to form a shield around the electric bulb. An aperture of proper size may be cut into the side of the cylinder to form an outlet for light used in the experiment.

- Red piece of paper is to be folded to form a double layer.

The following four questions will be answered by performing the experiment:

**Question 1.1**

a. Locate the axis of light trasmission of the polaroid film. This may be doneby observing light relflected from the surface of the rectangular glass plate provided. (Light trasmitting axis is the direction of vibration of elecric field vector of light wave transmitted through the polaroid.) Draw a straight line along the light transmission axis as exactly as possible \on the polaroid film. (#)

b. Set up the apparatus on the graph paper for the experiment to determine the refractive index of the glass plate for white light.

When unpolarized light is reflected lat the glass plate, reflected light is partially polarized.Polarization of the reflected light is maximum if the incident angle is equal to the refractive index of the glass plate, or tan i = n .
Draw lines or dots that are related to the determination of refractive index on the graph paper.(#)

**Question 1.2**

Assemble a polariscope to observe birefringence in birefringent glass plate when light is normally incident on the plastic sheet and the glass plates.

A birefringent object is the object which splits light into two components, with the electric field vectors of the two components perpendicular to each other. The two directions of the electric field vectors are known as birefringent axes characteristic of birefringent material. The two components of light travel with different velocity.

Draw a simple sketch depicting design and functions of thepolariscope assembled.

Insert a sheet of clear cellophane in the path of light in the polariscope. Draw lines to indicate birefringent axes (#). Comment briefly but concisely on what is observed, and describe how birefringent axes are located.

**Question 1.3**

a. Stick ten layers of self-adhesive tape provided on the glass plate as shown below. Make sure that each layer recedes in equal steps.

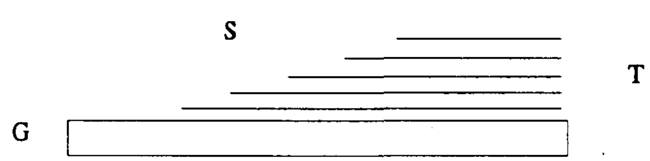



Fig. 19.10

G square glass plate as a substrate for the celophane layers

T 10 layers of cellophane sheet

S steps about 3~4 mm wide.

Insert the assembled squre plate into the path of light in the polariscope. Describe conditions for observing colours. How lcan these colours be changed ? Comment on the observations from this experiment.

b. Prepare monochromatic red by placing doubly-folded red plastic sheet in the path of white light. Mark on the asssembled square plate to show the steps which allow determination of the difference of the optical paths of the two components of light from birefringent phenomenon. described under 1.2 (#).
Estimate the difference of the optical paths from two consecutive steps

**Question 1.4**

a. With the polariscope assembled, examine the central part of the drawing triangle provided. Describe relevant optical properties of the drawing triangle pertaining to birefringence.

b. Comment on the results observed. Draw conclusions about the physical properties of the material of which the triangle is made.

**Additional Cautions**

Be sure that the folowing items affixed with the coded labels provided accompany the report.

#1 Polarized film with the position of trasmission axis clerly marked.

#2 Graph paper with lines and dots denoting experimental setup for determining refractive index.

#3 Sheet of cellophane paper with marking i ndicating the positions of birefringent axes.

#4 Square glass plate affixed with sellf-adhesive tape with markings to indicate the positions of birefringent axes.

# Solution

In this experiment the results from one experimental stage are used to solve problems in the following experimental stages. Without actually performing all parts of the exeriment, solution cannot be meaningfully discussed.

It suffices to add that some transparent crystals are anisotropically anisotropic, meaning their optical properties vary with the direction . Cystals wahich have this property are said to be doubly refractin or exhibit birefringence.

This phenomenon can be understood on the basis of wave theory. When a wavefront enters a birefringent material, two sets of Huygens wavelets propagate from every point of the entering wavefront causing the incident light to split into two components of two different velocities. In some crystal there is a particular direction ( or rather a set of parllel directions )in which the velocities of the two components are the same. This direction is known as optic axis. In other crystals there are two optic axes. The former is said to be uniaxial, and the latter biaxial.

If a plane porized light ( which may be white light or monchromatic light is allowed to enter a uniaxial birefringetn material, with its plane of polarization making some angle, say 45° with the optic axis, the incident light is splitted into two componments( ordinary and extraordinary ) travelling with two different velocities. Because of diffaerent velocities, their phases different.

Upon emerging fgrom the crystal, the two components recombine to from a resultant wave. The phase difference between the two components causes the resultant wave to be either linearly, or curcularly or ellitically polarized depending on the phase difference between the two components. The type of polarization can be determined by means of an analyser which is a second polaroid sheet provided for this experiment.

## Problem 2

**Introduction.** Freeelectrons in a metal may be thought of as being "electron gas" confined in potential or energy wells. Under normal condition or even when a voltage is applied near the surface of the metal, these electrons cannot leave the potential well ( See Fig 19.11) If however the metal or the electron gas is heated, the electrons have enough thermal energy (kinetic energy) to overcome the energy barrier W ( W is known as " work function"). If a voltage is applied across the metal and the anode, these thermally activated electrons may reach the anode.
The number of electrons arriving at the anode per unit time depends on the nature of the cathode and the temperaturre, i.e. all electron s freed from the potential will reach the anode no longer increase with aapplied voltage.(See Fig 19.11)
The saturated current corresponding to the number of thermally activated electrons freed from the metal surface per unit time obeys what is generally known as Richardson's equation i.e.

$$I_B = CT^2 e^{-\frac{W}{kT}}$$

where C is a constant

T temperature of the cathode in Kelvin

k Boltzmann's constant = $1.38 \times 10^{-23}$ ($Jk^{-1}$)

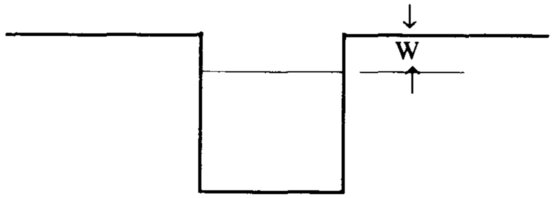



Fig 19.11

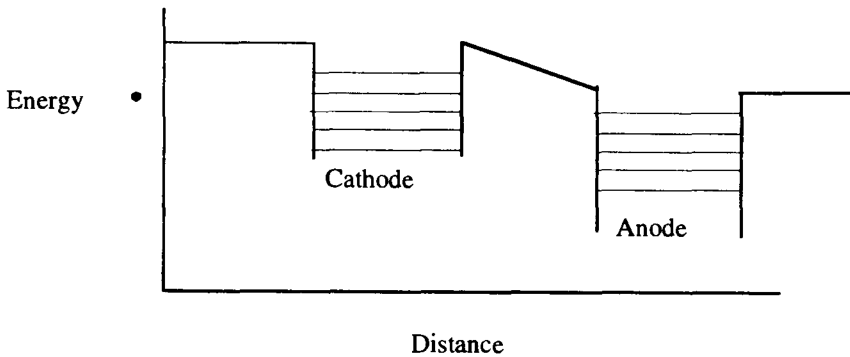



Fig.19.12

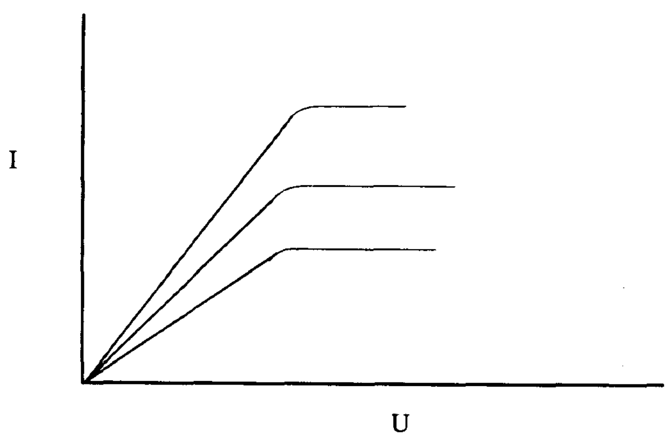



Fig 19.13 Graph of current as a function of voltage across anode-cathode

Determine the value of work function W of tungsten metal in the form of heating filament of the vacuum tube provided.
The following items of equipment are placed at the disposal of the contestant:

- Electron tube AZ 41 which is a high-vacuum, full-wave rectifying diode. The cathode is made from a coated tungsten filament the work function of which is to be ascertained. According to the manual prepared by its manufacturer, no more than 4 V should be used when applying heating current to the cathode. Since the tube has two anodes, it is most desirable to have them connected for all measurements. The diagram in Fig 19.14 is a guide to identifying the anodes and cathodes.

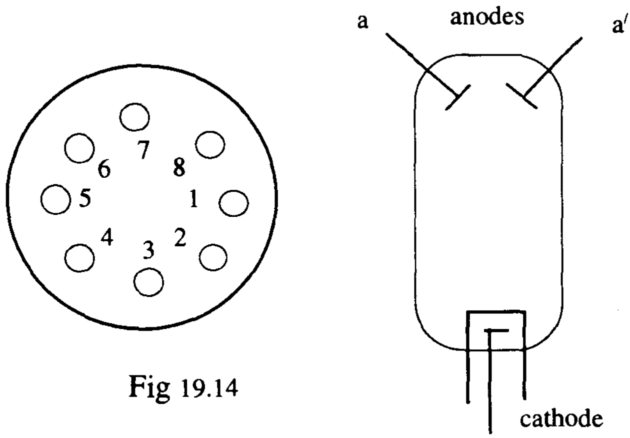



Fig 19.14

- Multimeter 1 unit, internal resistance for voltage measurement: 10 MΩ
- Battery 1.5 V (together with a spare.)
- Battery 9 V; 4 units can be connected in series as shown in Fig 19.15
- Connectors; 2 units
- Resistors; each of which has specifications as follows:
    - 1000 Ω ±2% (Brown, black, black, brown,brown, red)
    - 100 Ω ±2% (Brown, black, black, black,brown, red)
    - 47.5 Ω ±1%(yellow, violet, green, gold, brown)
- Resistors; 4 units, each of which has resistance of about 1 Ω and coded.
- Connectors; 12 units
- Connecting wire 20 cm in length; 6 units
- Screw driver; 1 unit
- Graph paper; 1 sheet
- Graph of specific resistance of tungsten as a function of temperature; 1 sheet

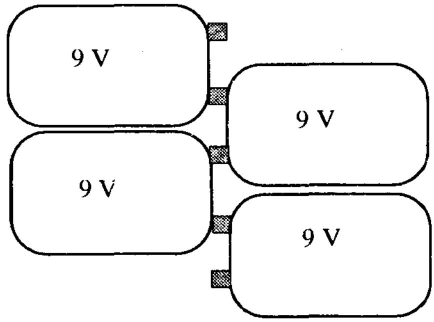



Fig 19.15

Solve the following problems:

5.1 Determine the resistance of 4 numerically -coded resistors. Under no circumstance must the multimeter be used as an ohmmeter.

5.2 Determine saturated current for 4 different values of cathode temperature,using 1.5 V battery to heat the cathode filament. A constant value of voltage between 35-40 V between the anode and the cathode is sufficient to produce saturated current. Obtain this value of voltage by connecting the four 9 V batteries in series. Describe how the different values of temperature are determined?

5.3 Determine the value of W. Explain the procedures used.

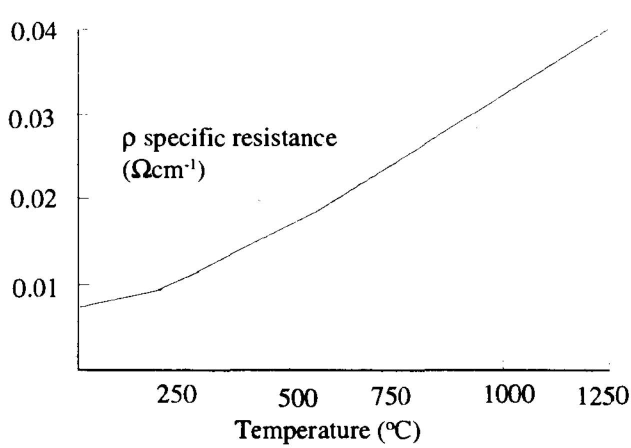



Fig 19.16

# Solution

5.1 Connect the circuit as shown in Fig 19.17

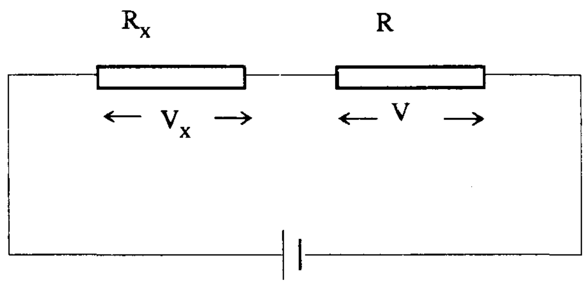



Fig. 19.17

$R_X$ is resistance to be determined
R known value of resisrtance

Measure potential difference across $R_X$ and R. Choose the value of R which gives comparable value of potential difference across $R_X$.

In this particular case R = 47.5 Ω

$$\frac{R_X}{R} = \frac{V_X}{V_R}$$

where $V_X$ and $V_R$ are values of potential difference across $R_X$ and R respectively.
$R_X$ can be calculated from the above equation.
(The error in $R_X$ depends on the errors of $V_X$ and $V_R$)

5.2 Connect the circuit as shown in Fig 19.18

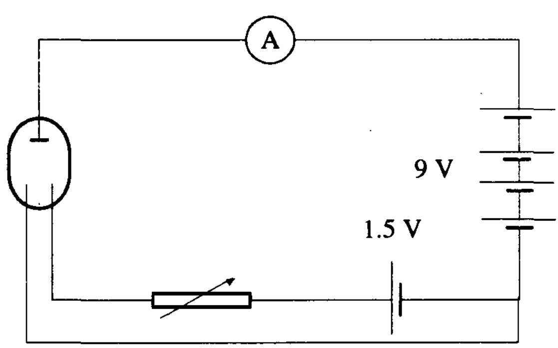



Fig 19.18

- Begin the experiment by measuring resistance $R_0$ of tungsten cathode when there is no heating current
- Add resistor $R = 1000\,\Omega$ into the cathode circuit, determine resistance $R_0$ of tungsten cathode with heating current in the circuit. Measure voltage across cathode and known resitance, calculate resistance of the current -carrying cathode
- Repeat the experiment ,using rsistor $R = 147.5\,\Omega$ in the cathode circuit, determine resistance $R_2$ of tungsten cathode with heating current in the circuit.
- Repeat the experiment , using resistor $R = 47.5\,\Omega$ in the cathode circuit, determine resistance $R_3$ of tungsten cathode with heating current in the circuit.
- Plot a graph of $1, \frac{R_1}{R_0}, \frac{R_2}{R_0}, \frac{R_3}{R_0}$ as a function of temperature, put the value of 1 to coincide with room temperature i.e 18° C approximately and draw the remaining part of the graph parallel to the graph of specific resistance as a function of temperature provided in the problem. From the graph, read the values of the temperature of the cathode $T_1$, $T_2$, $T_3$ in Kelvin.

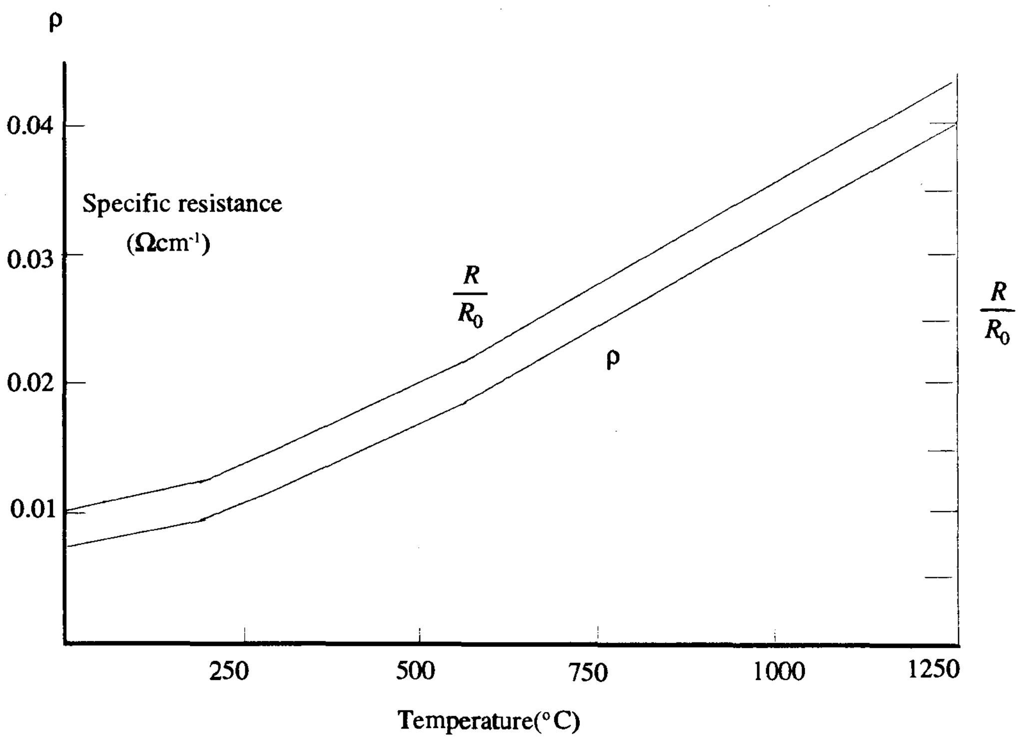



Fig 19.19

From the equation $I = CT^2 e^{-\frac{W}{kT}}$

$$\ln\frac{I}{T^2} = -\frac{W}{kT} + \ln C$$

Plot a graph of $\ln\frac{I}{T^2}$ against $\frac{1}{T}$.

The curve is linear. Determine slope m from the graph.

$$-m = -\frac{W}{k}$$

Work function W can be calculated using known values of m and k (given in the problem)

Error in W depends on error of T which in turn depends on the error of measured R.

# International Physics Olympiad XX

# 1989

# Warsaw, Poland

## Theory

### Problem 1

Liquids A and B do not mix and cannot be mixed to form a homogeneous mixture.The pressure of saturated water vapour of each individual liquid varies with temperature in accordance with the formula given below:

$$\ln\frac{P_i}{P_o}=\frac{a_i}{T}+b_i$$

where $p_0$ is normal atmospheric pressure
$p_i$ saturated vapour pressure of ith liquid
T absolute temperature of vapour
$a_i$ and $b_i$ are constants characteristic of ith liquid
ln natural logarithm or logarithm of base e ( e = 2.7182818....)

The values of ratio $\frac{P_i}{P_o}$ for liquids A and B at temperature 40° and 90° C are given in the table below:

| t (° C) | $\frac{P_i}{P_o}$ | |
|---|---|---|
| | i =A | i = B |
| 40° | 0.284 | 0.07278 |
| 90° | 1.476 | 0.6918 |

Errors of all values are negligible

1.1 Find boiling points of liquids A and B at pressure $p_0$ .

1.2 When liquids A and B are poured into a container, liquid layers formed are shown in Fig. 20.1.
The upper surface of liquid B is covered by a thin layer of liquid C which does not dissolve in liquid A nor liquid B. (Also neither liquid A nor liquid B dissolves in liquid C .) The effect is liquid B cannot evaporate freely. The ratio between the molecular masses of A and B in the gaseous stat is represented by $\gamma$ and

$$\gamma = \frac{M_A}{M_B}$$

At the start mass of A and of B are the same and equal m = 100 g.
The thickness of various layers is so thin that it can be assumed that the pressure at any point in the container is equal to normal atmospheric pressure, $p_0$.(The drawing has been exaggerated for clarity.)
Each th liquid system described is slowly heated in a uniform and continuous manner. It is found experimentally that the temperature of the system varies with time as shown in Fig. 20.2

Determine $T_1$ and $T_2$ which define the parallel part of the curve to the time axis i.e. T does not vary with time, and also mass of liquids A and B at time t. Give the values of $T_1$ and $T_2$ in round nearest numbers in ° C, and the values of mass to the accuracy of 1/10 g.

**N.B** Vapour pressure of the liquid

1. Obeys Dalton's law on the pressur e of a gaseous mixture, ie. the pressure of the mixture is the sum of partial pressure of individual gases which make up the mixture.
2. The gaseous mixture behaves as an ideal gas until its pressure induces saturated vapour.

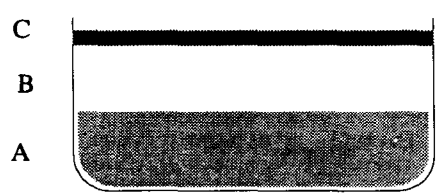



Fig. 20.1

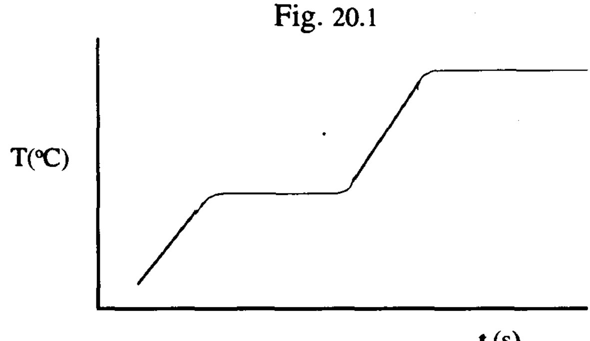



Fig. 20.2

## Solution

Each liquid begins to boil when its saturated vapour pressure is equal to the atmospheric pressure or pressure outsice and in contact with the liquid. In order to determine the boiling point of ith liquid ( i = A or B), we calculate T when $p_i = p_0$ or $\frac{P_i}{P_o} = 1$

Hence $\ln \frac{P_i}{P_o} = 0$

$$T_i = \frac{a_i}{b_i}$$

From the information given by the problem, we know the values of $P_i/P_0$ at 40° and 90° C, and we use these values to calculate $a_i$, $b_i$ by solving two simultaneous equations. From the values of $a_i$, $b_i$ calculated, the value of $T_i$ is obtained for liquid ith.

At 313 K $\ln 0.284 = \frac{a_A}{313} + b_A$

and 363 K $\ln 1.4766 = \frac{a_A}{363} + b_A$

$$\ln 0.284 - \ln 1.4766 = a_A(\frac{1}{313} - \frac{1}{363})$$

$$a_A = \ln\frac{0.284}{1.477} + \left(\frac{1}{313} - \frac{1}{363}\right)$$

$$= -3748.5$$

$$b_A = \ln 0.0284 - \frac{a_A}{363}$$

$$= 10.71$$

Boiling point of liquid A $T_A = \frac{3748.5}{10.71}$

$= 349.55 \approx 350$ K or 77° C **Ans**

Similarly, boiling point of liquid B

At 313 K $\ln 1.476 = \frac{a_B}{313} + b_B$

and at 363 K $\ln 0.6918 = \frac{a_B}{363} + b_B$

$$\ln 1.476 - \ln 0.6918 = a_B\left(\frac{1}{313} - \frac{1}{363}\right)$$

$$a_B = -5122$$

$$b_B = 13$$

$$T_B = 372.9 \text{ K or } 99.9^\circ \text{ C}$$

$$\sim 100^\circ \text{ C} \qquad \textbf{Ans}$$

b. Since all liquids are in the same container and in thermal contact with one another, their temperature values are equal and change at the same rate. At the beginning part of the experiment corresponding to linear increase in temperature, no evaporation of any liquid takes place. Also the evaporation of liquid B into the outside atmosphere cannot take place because it is covered by a layer of liquid C.

However the cover provided by liquid C cannot prevent evaporation or boiling inside liquids A and B. Internal boiling in the form of small bubbles inside the liquid is therefore possible and can take place when the pressure inside the bubble(saturated vapour pres sure) is equal to the pressure outside the bubble.

We may ask the question what is a condition for bubbles to be formed at intersurface of liquids A and B. The answer is: since a bubble is in contact simultaneously with both liquids, boiling takes place when the pressure inside the bubble is equal to the sum of saturated vapour pressure of liquid A and liquid B.( say $p_0$ )

Let us suppose that the liquid sysem is gradually heated until the bubbles of pressure $p_0$ and temperature $T_1$ are formed at the intersurface beween liquids A and B. Temperature $T_1$ is obviously the common boiling point of liquids A and B.

Under this condition, temperture $T_1$ is lower than the boiling points of both liquids A and B. At any given temperature the values of saturated vapour pressure of liquids A and B are different. At their common boiling point $T_1$, the sum of the values of saturated vapour pressure of liquids A and B is equal to $p_0$. Our next task is to find $T_1$ .

From the formula $$\frac{P_A}{P_O} = e^{\frac{a_A}{T} + b_A}$$

$$\frac{P_B}{P_O} = e^{\frac{a_B}{T} + b_B}$$

$$\frac{P_A}{P_O} + \frac{P_B}{P_O} = e^{\frac{a_A}{T} + b_A} + e^{\frac{a_B}{T} + b_B}$$

let $$y(T)=\frac{P_A}{P_O}+\frac{P_B}{P_O}=e^{\frac{a_A}{T}+b_A}+e^{\frac{a_B}{T}+b_B}$$

Boiling at the interssurface takes place when $\frac{P_A+P_B}{P_O}=\frac{P_O}{P_O}=1$

The value of T can be solved from the above equation graphically, ie. a graph of y(T) is plotted as a function of T and read the value of T that gives y(T) = 1.

By this means T = 67° C

The values of vapour pressure of liquids A and B are calculated from,

$$P_A=e^{\frac{a_A}{340}+b_A}.P_O=0.734P_O$$

$$P_B=e^{\frac{a_B}{340}+b_B}.P_O=0.267P_O$$

The value of vapour pressure in both cases depends on temperature only. Hence the size of the bubble remains the same while lthe bubble moves around in either liquid A or B.The volume of the bubble at the intersurface between the two liquids depends on the value of $p_0$ or $(p_A + p_B.)$ which is also constant for constant T.
This implies that the ratio between the mass of saturated vapour liquid A and the same of liquid B is the same for fixed value of T.
After the first liquid is exhausted by evaporation, the system of liquid turns into the system of liquid and gas( or vapour), and the temperature of this system begins to rise again.

Let $T_2$be the boiling point of the second liquid. The ratio between the mass of saturated vapour of liquid A and the same for liquid B in the bubble at the intersurface i.e. $\frac{m_A}{m_B}$ at $T_1$ is equal to the raio between the density of liquid A and liquid B i.e. $\frac{\rho_A}{\rho_B}$

Assume that saturated vapour behaves as an ideal gas.

$$P_AV_A=\frac{m_A}{M_A}RT \qquad \frac{P_A}{\rho_A}=\frac{RT}{M_A}$$

$$P_BV_B=\frac{m_B}{M_B}RT \qquad \frac{P_B}{\rho_B}=\frac{RT}{M_B}$$

$$\frac{m_A}{m_B}=\frac{\rho_A}{\rho_B}=\frac{P_AM_A}{P_BM_B}$$

$$= \frac{.743}{.267} \times 8$$

$$= 22.0$$

This result tells us that liquid A evaporates 22 times as fast as liquid B. If we start off with 10 g of liquid A and 10 g of liquid B. With all 10 g of liquid A gone, about 10/22 g or about 5 g of liquid B still remain.

In the graph provided by the problem, $T_2$ is the boiling point of liquid B. **Ans**

# Problem 3

Three mass points $P_1$, $P_2$ and $P_3$ of mass $m_1$, $m_2$ and $m_3$ respectively interact with one another through gravitational attracting force as the only force in operation in this problem.

Let $\alpha_{ij}$ be distance between i and jth mass points
$\sigma$ represent the axis through the Centre of Mass of the mass point system and normal to $\Delta P_1P_2P_3$

Find the condition that angular speed $\omega$ of the system of mass points about axis s and distances

$$P_1P_2 = \alpha_{12} \;;\; P_2P_3 = \alpha_{23} \;;\; P_3P_1 = \alpha_{31}$$

are constant and the triangle $\Delta P_1P_2P_3$ remains unchanged through the rotating motion i.e. the condition for which the system of mass points behaves as a rigid body.

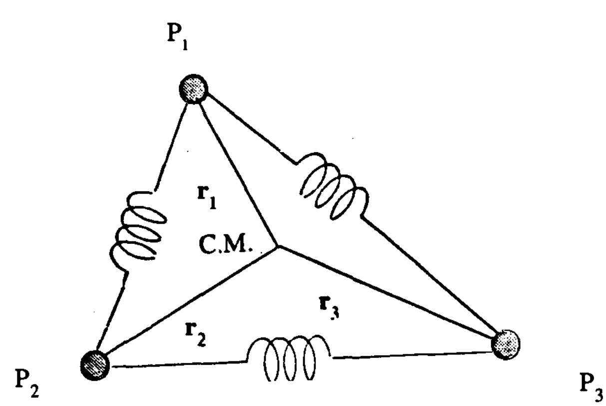



Fig 20.3

## Solution

Choose a coordinate system in which $P_1$ $P_2$ and $P_3$ are coplanar in xy plane. Since the origin of the coordinate system coincides with the centre of mass

$$m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 = 0$$

Consider mass point $m_1$ at $P_1$. There are two forces acting on $m_1$ i.e.

$$\mathbf{F}_{21} = \frac{Gm_1m_2}{|\mathbf{r}_2 - \mathbf{r}_1|^2}\cdot\frac{(\mathbf{r}_2 - \mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|} = \frac{Gm_1m_2}{\alpha_{12}^3}(\mathbf{r}_2 - \mathbf{r}_1) \quad (1)$$

$$\mathbf{F}_{31} = \frac{Gm_3m_1}{|\mathbf{r}_3 - \mathbf{r}_1|^2}\cdot\frac{(\mathbf{r}_3 - \mathbf{r}_1)}{|\mathbf{r}_3 - \mathbf{r}_1|} = \frac{Gm_3m_1}{\alpha_{31}^3}(\mathbf{r}_3 - \mathbf{r}_1) \quad (2)$$

The centripetal force( viewed from the inertial system i.e. the system in which an observer does not move with the rotating system) on m and causes m to move in uniform circular motion about O ie

$$\mathbf{F}_{r1} = -m_1\omega^2\mathbf{r}_1 \quad (3)$$

Hence $\mathbf{F}_{21} + \mathbf{F}_{31} = \mathbf{F}_{r1}$

[Alternatively, in non-inertial frame ( The observer is at rest in the frame which rotates with angular speed $\omega$ along with the the system of mass points ) equation of motion of $P_1$ is

$$\mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_{r1}' = 0 \quad (4)$$

Centrifugal force pointing in the direction of increasing **r** is

$$\mathbf{F}_{r1}' = m\omega^2\mathbf{r}_1 = -\mathbf{F}_{r1}$$

(3) and (4) give the same result.

Substitute $\mathbf{F}_{21}$ $\mathbf{F}_{31}$ and $\mathbf{F}_{r1}$ in (3)

$$\frac{Gm_1m_2}{\alpha_{12}^3}(\mathbf{r}_2 - \mathbf{r}_1) + \frac{Gm_3m_1}{\alpha_{31}^3}(\mathbf{r}_3 - \mathbf{r}_1) + mw^2\mathbf{r}_1 = 0$$

$$\frac{Gm_1m_2}{\alpha_{12}^3}\mathbf{r}_2 + \frac{Gm_3m_1}{\alpha_{31}^3}\mathbf{r}_3 + m_1\mathbf{r}_1(\omega^2 - \frac{Gm_2}{\alpha_{12}^3} - \frac{Gm_3}{\alpha_{31}^3}) = 0$$

Substitute $m_2\mathbf{r}_2 = -m_1\mathbf{r}_1 - m_3\mathbf{r}_3$

$$\frac{Gm_1}{\alpha_{12}^3}(-m_1\mathbf{r}_1 - m_3\mathbf{r}_3) + \frac{Gm_1}{\alpha_{31}^3}m_3\mathbf{r}_3 + (\omega^2 - \frac{Gm_2}{\alpha_{12}^3} - \frac{Gm_3}{\alpha_{31}^3})m_1\mathbf{r}_1 = 0$$

$$m_1\mathbf{r}_1(\omega^2 - \frac{Gm_1}{\alpha_{12}^3} - \frac{Gm_2}{\alpha_{12}^3} - \frac{Gm_3}{\alpha_{31}^3}) + Gm_1m_3\,\mathbf{r}_3\,(\frac{1}{\alpha_{31}^3} - \frac{1}{\alpha_{12}^3}) = 0$$

Since $\mathbf{r}_1$ and $\mathbf{r}_3$ are two linearly independent vectors, the above equation can be true only if

$$(\frac{1}{\alpha_{31}^3} - \frac{1}{\alpha_{12}^3}) = 0$$

and
$$(\omega^2 - \frac{Gm_1}{\alpha_{12}^3} - \frac{Gm_2}{\alpha_{12}^3} - \frac{Gm_3}{\alpha_{31}^3}) = 0$$

Hence
$$\alpha_{13} = \alpha_{31} = \alpha_{12} = \alpha$$

$$\omega^2\alpha^2 = G(m_1+m_2+m_3)$$

If we starts off with $P_2$ we should obtain

$$\alpha_{23} = \alpha_{21} = \alpha$$

And with $P_3$ we should arrive at

$$\alpha_{31} = \alpha_{32} = \alpha$$

In all three cases $\omega^2\alpha^2 = G(m_1+m_2+m_3)$

It is conluded that for the system of three mass points to rotate as a rigid body, the distance beween any two mass points must be the same( = $\alpha$). and the angular speed

$\omega = (GM/\alpha^3)^{1/2}$ where $M = m_1+m_2+m_3$ **Ans**

# Problem 3

This problem is concerned with the investigation of the way in which an electron microscope can be converted to a proton microscope. The accelerating potential for the electron microscope is U = 511 kV and for the proton microscope the accelerating potential is -U. Towards this objective, the following problems will be investigated and answered:

3.1 An electron after going through accelerating potential U finds itself in non-uniform magnetic field **B** generated by coils $L_1$, $L_2$, $L_3$, $L_4$, ...... ........ $L_n$ carrying currents $I_1$, $I_2$ $I_3$, $I_4$ ..........$I_n$ respectively.

What would be the currents $I'_1$,$I'_2$, $I'_3$, $I'_4$.....$I'_n$ in coils $L_1$, $L_2$, $L_3$, $L_4$,......$L_n$ in in order to keep the proton (accelerated by potential -U ) on the same path of the electron in the electron microscope?.

**Hints** . In trying to solve this problem, consideration must be given to the condition for which the path of the proton will be the same as that of electron. Application of the formula

$$\mathbf{p}.\frac{d\mathbf{p}}{dt} = \frac{1}{2}\frac{d\mathbf{p}^2}{dt} = \frac{1}{2}\frac{dp^2}{dt}$$

should help facilitate the analysis of the problem.

3.2 Would the resolving power of the microscope increase or decrease if electron current is replaced by proton current ? Resolving power can be defined as smallest distance between two dots in the image that renders the dots can still be seen as two separate dots.

Resolving Power or $\quad$ R.P. $\propto \dfrac{1}{\lambda}$

## Solution

Since the electron is acceelerated by potential of 511 keV or .511 MeV, it receives kinetic energy of .511 MeV . This value of kinetic energy is of the same order of electron's rest mass energy, the calculation must be based on the concepts of Special Relativity. Equation of motion of the electron may be written as

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}_L$$

**p** momentum of electron $\quad = m\mathbf{v} = \dfrac{m_o \mathbf{v}}{\sqrt{1-\dfrac{v^2}{c^2}}}$

**v** velocity of electron

$\mathbf{F}_L$ Lorentz force $\quad = -e\,\mathbf{v} \times \mathbf{B}$

Lorentz's force acts in the direction normal to the momentum.

$$\mathbf{F}_L \cdot \mathbf{v} = \mathbf{v}.\, \mathbf{F}_L = 0$$

$$\mathbf{p}.\frac{d\mathbf{p}}{dt} = \mathbf{p} \cdot \mathbf{F}_L = 0$$

$$\frac{dp^2}{dt} = 0$$

$$\mathbf{p}^2 = \text{constant}$$

i.e. momentum is constant

From $\quad \dfrac{d\mathbf{p}}{dt} = \mathbf{F}_L$

$$\frac{d\mathbf{p}}{ds}.\frac{ds}{dt} = \mathbf{F}_L$$

ds is elemental distance measured along the path of motion of electron(geodesic) and

$$\frac{ds}{dt} = v$$

and $$v\frac{d\mathbf{p}}{ds}. = \mathbf{F}_L$$

$$\frac{d\mathbf{p}}{ds}. = \frac{\mathbf{F}_L}{v}$$

$$\frac{d}{ds}\left(\frac{\mathbf{p}}{|\mathbf{p}|}\right) = \frac{\mathbf{F}_L}{v|\mathbf{p}|}$$

$$\frac{d\mathbf{1}_p}{ds} = \frac{\mathbf{F}_L}{v|\mathbf{p}|}$$

$\mathbf{1}_p$ is a unit vector in the direction of p or the path of the electron.
The equation above shows that the path of motion of the electron depend on Lorentz's force $\mathbf{F}_L$ and the magnitude of $\mathbf{v}$ or $\mathbf{p}$ only.

If we want the path of proton to be the same as that of electron

$\frac{\mathbf{F}_L^{\prime}}{v^{\prime}|\mathbf{p}^{\prime}|}$ for proton must be equal to $\frac{\mathbf{F}_L}{v|\mathbf{p}|}$ for electron or

$$\frac{-e\mathbf{v}\times\mathbf{B}}{v|\mathbf{p}|} = \frac{-e\mathbf{v}^{\prime}\times\mathbf{B}^{\prime}}{v^{\prime}|\mathbf{p}^{\prime}|}$$

Observe that $\mathbf{v}$ is perpendicular to $\mathbf{B}$ as $\mathbf{v}^{\prime}$ is to $\mathbf{B}^{\prime}$

The equation above implies that the magnetic fields of both cases must be in the same direction ,and the magnitude of the field for proton must increase in the ratio

$$|\mathbf{B}^{\prime}| = \frac{|\mathbf{p}^{\prime}|}{|\mathbf{p}|}|\mathbf{B}|$$

while the current must be stepped up by the factor of $\frac{|\mathbf{p}^{\prime}|}{|\mathbf{p}|}$.

## Determination of p and p$^{\prime}$

Special Theory of Relativity gives total energy of the electron in a formula

$$E^2 = E_0^2 + p^2c^2$$

E is total energy
$E_0$ is rest mass energy of electron and
$E_0 = m_0c^2 = .511$ eV

Hence $$E = E_0 + E_K$$

$E_K$ is kinetic energy and $E_K = .511$ MeV

$$(.511+.511)^2 = .5112 + p^2c^2$$

$$p = \sqrt{3} \times \frac{0.511}{c}$$

For proton, classical formula may be used to determine p$'$ without loss in accuracy.

Kinetic energy of proton $.511 = \dfrac{(p')^2}{2m_p}$

$$m_p c^2 = 938 \text{ eV}$$

Hence $cp' = \sqrt{2 \times 0.511 \times 938}$

$$\frac{p'}{p} = \frac{\sqrt{2 \times 0.511 \times 938}}{\sqrt{3} \times 0.511}$$

$$= 35$$

We conclude that the current must be stepped up 35 times and the direction of the current reversed to allow for the positive charge of the proton and enable the protons to travel the same path of the electron. **Ans**

3.2 Resolving Power of the electron microscope is i nversely proportional to wave-length $\lambda$.

or $\quad R.P. \propto \dfrac{1}{\lambda}$

For the electron microscope $\lambda = \dfrac{h}{p}$

For the proton microscope $\lambda = \dfrac{h}{p'}$

Ratio of resolving power of the two types of microscope

$$\frac{(R.P.)_{proton}}{(R.P.)_{Electron}} = \frac{p}{p'}$$

$$= \frac{1}{35}$$

The proton microscope under discussion can differentiate between two points which are at minimum distance of 35 times shorter than that of the electron microscope. **Ans**

# Experiment

## Problem 1

### Apparatus

- One pair of piezo-electric discs (emitt signals when compressed and vibrate when fed by current.) The disc is 10 mm. thick and equipped with two electrodes.

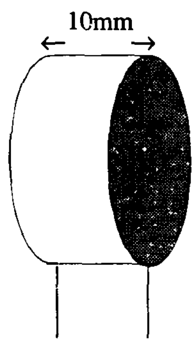



Fig. 20.5

- Signal generator for providing sine-wave signals, complete with a photogaph of its controlling panel and description of the function of the switches and control knobs.
- Dual or 2-channeled oscilloscope, with a photograph of its dial and description of the function of the switch and all contolling knobs.
- Two plastic bags containing two different types of liquid.
- A small beaker containing glycerine for use on piezo-electric discs, in order to enhance transmitted vibration of the material in contact with the disc.
- Connecting wires and 3-way adaptor.
- Stands for mounting plastic bags.
- Supporting mounter for a caliper.

Piezo-electric material subject to external electric field undergoes changes in it size, vice versa if its dimension is changed by an external force, electric field is generated giving electric signal. Based on this principle, we can design a piezo-electric disc which oscillates when AC signal is sent through the disc and also gives AC signal when forced to vibrate mechanically, or by other similar means.

a.Given the speed of ultrasonic wave through the piezo-electric material of the discs is approximately $4 \times 10^3$ m/s, determine resonant frequency of mechanical vibration

in the direction parallel to the axis of the disc.
Note that vibration may take place at any other frequencies (eigen frequencies) which may be higher or lower than the resonant frequency.

b. Find the speed of ultrasonic wave in the two liquids provided and estimate the errors of the answers.

c. Find the ratio of the speeds of ultrasonic wave in the two liquids and estimate the error of the answer.
Complete the summarized form by filling in experimental results. In addition to the filled form, description of the calculation of resonant frequency, meauring methods used and estimation of error of the final answers must be reported in a concise and clear manner.

Give definitions of the variables introduced into the calculation and proper attending units.

**Summarized Form of Report**

| | |
|---|---|
| a.Formula used in the calculation of ............................................ | result of calculation(unit)...................... |
| ............................................ | ...................................................... |
| Freqency at which vibration is best transmitted..................................... ...................................................... | ................................................... |
| .............etc. | etc. |

| b.definition of quantity measured | Symbol | Experimental Results |
|---|---|---|
| .................................... | ...................... | ................................... |
| .............................. . | .................... | |
| ............................. .......... | ......................... | . .................. |
| .............................. | | |
| etc. | etc. | etc |

- Final formula for calculation of the speed of ultra-sonic sound..

......................................................................................................................................

......................................................................................................................................

| | Speed of ultrasound | Errors |
|---|---|---|
| Liquid A | ........................... | ........................... |
| Liquid B | ........................... | ........................... |

c. Ratio of speeds of ultra sound Error

...................................................... ......................................................

## Solution

a. Since the cross-section area of the piezo-electric disc is not rigidly fixed, the position of the frontal area of the disc on either side becomes an anti-node, and the midpoint between the two positions a node, as illustrated in Fig.20.6

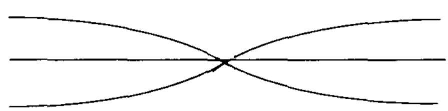


Fig 20.6

The resonant frequency can be calculated using the formula

$$l = \frac{\lambda}{2} = \frac{v}{2\nu}$$

where $\nu$ is the frequency measured in the unit of Hz or cycle per second

$v$ is the speed of sound wave

$\lambda$ wavelength

$l$ thickness of the piezo-electric disc

Hence

$$\nu = \frac{v}{2l} = \frac{4 \times 10^3}{2 \times 10^{-2}}$$

$$= 2 \times 10^5 \text{ Hz (approximately)}$$

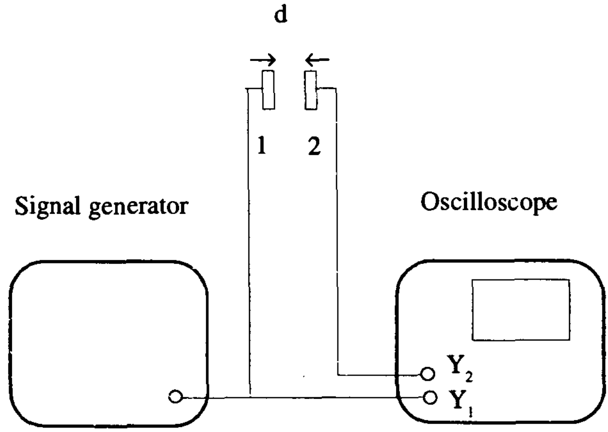



Fig. 20.7

## Experimental Procedures

Set up the apparatus as shown in the figure above.

Split the signal from the sinewave signal generator into two components one of which is connected to the oscilloscope at Channel 1 ($Y_1$) and other to the first piezo-electric disc.

Apply the signal from the second piezo-electric disc as an input to Channel 2 ($Y_2$) of the oscilloscope.

Sandwich the bag containing the liquid (See Fig. 20.8 ) between the two piezo-electric discs.

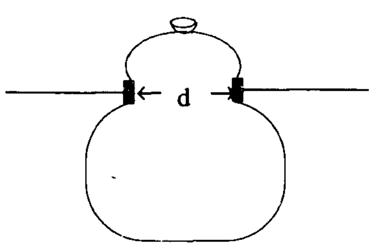



Fig 20.8

Adjust the frequency of ultrasonic sound from 100 to 1,000 kHz
At the resonant frequency of piezo-electric disc ( n ~ 220 Hz), the amplitude of the signal from the second piezo-electric disc is maximum.
Other frequencies which also give maximum amplitudes are 110 Hz and 670 Hz.

## Determination of resonant frequency of piezo-electric discs

**Method 1** Take frequency reading on the dial of sine-wave generator. In this case the error is estimated to be about 0.5 kHz.

**Method 2** Calculate the value of of the frequency from time-base reading at the oscilloscope using the formula $\nu = 1/t$ . In this case the error is about 5 kHz .(i.e. half of the smallest scale division.)Measurement of phase shift from phase difference of signals from Channels 1 and 2.
Use the same experimental setup as shown in Fig. 20.7

## Calculation in Method 1

Signal of Channel 2 is due to sound wave going through the thickness d cm in the liquid

Hence phase shift represented by $-\Delta\phi$ is given by

$$-\Delta\phi = \frac{2\pi d}{\lambda} = \frac{2\pi d}{v}$$

$$\frac{d}{\lambda} = n \quad (n = 0,1,2,3\ldots\ldots\ldots \text{ at resonant frequencies })$$

$$\Delta\phi = -2\pi n$$

By comparing phases of signals of Channels 2 ( n cycles or waves ) and Channel 1 , $\Delta\phi$ is obtained.

## Calculaion in Method 2

Connect the signal which passes through the liquid to X terminal of the oscilloscope ( X input ).

Connect the signal direct from the signal generator to Y terminal of the oscilloscope ( Y input ).

At Channel X $\qquad X = X_0 \sin(\omega t - \Delta\phi)$

At Channel Y $\qquad Y = Y_0 \sin \omega t$

where

$$-\Delta\phi = \frac{2\pi d}{\lambda} = 2\pi n$$

$$\frac{Y}{X} = \frac{Y_0}{X_0}$$

The curve traced out on the screen is linear. This phenomenon allows us to find $\lambda$

(corresponding to resonant frequency) with high accuracy by measuring d as a function of n.

We begin by applying one convenient value of resonant frequencies (such as 220 Hz) and choose the value of d for example from 10 to 15 cm.

Connect the signal through the liquid to Channel Y.
Connect the signal direct from the signal generator to Channel X.

Adjust d until obtaining $\Delta\phi = 0$ i.e. the curve on the oscilloscope screen is linear. Refer to this experimental setup by $n = 0$.
Lower the value of d until another $\Delta\phi = 0$ is obtained and the curve on the oscilloscope screen is once again a straight line. The experimental setup corresponds to $n = 1$. From the measurement of d as a function of n, make a table of the values of d versus n and plot a graph of d as a function of n.

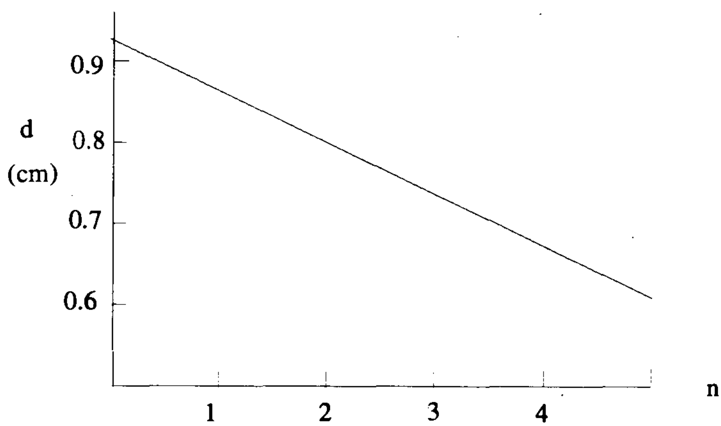



Fig. 20.9 (a) Graph of d as a function of n (for liquid A)

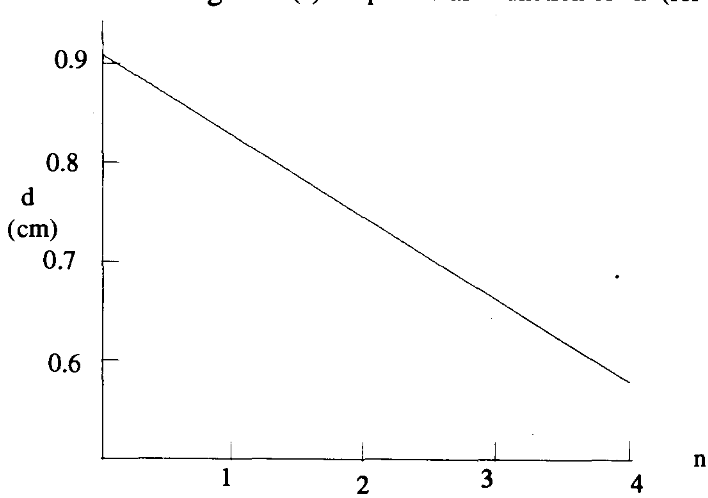



Fig 20.9 (b) Graph of d as a function of n (for liquid B)

For liquid A

$$\text{Slope} \quad \lambda_A = \frac{9.25 - 5.9}{5}$$
$$= 0.67 \text{ cm}$$

The chief source of error in this case depends on the longitudinal dimension of the piezo electric disc which is half wavelength at lowest resonant frequency i.e. 0.10/2 = .05 cm

$$\lambda_A = 0.67 \pm 0.05 \quad \text{cm}$$

For liquid B

$$\text{Slope} \quad \lambda_B = \frac{9.10 - 5.70}{4}$$
$$= 0.85 \pm 0.05 \quad \text{cm}$$

Speed of sound wave in liquid A =wavelengtre × resonant freqency employed

$$= (0.67 \pm 0.05) cm \times 220 Hz$$
$$= 147 \pm 11 \quad cm.s^{-1} \ (8\%)$$

Speed of sound wave in liquid B = wavelengtre × resonant freqency employed

$$= (0.85 \pm 0.05) cm \times 220 Hz$$
$$= 187 \pm 11 \quad cm.s^{-1} \ (6\%)$$

Ratio between speed of sound wave in liquid A and speed of sound wave in liquid B

$$= \frac{\lambda_A}{\lambda_B} = \frac{85}{87} \qquad (\text{ 14\% error})$$

The value of the required ratio = 0.99 ± .14 **Ans**

# International Physics Olympiad XXI

# 1980

# Groningen, The Netherlands

## Theory

### Problem 1. X-ray Diffraction in the Crystals

This problem is concerned with the investigation of X-ray diffraction in simple cubic crystals. For this purpose, we shall begin with the study of diffraction by the crystal lattice.

If a plane wave is sent in the direction normal to the plane of the crystal which in effect is a two- dimensional grating consisting of $N_1$ and $N_2$ slits (or atoms ) -separated by distance $d_1$ for the side consisting of $N_1$ slits and $d_2$ , for the side consisting of $N_2$ slits.
The diffraction pattern is observed on the screen located at distance L from the crystal, given $L >>> N_1 d_1$ and $N_2 d_2$.

1.1 Determine the position and the width of principal maxima of various orders on the screen. The width here means the distance between the positions of minimum intensity on each side of the principal maxima.
The next step is to study a whole cubic crystal with the lattice constant a i.e. smallest distance between two atoms of the same kind. The crystal constitutes a three- dimensional grating characterized by $N_{0a} \times N_{0a} \times N_{1a}$ and $N_1 <<<< N_0$.
The crystal is placed in the path of an x-ray beam which travels parallel to z axis, and the surface of the crystal makes angle $\theta$ with y axis, while the low edge of the crystal is parallel to x axis. (See Fig. 21.1)

1.2 Determine the width and position of principal maxima as a function of $\theta$ ( for small $\theta$. )
Explain what would happen as a consequence of $N_1 <<< N_0$.

1.3 We could calculate various quantities by applying Bragg' Diffraction Theory, that is to say we consider the reflection of x-ray beam at the lattice planes of the crystal, and the diffraction pattern is caused by the interference of the beams reflected at the lattice planes. Show that Bragg's reflection gives the same condition for the constructive interference leading to formation of principal maxima as calculated and obtained in 1.2.

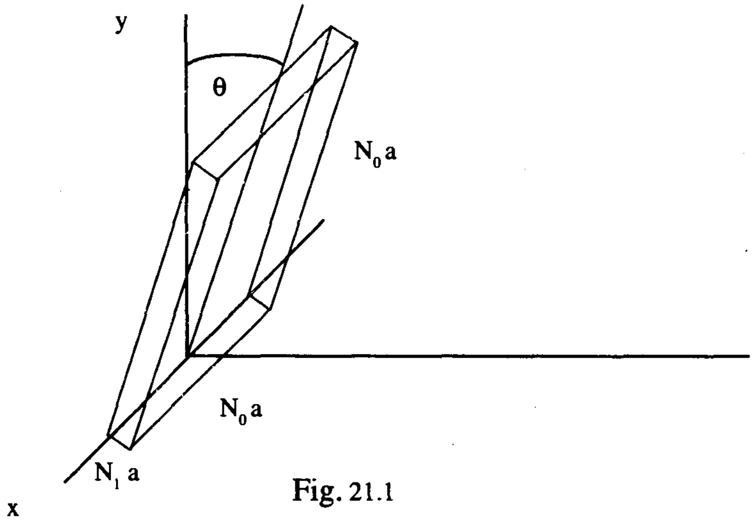



Fig. 21.1

In another experiment, crystal powder is used to scatter the x-ray beam. In this situation, the size of the powder particles which come in large number is much greater than the distance between the nearest atoms in the crystal .

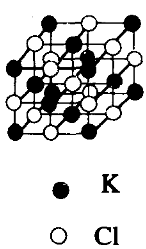



Fig. 21.2

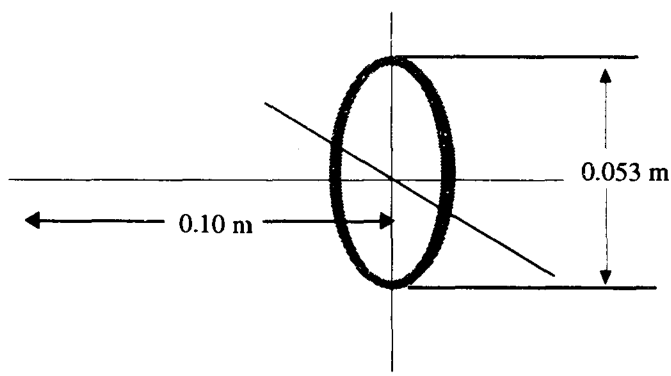



Fig. 21.3

In this scattering experiment, an X-ray beam of wavelength 0.5 nm , and powder of KCL crystal are used. The structure of KCl crystal is shown in the figure above. Bragg's scattering gives a pattern of concentric rings on the photographic film, placed at distance 0.10 m from the powder. The radius of the smallest ring is 0.53 m (See Fig 21.3).

The ions $K^+$and $Cl^-$ have approximately the same size and are assumed to have the same scattering property.

1.4 Calculate of the distance between two nearest $K^+$ ions in the crystal.

## Solution

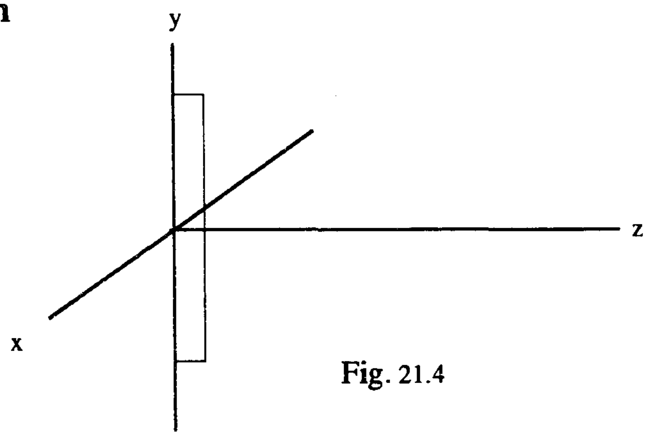



Fig. 21.4

1.1 Consider atoms or slits along the x axis and their effects on the incident x-ray beam. The difference in phases of the beams scattered at two consecutive atoms located on x-axis observed in the direction making an angle $\alpha$ with z axis ( and 90° with y axis ) is

$$\frac{2\pi}{\lambda} d_1 \sin\alpha$$

The condition for constructive interference and maximum intensity is,

$$\frac{2\pi}{\lambda} d_1 \sin\alpha = 2n_x\pi$$

$$n_x = 0,1,2,3,\dots\dots\dots$$

For small values of $\alpha$ $\sin\alpha \sim \tan\alpha \sim \frac{x}{L}$

$$d_1 \frac{x}{L} \cong n_x$$

$$x \approx \frac{n_x \lambda L}{d_1} \qquad (1)$$

$n_x = 0,1,2,3$.......corresponding to positions of principal maxima.

In the case of a large number of slits or atoms serving at secondary x-ray sources, we need to combine vectorially the amplitudes of interfering beams from all secondary sources. We note that the phase difference between beams from two consecutive slits is given by $\delta$

where $$\delta = \frac{2\pi}{\lambda} d_1 \sin\alpha$$

The amplitudes of the scattered beams are vectorial quantitiies and add vectorially.

Fig 21.5 demonstrates the way in which these vectors are added.

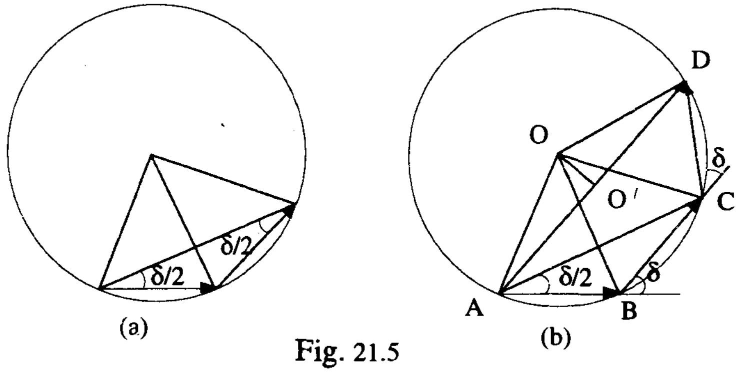



Fig. 21.5

In Fig 21.5 a addition of two vectors representing the amplitues of two beams gives the resultant amplitude with resultant phase of δ/2 .

Likewise, vectorial addition of the amplitudes of N beams givesthe resultant amplitude with a net phase represented by Δ or ∠ BAD

where $$\Delta = (N-1)\frac{\delta}{2}$$

The above relation can be proved by drawing a circle of radius OA = OB = OC = OD, and apply the principle that the angles at the centre supported by arcs of equal length are equal. From the diagram it can be induced that, the resultant phase expessed as angle ∠BAD

and $$\angle BAD = (N\text{-}1).\angle ACB = (N\text{-}1)\frac{\delta}{2}$$

$$\Delta = N\frac{\delta}{2}$$

At the positions of principal maxima

$$\Delta = N\frac{\delta}{2} = 2n\pi \qquad n = 0,1,2,3,\ldots\ldots$$

Condition for destructive interference and minimum intensity adjacent to principal maxima is

$$\Delta = N\frac{\delta}{2} = \pm\pi \qquad (2)$$

$$\delta = \pm\frac{2\pi}{N} \qquad (3)$$

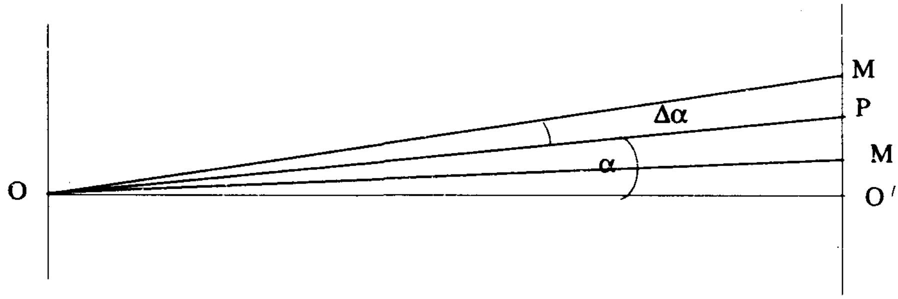



Fig. 21.6

At the position of minimum intensity M, adjacent to the position of principal maxima at P,

$$\delta_M = \frac{2\pi}{\lambda} d_1 \sin(\alpha + \Delta\alpha) = \frac{2\pi}{N} \tag{3}$$

Likewise, at the position of minimum intensity M′ adjacent to the position of principal maxima at P on the other side, is

$$\delta'_M = \frac{2\pi}{\lambda} d_1 \sin(\alpha - \Delta\alpha) = -\frac{2\pi}{N} \tag{4}$$

For small values of α

$$\sin(\alpha \pm \Delta\alpha) = \alpha \pm \Delta\alpha$$

Width of principal maxima at P is given by (3)-(4)

$$\frac{2\pi}{\lambda} d_1 2\Delta\alpha = \frac{4\pi}{N}$$

$$2\Delta\alpha = \frac{2\lambda}{Nd_1} \tag{5}$$

From the diagram $\Delta\alpha = \dfrac{\Delta x}{L}$

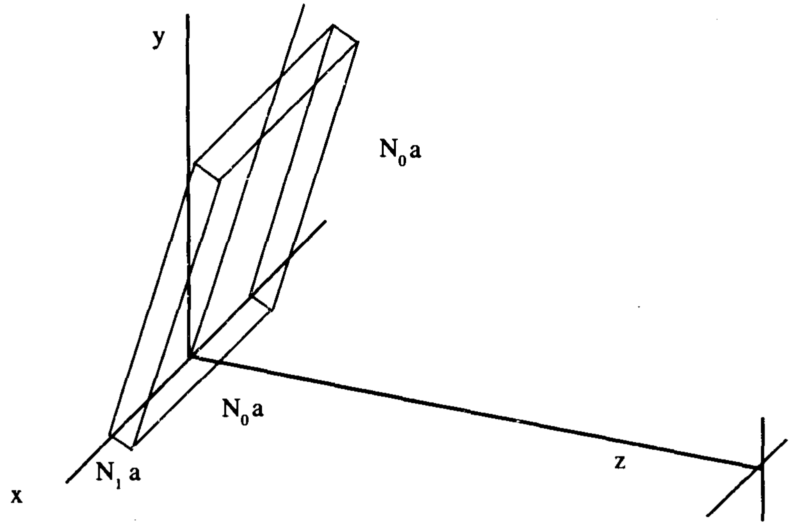
Width of the band $2\Delta x = \dfrac{2\lambda L}{Nd_1}$ **Ans**

By the same spirit of the argument , it can be induced that , diffraction pattern due to atoms along y axis gives the band width of principal maxima as:

$$2\Delta y = \frac{2\lambda L}{Nd_2}.$$ **Ans**

1.2 Consider diffraction when the crystal makes angle θ with y axis ( θ is small) as shown in the figure below.

(a)




Consider diffraction caused by atoms along x axis.

(b)

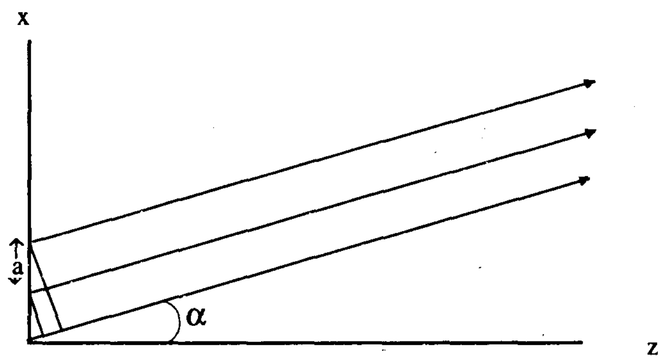



Fig. 21.7

Since the beams that are scattered at atoms along x axis are in the same phase, the situation in this case is the same as that discussed under 1.1, i.e. principal maxima occurs at

$$x_n = \frac{n_x \lambda L}{a} \qquad n_x = 0,1,2,3,.....$$

Width of principal maxima

$$2\Delta x_n = \frac{2\lambda L}{N_0 a}$$

The pattern of diffraction caused by atoms along x axis does not depend on the diffraction pattern caused by atoms along y-axis. **Ans**

Consider diffraction pattern caused by atoms on the crystal edge which makes angle θ with y axis.

This direction will be referred to as y′ axis of the crystal.

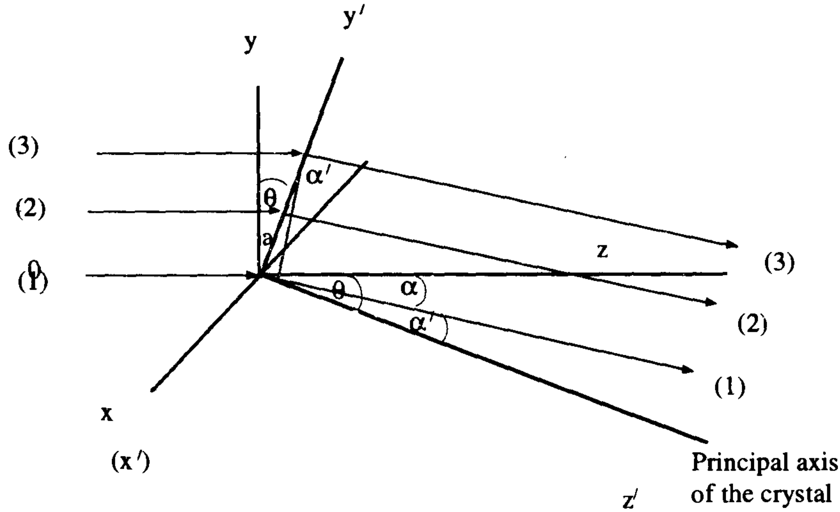



Fig. 21.8

Difference in the optical path of x-ray beams scatterd at two consecutive atoms is a sinθ.

Draw a straight line normal to the surface of the crystal to obtain the direction of the principal a xis of the crystal.

In the direction making angle α′ with the principal axis of the crystal, the condition for constructive interference

$$a \sin\theta - a \sin\alpha' = \frac{2n\lambda}{2}$$

From the diagram $\alpha' = \theta - \alpha$

where α is the angle which the scattered beam makes with z axis of the reference coordi nate system, and θ is the angle which the principal axis of the crystal makes with z axis of the reference coordinate system.

Hence $a \sin\theta - a \sin(\theta - \alpha) = \dfrac{2n\lambda}{2}$

$$a\,[\sin\theta - (\sin\theta\cos\alpha - \cos\theta\sin\alpha)] = \frac{2n\lambda}{2}$$

Substitute $\cos\alpha \approx 1$ to obtain

$$a\cos\theta\sin\alpha \approx n\lambda$$

or $$\frac{2\pi}{\lambda}(a\cos\theta)\sin\alpha \approx 2n\pi \qquad \alpha \approx \frac{n_y\lambda}{a\cos\theta}$$

$$y = \frac{Ln_y\lambda}{a\cos\theta}$$

It may be interpreted from the above equation that in such orientation of the crystal, the crystal is equivalent to a crystal of lattice constant = a cosθ placed normal to the beam. The two give the same patttern of diffraction pattern.

From 1.1 width of principal maxima is given by

$$2\Delta y = \frac{2\lambda L}{N_1 a\cos\theta} \quad \textbf{Ans}$$

Consider diffraction pattern caused atoms lined up in the direction at angle 90° - θ with - y axis. This direction is referred to as z′ axis of the crystal.

In this case the difference in the optical path betwwen beams scattered at two consecutive atoms is - acosθ.

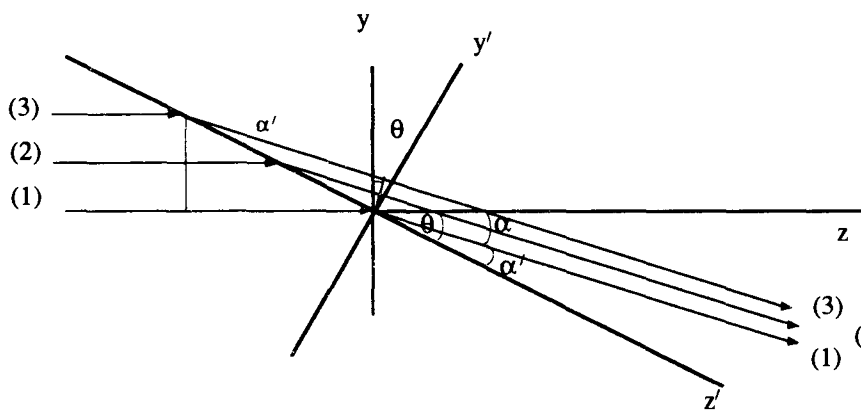



Fig. 21.9

as with the preceding case, a straight line is drawn normal to z′ axis of the crystal to obtain the principal axis of the crystal.

In the direction making angle α′ with the principal axis of the crystal, the condition for principal maxima to be formed is

$$-\,a\cos\theta + a\cos\alpha' = \frac{2n\lambda}{2}$$

From the diagram $\alpha' = \theta - \alpha$

where α is the angle which the scattered beam makes with z axis of the reference coordinate system.

Hence $\quad - a\cos\theta + a\cos(\theta - \alpha) = \frac{2n\lambda}{2}$

$$- a\cos\theta + a\cos\theta\cos\alpha + a\sin\theta\sin\alpha = n\lambda$$

For small α, $\quad \cos\alpha \approx 1$

$$- a\cos\theta + a\cos\theta + a\sin\theta\sin\alpha = n\lambda$$

$$a\sin\theta\sin\alpha = n\lambda$$

or $\quad \frac{2\pi}{\lambda}(a\sin\theta)\sin\alpha \approx 2n\pi \qquad \alpha \approx \frac{n_{y'}\lambda}{a\sin\theta}$

The crystal in this orientation gives the same diffraction pattern as given by a crystal with the lattice constant a sinθ placed normal to the direction of the incident x-ray beam.

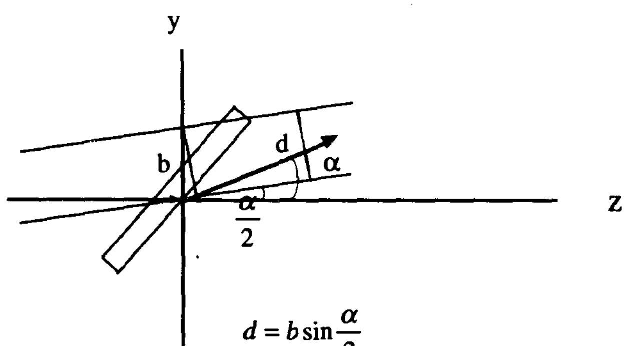
From 1.1 the position of principal maxima $y' = L\sin\alpha \approx L\alpha$

$$y' = \frac{n'_y \lambda L}{a\sin\theta}$$

Width of principal maxima $\qquad 2\Delta y' = \frac{2\lambda L}{N_1 a\sin\theta} \qquad$ **Ans**

If $N_1 <<< N_0$ the diffraction pattern caused by x-ray beams scattered at atoms along y′ axis of the crystal will be overshadowed by diffraction pattern s produced by atoms in other two orientattions. (Width of Δy′ will be very large)

1.3 Application of Bragg's theory to determine diffraction pattern and positions of principal maxima.




$d = b\sin\frac{\alpha}{2}$

$b = a\cos\theta$ for scattering from atoms along y′ axis

$b = a\sin\theta$ for scattering from atoms along z′ axis

$b = a$ for scattering from atoms along x′ axis

Fig 21.10

Consider diffraction pattern in yz plane caused by atoms along y′ of the crystal.

A straight line is drawn dividing angle $\alpha$ to obtain $\frac{\alpha}{2}$ as " glancing angle".

Hence $$2d\sin\frac{\alpha}{2} = n_y\lambda \quad \text{(Bragg's formula)}$$

where d is the distance between two nearest crystal planes at which Bragg's scattering takes place.
It has already been shown that the crystal in this orientation behaves as though it has lattice constant of a.cosθ.
and by straightforward geometry it can be shown that distance between two scattering planes is given by $d = a\cos\theta\cos\frac{\alpha}{2}$

Hence $$2a\cos\theta\cos\frac{\alpha}{2}\sin\frac{\alpha}{2} = n_y\lambda$$

$$\sin\alpha = \frac{n_y\lambda}{a\cos\theta}$$

Distance of bright fringe from the centre of the screen

$$y = L\tan\alpha \approx L\sin\alpha$$

$$y = \frac{n_y\lambda L}{a\cos\theta}$$

Consider diffraction caused by atoms oriented along z′ axis of the crystal. In this case d represents the distance between two nearest planes at which scattering take place, and

$$d = a\sin\theta\cos\frac{\alpha}{2}$$

which gives rise to Bragg's formula

$$2a\sin\theta\cos\frac{\alpha}{2}\sin\frac{\alpha}{2} = n_{y'}\lambda$$

Distance of bright fringe from the centre of the screen

$$y' = \frac{n_y'\lambda L}{a\sin\theta}$$

For scattering from atoms along x′ axis of the crystal

$$d = a\cos\frac{\alpha}{2}$$

$$2a\cos\frac{\alpha}{2}\sin\frac{\alpha}{2} = n_x\lambda$$

$$\sin\alpha = \frac{n_x\lambda}{a}$$

$$x = \frac{n_x\lambda L}{a}$$

The application of Bragg's formula gives the same results as in 1.2

1.4 Calculation of lattice constant.

Referring to Fig 21. 10 a line is drawn dividing angle $\alpha$ subtending radius 0.053 m to obtain $\frac{\alpha}{2}$ as the glancing angle.

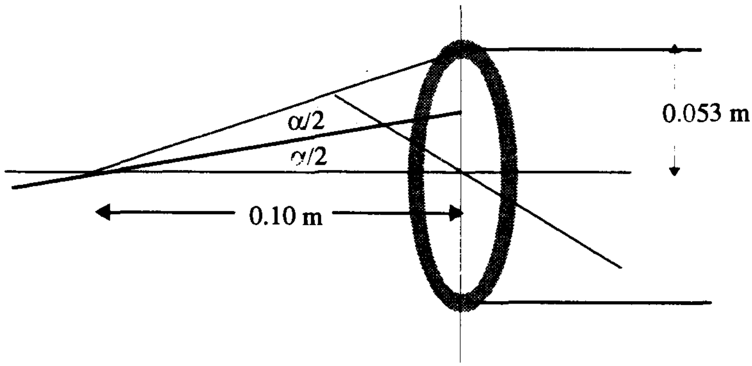



Fig. 21.10

Apply Bragg's formula $2d\sin\frac{\alpha}{2} = n\lambda$

where d is the distance between the two adjacent parallel planes at which Bragg's scattering takes place.

$\frac{\alpha}{2}$ glancing angle

$\lambda$ wavelength of x-ray beam = $0.15\times10^{-9}$m

From the diagram $\tan\alpha = \frac{0.053}{0.10} = 0.53$

$$\alpha = 27.9^o$$

$$\frac{\alpha}{2} = 14^o$$

Substitution of the above gives

$$2d\sin 14^o = 0.15\times10^{-9}$$

$$d = 0.313\times10^{-9} \quad \text{m}$$

$$d = .313 \quad \text{nm}$$

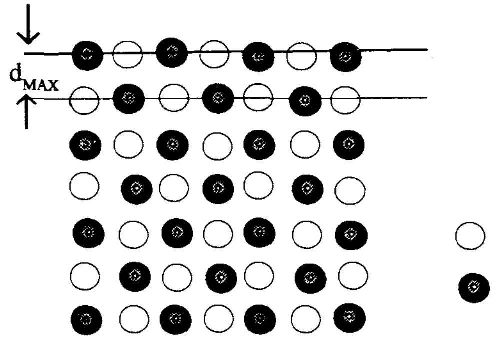



Fig.21.11

All the rings shown on the film represent n =1 or first order difrraction for different values of distance between the scattering planes ($n\lambda$= constant). Since $\alpha$ is the smallest angle, d must be maximum corresponding to distance between K and Cl atoms.

Hence distance between $K^+$ - $K^+$ ions

$$= 2 \times .313 \times 10^{-9} \text{ m}$$

$$= 0.63 \times 10^{-9} \text{ m} \quad \text{or} \quad 6300 \text{ A} \quad \textbf{Ans}$$

## Problem 2

In 1991 USA sent a space craft "Atlantis" (A) to orbit the earth. To facilitate the analysis, it will be assumed that the plane of the orbit coincides with the plane that defines the earth's equator.

At a certain moment of time planned, the craft releases satellite S linked to the mother craft A by a light, strong and flexible cable made of a good electric conductor of length L. Assume that all frictions in this problem are negligible.

Let $\alpha$ be the angle between the cable of length L and the orbital radius R of the spacecraft A.

S and the cable are also in the plane through the equator.

If m, mass of satellite S is much smaller than mass M of the space craft A. ( i.e. m <<<< M ) and L is much shorter than R ( i.e.L <<<< R )

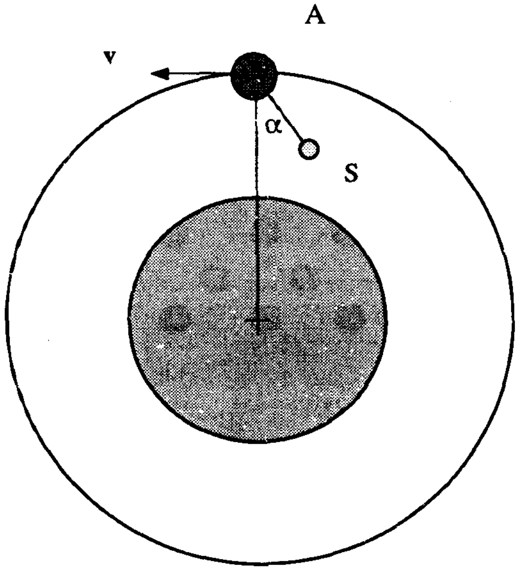



Fig. 21.10

2.1 Determine a set of values of α which results in the spacecraft and the satellite moving together in a stable equilibriumcondition, i.e. the angle α measured from the vertical direction at A is constant. All values of such α must be considered.
Given that at one point in time the cable is displaced from its equilibrium position by a small angle and thus begins to oscillate like a pendulum.

2.2 Determine the period of the oscillation of the satellite as a function of the period of the space craft's circular motion about the earth.

2.3 In Fig. 21.1 the direction of the earth's magnetic field points in the direction out of and normal to the paper. Since the cable moves in the direction normal to the earth's magnetic field, a voltage difference is generated across the ends of the cable.

The space around the space craft is not only subjected to the earth's magnetic field but also consists of rarified gases in the form of ions which serve as charge carriers. The space craft A and the satellite S constitute electrical electrodes for current flow.

Due to motion of a conductor in the magnetic field, let the current generated be I.
In which direction does the current flow? Asssume the value of α = 0.

## Data

Period of the craft orbital motion around the earth $T = 5.4103$ s

Length of cable $L = 2.0 \times 10^4$ m

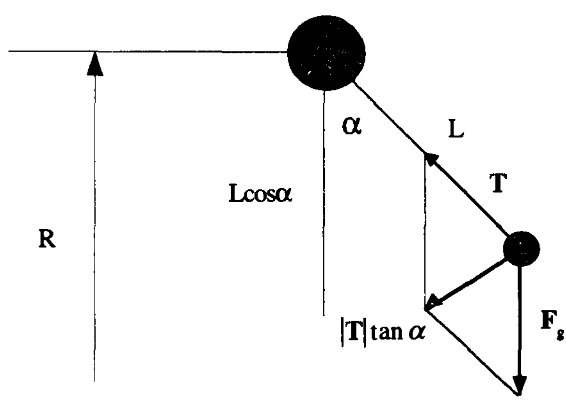
Strength of the earth's magnetic field $B = 5.0 \times 10^{-5}$ Wb $m^{-2}$

Mass of Atlantis space craft $M = 1.5 \times 10^5$ kg

2.3 If a generator is connected in the circuit which draws current from the space, the current generated by the generator in the circuit is constant and equal to 0.1 A in the opposite sense to the original current:
How long can the value of the current remain constant if the altitude of the orbit increases by 10 m ?
Suppose $\alpha$ remains constant at the value of 0 and ignore the effects of the ionic current in the rarified conducting gas, would the altitude of the orbit increase or decrease ? Explain your reason.

# Solution

2.1




To the center of the earth

Fig. 21.11

Resultant force between the tension, **T** in the cable and the pulling force of the gravitation, $\mathbf{F}_g$ on mass m of the satellite produces torque about A normal to $\mathbf{F}_g$.

Since m <<<< M, the tension in the cable produces no effect on mass m of A. The space craft A continues to move in the orbit around the earth with uniform speed, irrespective of what happens to S.
The motion of satellite S consists of rotating motion along with space craft A around the earth, and also rotating motion about A.( Or simple harmonic motion relative to the space craft A.)

Equation of motion of space craft A is,

$$M\Omega^2 R = \frac{GM_E M}{R^2}$$

where $M_E$ is the mass of the earth
G gravitational constant
$\Omega$ angular speed of space craft of mass M

$$\Omega^2 = \frac{GM_E}{R^3}$$

$$GM_E = \Omega^2 R^3 \tag{1}$$

Equation of motion of satellite S of mass m

$$F_g - \frac{T}{\cos\alpha} = m\Omega^2(R - L\cos\alpha) \tag{2}$$

And $$F_g = \frac{GM_E m}{(R - L\cos\alpha)^2}$$

Substitute $GM_E$ from (1) in (3) to obtain

$$F_g = \frac{\Omega^2 R^3}{(R - L\cos\alpha)^2} \tag{3}$$

Substitute $F_g$ from (4) in (2) to obtain

$$\frac{T}{\cos\alpha} = \frac{\Omega^2 R^3 m}{(R - L\cos\alpha)^2} - m\Omega^2(R - L\cos\alpha) \tag{4}$$

$$\frac{T}{\cos\alpha} = \frac{\Omega^2 R^3 m - \Omega^2 m(R - L\cos\alpha)^3}{(R - L\cos\alpha)^2}$$

Make use of $(R - L\cos\alpha)^3 = R^3\left(1 - \frac{L\cos\alpha}{R}\right)^3 \approx R^3\left(1 - \frac{3L\cos\alpha}{R}\right)$

$$(R - L\cos\alpha)^2 = R^2\left(1 - \frac{L\cos\alpha}{R}\right)^2 \approx R^2\left(1 - \frac{2L\cos\alpha}{R}\right)$$

$$\frac{1}{(R - L\cos\alpha)^2} \approx \frac{1}{R^2}$$

$$\frac{T}{\cos\alpha} = \frac{\Omega^2 R^3 m - \Omega^2 m(R^3 - 3R^2 L\cos\alpha)}{R^2}$$

$$T = 3m\Omega^2 L\cos^2\alpha$$

Consider the torque on S moving in the orbit around the earth with space craft A i.e. in the non-inertial frame which rotates about the earth with angular speed Ω.

$$\tau = I\ddot{\alpha}$$

$$-T\tan\alpha = mL^2\ddot{\alpha}$$

The negative sign indicates that torque is in the direction of decreasing α.

$$\ddot{\alpha} + 3\Omega^2 \sin\alpha\cos\alpha = 0$$

If α is constant both $\ddot{\alpha}$ and $\dot{\alpha}$ are 0.

$$3\Omega^2 \sin\alpha\cos\alpha = 0$$

If sinα = 0, then α = 0, π

If cosα = 0 then $\alpha = \frac{\pi}{2}, \frac{3\pi}{2}$

It is noted from the diagram that at $\alpha = \frac{\pi}{2}$ and $\frac{3\pi}{2}$, there is still torque on the satellite, while at α = 0 and π, torque is 0. Hence stable orientation of the satellite is at α = 0, π **Ans**

2.2 For small value of α measured from the vertical

$$\ddot{\alpha} + 3\Omega^2\alpha = 0$$

Satellite S executes simple harmonic motion with angular frequency $\Omega' = \sqrt{3}\Omega$

If T is periodic time of the motion of space craft A around the earth
T′ periodic time of the simple harmonic motion of satellite S relative to space craft A,

$$T' = \frac{T}{\sqrt{3}}$$

$$= 0.58T$$

2.3 The motion of the conducting cable of length L normal to the earth's magnetic field causes conventional current to flow from satellite S to spacecraft A. (The current makes return trip by way of ions around the satellite..**Ans**

2.4 When the generator on board sends conventional current from the space craft A to the satellite S, the value of the current is 1 A.
The current produces Lorentz force which tends to push the cable in the direction of motion

of the space craft A.

The magnitude of Lorentz force $F_L$ = I.L.B

Lotentz force produces change in the motionof the space craft by increasing the total mechanical energy of the space craft. When Lorentz force moves the distance of one orbit, increase in mechanical energy is

$$\Delta E = I.L.B2\pi R$$

Also
$$\Delta E = \frac{1}{2}Mv_f^2 - \frac{GM_EM}{R_f} - \left(\frac{1}{2}Mv_i^2 - \frac{GM_EM}{R_i}\right)$$

where $v_f$ is the value of the velocity of the space craft A in the orbit of radius $R_f$

$v_i$ the value of the velocity of the space craft A in the orbit of radius $R_i$

Substitute $\frac{Mv^2}{R} = \frac{GM_EM}{R^2}$ or $Mv^2 = \frac{GM_EM}{R}$

to obtain
$$\Delta E = \frac{GM_EM}{2}\left(\frac{1}{R_i} - \frac{1}{R_f}\right)$$
$$\approx \frac{GM_EM}{2}\frac{\Delta R}{R^2}$$
$$\approx \frac{GM_EM}{2}\frac{\Delta R}{R^2}$$

Substittute $\Omega^2 = \frac{GM_E}{R^3}$, $\Delta E = \frac{1}{2}\Omega^2 MR\Delta R$

Within the time interval of 1 period or T s into the motion of the spacecraft A around the earth, the system of space craft and satellite gains mechanical energy I.L.B 2πR.

Energy gained by the system per 1 s is $\frac{ILB.2\pi R}{T}$

Hence the energy gained by the system in t s is $I.L.B.2\pi R.\frac{t}{T}$

Let the altitude of the orbit change by ΔR

$$I.L.B.2\pi R.\frac{t}{T} = \frac{1}{2}\Omega^2 MR\Delta R$$
$$= \frac{4\pi^2}{2T^2}MR\Delta R$$
$$t = \frac{\pi M\Delta R}{T.I.L.B}$$

Substitute T = $5.4 \times 10^3$ s
L = $2.4 \times 10^4$ m
B = $5.0 \times 10^{-5}$ Wb $m^{-2}$
M = $10^5$ kg
ΔR = 10 m

$$t = \frac{3.14\times10^5\times10}{6.4\times10^3\times1\times2.4\times10^4 5.0\times10^{-5}}$$
$$= 5.82\times10^3 \quad s$$

The current will flow for the period of $5.82\times10^3$ s in order to raise the altitude of the system of space craft and satellite by 10 m. If $\alpha$ =0 The altitude of the satellite's orbit will keep increasing until the end of the ion's layer.**Ans**

# Problem 3 Self Rotating Neutron Star

Pulsar is a type of star and a source of short-wavelength electromagnetic waves emitted in pulses or periodically. The period of emission of electromagnetic waves is of the order of ms. Radio recievers of appropriate type under favourable conditions can detect these electromagnetic radiations, and can also be used to analyze pulses and accurately measure the period.

The electromagnetic waves sent out in pulses are generated by the star which consists entirely of neutrons, hence the term "neutron star". Neutron stars are extremely dense.While their mass is of the same order with the sun, their radii are mere 20 to 30 km. Another characteristic of the neutron star is the rapid rotating motion about its own axis. Because of this rapid rotating motion, th neutron star is slightly flattened at the two poles and bulges at the equator.

Assume that the cross sectional area cutting through the two poles is an ellipse with two axes of almost the same length.

Let $r_P$ be the radius of the neutron star measured from the centre to the pole.
$r_E$ the radius of the neutron star measured from the centre to the equator.
$\varepsilon$ index of elongation and $\varepsilon = \frac{r_E - r_P}{r_E}$
M mass of the neutron star = $2.0\times10^{30}$ kg
$<r>$ averaged radius of the star = $1.0\times10^4$ m
T period of rotation = $2.0\times10^{-2}$ s

3.1 Determine $\varepsilon$, given G = $6.67\times10^{-11}$ N m$^2$ kg$^{-1}$.

3.2 After some several years of the existence as a neutron star, the rotating star begins to slow down due to loss in its energy. As a result, the value of the index of elongation decreases. However, the solid crust of the star still floats on the sea of liquid neutron in the interior and tends to resist any trend towards equilibriium. Instead of continuous and gradual change, a "star quake" takes place causing the star to undergo sudden change before settling down in an equilibrium state.
Between and after the quake, the angular speeds of the nuetron star are recorded and prensented in Fig.21.12 .

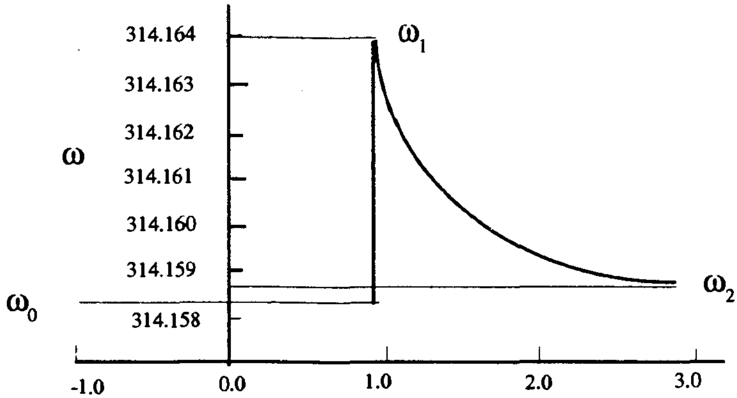



Fig. 21.12

Calculate the radius of the liquid mass in the interior of the star using data given in the figure above. Assume that the density of the crust is the same as that of the liquid part, and also the liquid core suffers no change in form.

## Solution

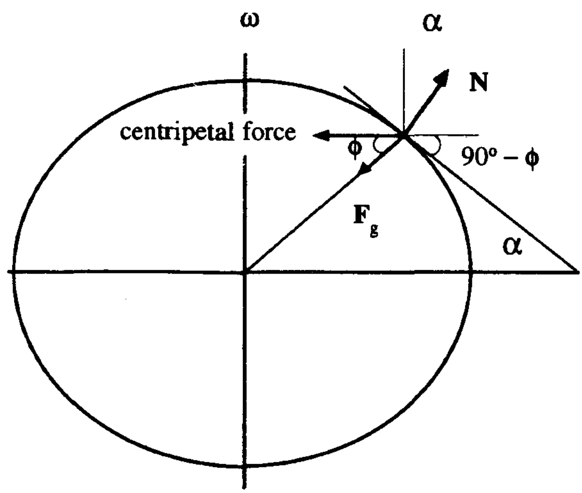



Fig. 21.13

**Method 1** Consider 1 unit mass at a position defined by the latitude making angle $\phi$ with the plane defining the equator.

In the inertial frame (the frame which does not rotate with the star), forces acting on 1 unit mass are:

gravitaional pull $\mathbf{F}_g$ and normal force $\mathbf{N}$

Let $\alpha$ be the angle which N makes with the axis of the star
Equation of motion of the unit mass is

$$\omega^2 x = F_g \cos\phi - N\sin\phi$$

$$N\cos\phi = F_g \sin\phi$$

From (2) $$N = F_g \frac{\sin\alpha}{\cos\alpha}$$

Substitute N to obtain $$\omega^2 x = F_g \cos\phi - F_g \sin\phi \tan\phi$$

$$F_g = |\mathbf{F}_g| = \frac{GM}{\langle r\rangle^2}$$

where M is mass of neutron star

Let $$x = \langle r\rangle \cos\phi$$

$$y = \langle r\rangle \sin\phi$$

From the diagram $$\tan\phi = \frac{dy}{dx}$$

Substitute cosϕ sinϕ and tanϕ in (1)

$$\omega^2 x = -\frac{GM}{\langle r\rangle^2}\left[\frac{x}{\langle r\rangle} - \frac{y}{\langle r\rangle}\frac{dy}{dx}\right]$$

$$= \frac{GM}{\langle r\rangle^3}(y\frac{dy}{dx} - x)$$

$$y\frac{dy}{dx} + (1 - \omega^2 \frac{\langle r\rangle^3}{GM})xdx = 0$$

$$\omega^2 \frac{\langle r\rangle^3}{GM} = \frac{(2.0\times10^{-2})(1.0\times10^4)^3}{6.67\times10^{-11}\times2.0\times10^{30}}$$

$$= 7.39\times 10\text{-}4$$

From (4) $y + \gamma x\, dx = 0$
where $\gamma = 1 - 7.39\times10^{-4}$

The equation defines the cross sectional area of the plane through the two poles is

$$\left(\frac{y}{r_P}\right)^2 + \left(\frac{x}{r_E}\right)^2 = 1$$

Differentiation of the equation above gives

$$\frac{2ydy}{r_P^2} + \frac{2xdx}{r_E^2} = o$$

Comaparing (4) and (6) gives

(6)

$$\left(\frac{r_P}{r_E}\right)^2 = 1 - \frac{\omega^2 \langle r \rangle^3}{GM}$$

$$\left(\frac{r_P}{r_E}\right)^2 = 1 - 7.39 \times 10^{-4}$$

$$\frac{r_P}{r_E} = 1 - 3.69 \times 10^{-4}$$

$$1 - \frac{r_P}{r_E} = 3.69 \times 10^{-4}$$

Index of elongation $\varepsilon = 3.69 \times 10^{-4}$ **Ans**

**Method 2** From data provided in the problem, the neutron star may be looked upon as a liquid drop or a huge liquid mass with the same value of surface tension throughout its entire surface.

Consider a neutron sheet of infinitesimally thin thickness forming a crust or skin encompassing the mass of neuron liquid.

Apply Guass's law for the neutron star that is not rotating

$$-4\pi r^2 F_g = 4\pi GM$$

$$F_g = -\frac{GM}{r^2}$$

where r is the radius of the star..

$F_g$ flux of gravitational field per unit area or gravitational force per unit mass.

Consider a neutron sheet of 1 $m^2$ areaand mass $\Delta m$ kg

Gravitational pull acting $\frac{GM\Delta m}{r^2}$ on this elemental surface is $\frac{GM\Delta m}{r^2}$ directed downward.

IN the neutron fluid, the pressure pushes the thin neutron sheet upward is also $\frac{GM\Delta m}{r^2}$.

This expression tells us that the pressure inside the netron star is inversely proportional to the square of the distance from the centre of the star.

Considering there is pressure acting on 1 $m^2$ area of the thin neutron sheet everywhere, the neutron sheet of 1 $m^2$ area may be seen exerting downward force of the same magnitude. This downward force may be understood as due to skin effect of the neutron sheet.

Consider the neutron star that is in rotation motion.

At the equator of the star, the thin neutron sheet of 1 $m^2$ surface area is subjected to the downward force due to skin effect equal to that at the pole ie $\frac{GM\Delta m}{r_P^2}$

Viewed in the rotating or non-inertial frame of reference, the forces acting on 1 $m^2$ are

- centrifugal force $\Delta m\omega^2 r_E$ ( $r_E$ radius of the star at the equator) and the pressure of liquid neutron $\frac{GM\Delta m}{r_E^2}$ - both acting in the direction away from the centre of the star.

- downward force due to skin effect of the neutron sheet $\frac{GM\Delta m}{r_P^2}$ acting in the direction toward the centre of the star.

The elemental surface is at equilibrium in the non-inertial frame.
Hence

$$\Delta m\omega^2 r_E + \frac{GM\Delta m}{r_E^2} = \frac{GM\Delta m}{r_P^2}$$

Consideration in the inertial frame gives lthe same oputcome as shown below:
In inertial frame, the downward force due to the ski n effect of the liquid neutron star and the pressure pushing the sheet upward provides the net force required for circular motion of the elemental neutron sheet of frequency ω.

$$\frac{GM\Delta m}{r_P^2} - \frac{GM\Delta m}{r_E^2} = \Delta m\omega^2 r_E$$

In either case , one arrives at

$$\omega^2 r_E = \frac{GM}{r_P^2} - \frac{GM}{r_E^2}$$

$$\left(\frac{r_P}{r_E}\right)^2 = 1 - \frac{\omega^2 r_E^3}{GM}$$

$$\left(\frac{r_P}{r_E}\right)^2 = 1 - 7.39 \times 10^{-4}$$

$$\frac{r_P}{r_E} = 1 - 3.69 \times 10^{-4}$$

$$1 - \frac{r_P}{r_E} = 3.69 \times 10^{-4}$$

$$\varepsilon = 1 - \frac{r_P}{r_E} = 3.69 \times 10^{-4} \quad \textbf{Ans}$$

3.2 Effects of the quake causes the moment of inertia of the crust represrnted by $I_S$ to decrease by $\Delta I_S$.

From the principle of the conservation of angular momentum,

$$I_S\omega_0 = (I_S - \Delta I_S)\omega_1$$

$$\Delta I_S = I_S\left(\frac{\omega_1 - \omega_0}{\omega_1}\right)$$

After the adjustment of the crust as a mechanism to move toward an equilibrium, the angular speeds of the outer crust and the liquid in the interior are the same.

Let $I_C$ be the moment of inertia of the liquid core of the star.

The principle of the conservation of angular momentum gives

$$(I_S + I_C)\omega_0 = (I_S + I_C - \Delta I_S)\omega_2$$

$$\Delta I_S = \frac{(I_S + I_C - \Delta I_S)(\omega_2 - \omega_0)}{\omega_2}$$

$$\frac{I_S(\omega_1 - \omega_0)}{\omega_1} = \frac{(I_S + I_C)(\omega_2 - \omega_0)}{\omega_2}$$

$$\frac{I_S}{I_S + I_C} = \frac{(\omega_2 - \omega_0)\omega_1}{(\omega_1 - \omega_0)\omega_2}$$

$$1 - \frac{I_S}{I_S + I_C} = \frac{I_C}{I_S + I_C} = 1 - \frac{(\omega_2 - \omega_0)\omega_1}{(\omega_1 - \omega_0)\omega_2}$$

Moment of inertia I is directly proportional to the square of the radius.

$$\frac{I_C}{I_S + I_C} = \left(\frac{r_C}{r_E}\right)^2 = 1 - \frac{(\omega_2 - \omega_0)\omega_1}{(\omega_1 - \omega_0)\omega_2}$$

where $r_C$ is the radius of the liquid core

$r_E$ the average value of the radius

Substitute the values of $\omega_1$ $\omega_2$ and $\omega_0$ read from the graph provided to obtain

$$\frac{r_C}{r_E} = 0.95$$

And radius of the liquid core $r_C = 0.95 \times 10^4$ m **Ans**

# Experiment

## Problem 1 Determination of the Efficiency of Light-Emitting Diode or LED

### Introduction

In this experiment, two types of semi-conductor are used i.e. light emitting diode(LED) and photo-diode (Photo-Diode or PD)
In LED, part of electrical energy is used to excite electron to higher states. When these electrons return to the lower states, photons of energy $E_p$ characteristic of the photons are emitted.

$$\text{with } E_P = \frac{hc}{\lambda}$$

where h is Planck's constant
c speed of light
λ photon's wavelength

Efficiency of LED is given by $\eta = \frac{\Phi}{P_{LED}}$

where $\Phi$ is power or energy emitted per 1 unit time and
$P_{LED}$ power consumed by LED

In the photo-detecting diode or PD, light energy is converted into electrical energy. The process begins with light falling on the photo-sensitive surface of the diode. Some photons (not all) are released from the crystal which is a photo-sensitive material and thus become free elctrons.

The ratio between the number of photons striking photo-sensitive surface per second ($N_p$) and the number of freed electrons ($N_E$) is referred to as quantum efficiency or $q_e$.

$$q_e = \frac{N_E}{N_P}$$

### Experiment

The objective of this experiment is to determine the efficiency of LED as a function of current through LED.Towards this end, we measure the intensity of light emitted from LED with a photo-diode.

LED and PD are housed in the respective boxes and connected in the circuit shown in Fig. 21.14. The potential across the diode and the current through both diodes i.e. LED and PD are obtained through measuring voltages across $R_1$ and $R_2$.

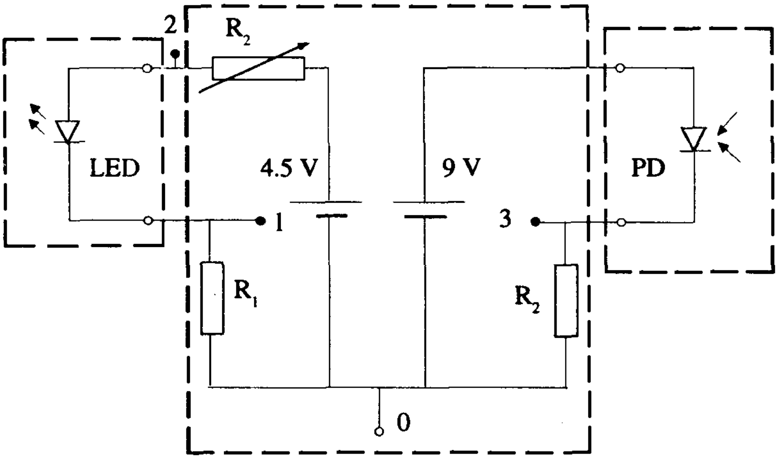



Fig. 21.14

The multimetre provided can be used for measuring voltage only.
To use the multimeter for this purpose, the knob is turned to position marked V. In this position, sensitivity and proper scale is set automatically, and the letters "auto" should appear on the screen. However , if the letters "auto" are not displayed,the knob is turned to "off" and back to position V again. The probes of the multimetre must be connected across terminals marked COM and V-Ω

The two boxes housing PD and LED can be moved freely along the board. When the two boxes are lined up facing each other, the apertures of LED and PD are in the same straight line normal to the cross-sectional areas of both apertures.

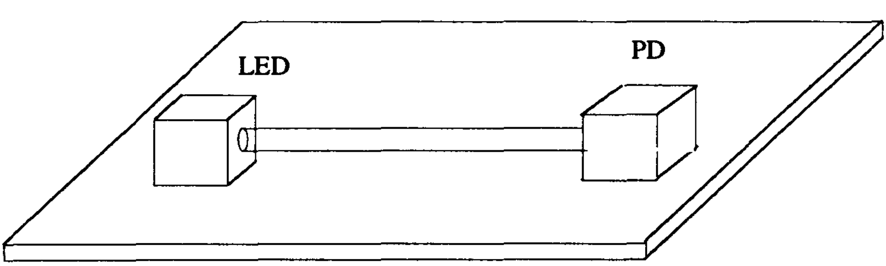



Fig 21.15

## Necessary Information

Quqntum efficiency of photodiode = 0.88
Light collecting area of photodiode = $2.75 \times 2.75\ mm^2$
Wavelength of light emitted by photodiode = 635 nm

Internal resistance of voltmeter
= 100 MΩ when measurement is carried out in the range not exceeding 200 mV
= 10 M Ω for any other range of measurement

The range of voltage being measured is displayed in small letters on the screen.

| | | | |
|---|---|---|---|
| Planck's constant | h = | $6.63 \times 10^{-24}$ | J.s |
| Electronic charge | e = | $1.6 \times 10^{-19}$ | C |
| Speed of light in vacuum | c = | $3.0 \times 10^{8}$ | m/s |

$R_1$ = 100 Ω
$R_2$ Adjustable

0 1

0 2

0 3 terminals for connecting with multimeter

## Hints

1. Before the quantum efficiency of LED can be determined, it is necessary to calibrate the scale for measuring light from the photo-detecting diode .

The immediate problem is there is no information about the photo-detecting diode. The contestant must therefore perform an experiment to demonstrate that the current in the PD is directly proportional to the intensity of light falling on the photo-sensitive surface of PD.

2. Find the value of the current through LED that gives the maximum value of quantum efficiency.

3. Perform an experiment to find the value of absolute quantum efficiency by investigating the cone of light emitted by LED.

Analysis of error is not required,nor there are any marks for such analysis. Experimental results must be clearly and concisely reported, using tables and graphs complete with proper units.

# Solution

1. Calibrate the light intensity readings expressed in terms of the current through the photo-detecting diode in the following ways:

1.1 Pass current through LED and choose proper value of current that is to be kept constant.

1.2 Meausre the intensity of light from LED expressed in terms of current through the photo-detecting diode. The current is calculated from the value of measured potential difference divided by resistance $R_3$

$$\text{i.e.} \quad I = c\frac{V_3}{R_3}$$

1.3 Since there is background light also recorded by the photo-detecting diode, each measurement of light intensity must be deducted by current reading of the background light.

1.4 Let x represent the distance between LED and PD.

Plot a graph of light intensity from LED expressed in terms of current in PD ( deducted by background current) as a function of $\frac{1}{x^2}$.

The curve is a straight line but does not go through the origin, because LED is located at some distance inside the box. The actual distance is therefore x + d , where d is the distance at which LED is located behind the aperture. The curve however shows that light intensity emitted by LED is directly proportional to current in PD.

If a graph of $\frac{1}{\sqrt{I}}$ as a function of x+d, the curve is a straight line. the intercept on the vertical axis represents the value of d.

Let $V_1$ be voltage across $R_1$ (unit in volt)
$V_1$ voltage across $R_1$ (in volt)
$V_2$ voltage across $R_2$ (in volt)
$V_3$ voltage across $R_3$ (in volt) when LED is emitting light.
$V_3'$ voltage across $R_3$ (in volt) when LED is not emitting light.
I Intensity of light from LED.

$$I \propto \frac{V_3 - V_3'}{R_3}$$

$$I = c\frac{V_3 - V_3'}{R_3} \qquad \text{where c is a constant.}$$

Record the intensity of light versus distance in the table below.

| x (cm) | I (in unit of cA) |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |

2. Determination of current in LED that results in $\eta_{MAX}$.

2.1 Set up the experiment as in 1, but in this case x or the distance between LED and PD is kept constant.

2.2 Adjust the intensity of light from LED by varying $R_2$.

Power consumed by LED = $I_{LED} \times V_{LED}$

where $I_{LED}$ is the current in LED,

$V_{LED}$ potental difference across LED,

$$V_{LED} = V_2 - V_1$$

$$I_{LED} = \frac{V_1}{R_1}$$

Power consumed by LED or $P_{LED} \propto \frac{(V_3 - V_1)V_1}{R_1}$

2.3 Measurement of the intensity of light at PD.

The intensity of light $\propto \frac{V_3 - V_3'}{R_3}$

In this part of the experiment, voltages to be measured are over 200 mV and the internal resistance of the voltmeter is 10 MΩ, while $R_3$ is 1 MΩ. The current through the voltmeter is stepped up by 10% , therefore when the voltmeter is used in this arrangement, the actual current is obtained by adding 10 % of the unadjusted value. Likewise when the voltmeter resistance is 1 MΩ , the actual current is obtained by adding 1% of unadjusted value.

Intensity of light $I = c\frac{V_3 - V_3'}{R_3}.\frac{110}{100}$ in thevoltage range > 200 mV

$I = c\frac{V_3 - V_3'}{R_3}.\frac{101}{100}$ in the voltage range < 200 mV

$R_1 = 100$ $R_2 = 1$ MΩ

Record $V_1$ $V_2$ $I_{LED}$ $P_{LED}$ I η in the table below.

| $V_1$ (V) | $V_2$ (V) | $I_{LED} = V_1/R$ (mA) | $P_{LED}$ = $(V_2 - V_1)/R_1$ (J.s) | I = $[(V_3 - V_3')/R_3]k$ | η |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

k = 1.1 or 1.01 depending on voltage range.

Plot a graph of h as a function of I. Read the value of the current at $\eta_{MAX}$ from the graph. From the data provided, $\eta_{MAX}$ occurs when the current through LED is approximately 6 mA.

3. Determination of Absolute Maximum Efficiency i.e. ratio between the light energy from LED and elctrical energy consumed by LED which results in maximum efficiency $\eta_{MAX}$.

Examination of the symmetry of LED reveals that light emitted from Led is in a conic form. It follows that light intensity from LED besides depending on distance from LED also depending on height measured from the horizontal plane.

Let $d \times d$ be the area of the square-shaped aperture of PD

r be horizontal distance measured from LED to the aperture of PD.

To find the net light intensity, we need to measure light intensity from different positions on the conic surface.

We thus divide the conic surface into several bands (See Fig 21.16 )

Let $r_i$ be horizontal distance of the ith band measured from LED to the position of the square aperture
and n the number of bands.

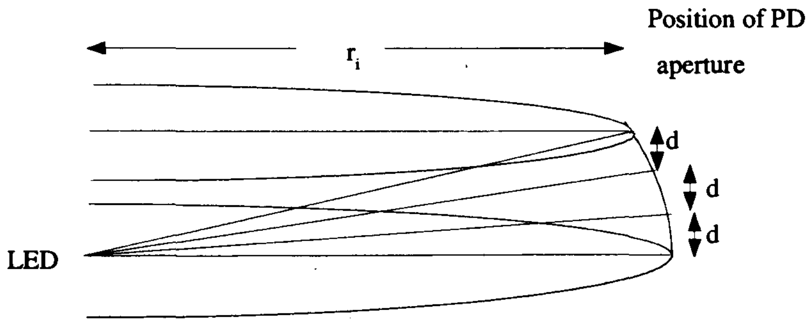



Fig 21.16

The area is divided into n bands having radii $r_1$ $r_2$ $r_3$...... $r_n$ measured from the vertical axis through LED.
At distance $r_i$ measured from the axis through LED to the aperture, light intensity at PD corresponds to current,

$$i(r_i) = N_e e = N_p . q_e . e = \frac{\phi(r_i).q_e.e}{h\nu} \tag{1}$$

where $\phi(r_i)$ is light energy at the aperture situated in the region of the band characterized by radius $r_i$
Hence total light energy emitted by LED is

$$\Phi = \sum_{i=1}^{n} \phi(r_i) \frac{2\pi r_i d}{d^2} = \frac{2\pi}{d} \sum_{i=1}^{n} \phi(r_i).r_i \tag{2}$$

From (1) $\qquad \phi(r_i) = \frac{h\nu}{q_e.e} i(r_i)$

Substitution of $\phi(r_i)$ in (2) to arrive at

$$\Phi = \frac{2\pi}{d} . \frac{h\nu}{q_e.e} \sum_{i=1}^{n} i(r_i).r_i$$

Measure intensity of light at the aperture of ith band i.e. $i(r_i)$ for each $r_i$ and calculate $i(r_i).r_i$. (This is done by moving PD vertically )

With the value of power consumed by LED (or $P_{LED)}$ known, maximum absolute efficiency can be calculated.

$$\eta_{AB} = \frac{\Phi}{P_{LED}}$$

# Problem 2 Calculation of Ratios of Strength of Magnetic Fields Between Pairs of Magnets

## Introduction

Whenever a conductor moves through magnetic field, electric current is induced in the conductor which forms a closed circuit. The current generated in the closed circuit in this manner is known as" Eddy Current". Interaction between induced current and magnetic field tends to resist the motion of the conductor in the magnetic field. An aluminum disc put to rotation about the axis through its centre normal to uniform magnetic field will be slowing down by the interaction between induced current and the magnetic field.

## Apparatus

Stand

Clamps

Uniform aluminum disc mounted on the axle through its centre and supported by the stand.

Two sets of magnets each set consists of two identical cylindrical magnets of equal pole strength( Let magnetic field between two magnets in each set be$B_1$ and $B_2$ respectively.)

Two weighing bobs, one bob wieghs twice the other.(Error of about1 %)

Stop watch

Ruler

## Experiment

An aluminum disc is designed to rotate about the horizontal axle held in position on the stand.(See Fig. 21.17). A light string has one end wound around the axle, and the other end attached to a bob. When the bob is allowed to descend vertically, the aluminum disc begins to turn with increasing angular speed until it reaches a terminal or constant angular value. The value of the final angular speed depends on the strength of magnetic field and other factors to be discussed.

Let $B_1$ and $B_2$ represent magnetic fields between two identical magnets of the two respective sets.

## Practical Qestions

1.Design an experiment to determine the best possible value of $\frac{B_1}{B_2}$

2. Explain concisely the theory or principle which allows determination of the ratio mentioned in 1 above.
3. Carry out the experiment to determine the value of the ratio .
4. Estimate the earror of the answer $\dfrac{B_1}{B_2}$.

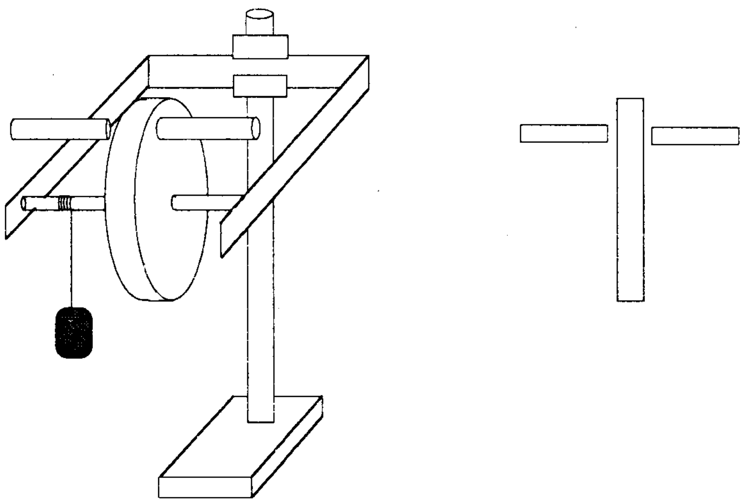


Fig. 21.17

# Solution

## Analysis of the relationship between relevant variables

Let I be the moment of inertia of the aluminum disc.

m mass of the bob attached to end of the string,

B magnetic field between the two cylindridal magnets facing each other in the end-on position

$\tau_F$ torque on the disc due to friction,

$\tau_B$ torque on the disc due to interaction beween induced current and the magnetic field,

The rate of change of angular momentum of the disc is equal to net torque on the disc,

$$I\ddot{\theta} = \left|\mathbf{r} \times \mathbf{T} - \tau_f - \tau_B\right| \tag{1}$$

where T is tension in the string,

$\ddot{\theta}$ angular acceleration of the disc, r radius of the axle

The resultant or net torque is in the same sense with angular acceleration.
Equation of motion of the bob,

$$mg - T = ma \qquad (2)$$

Substitute T from (2) in (1) to arrive at,

$$I\ddot{\theta} = (mg - ma)r - \tau_F - \tau_B$$

The value of the magnetic moment m of induced current is in direct proportion to the values of B and ω,

$$\mu \propto B\omega$$

Now that the torque τ due to the interaction between the induced current and the magnetic field is given by

$$\tau = \mu \times \mathbf{B}$$

and $$\tau \propto B^2 \omega R$$

( R is the distance fromthe centre of the disc to the mid point between the two cylindrical magnets,)

or $$\tau_B = cB^2 \omega R \qquad (\text{c = constant})$$

where $\tau_B$ is torque due to interaction between induced current and magnetic field.

Hence $$I\ddot{\theta} = (mg - ma)r - \tau_F - cB^2 \omega R$$

When the disc rotates with constant angular speed

$$\dot{\theta} = \text{constant} \qquad \ddot{\theta} = 0 \qquad \text{also } a = 0$$

$$0 = mgr - \tau_F - cB^2 \omega R$$

Substitute $$\omega = \frac{v_C}{r}$$

where $v_c$ is the final speed of the bob (constant)
r radius of the axle

thus $$0 = mgr - \tau_F - cB^2 R \frac{v_C}{r}$$

$$\frac{cB^2 v_C}{r} = mgr - \tau_F$$

$$v_C = \frac{gr^2}{cB^2 R}\left(m - \frac{\tau_F}{gr}\right) \qquad (3)$$

Equation above tells us that the magnitude of the final velocity of the bob is directly proportional to its mass m , but inversely proportional to $B^2$.

## Experimental Procedures

1. Set up the apparatus as shown in Fig.21.17 ,using the first bob. Release the bob and allow it to fall vertically. Make sure that in the range of height which we choose, the bob falls with a constant speed $v_C$. Conduct 4 experimental trials and in each trial calculate $v_C$ .Calculate average value $v_{c1}$ for the first experiment using the first bob.

2. Conduct the experiment similar to 1, using the second bob. As in 1, determine four readings $v_c$ from four experimental trials. Calculate the average value to obtain $v_{c2}$ for the second bob.

From the first experiment

$$v_{C1} = \frac{gr^2}{cB_1^2R}\left(m_1 - \frac{\tau_F}{gr}\right) \tag{4}$$

and the second experiment

$$v_{C2} = \frac{gr^2}{cB_1^2R}\left(m_2 - \frac{\tau_F}{gr}\right) \tag{5}$$

(4)-(5) $$v_{C2} - v_{C1} = \frac{gr^2}{cB_1^2R}(m_2 - m_1) \tag{6}$$

where $m_1$ and $m_2$ are weights of the first and second bobs respectively.

3. Repeat the experiment using the sencond set of magnets.

Likewise $$v_{C2}' - v_{C1}' = \frac{gr^2}{cB_2^2R}(m_2 - m_1) \tag{7}$$

where $v_{C1}'$ and $v_{C2}'$ represent the final speeds of the first and second bob when the second set of magnets is used.

(6)+(7) $$\frac{v_{C2} - v_{C1}}{v_{C2}' - v_{C1}'} = \left(\frac{B_2}{B_1}\right)^2 \tag{8}$$

The values $v'_{C2}$ , $v'_{C1}$ , $v_{C2}$ , $v_{C1}$ are obtained from the experiment, hence the value $\frac{B_2}{B_1}$ can be calculated in a straightforward manner.

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## Theory

## Problem 1

In Fig. 22.1, a solid ball of radius R rotates with angular speed $\omega_0$ about a horizontal axis which passes through its centre. The ball is dropped from height h measured from its lowest point to a flat horizontal floor. On rebounding the ball\ reaches the height of $\alpha h$ also measured from the lowest point of the ball to the floor. The ball stays in contact with the floor for a very brief interval of time before the rebound. The coefficient of friction between the ball and the floor is $\mu_k$.

Given m is the mass of the ball
g gravitational acceleration
I moment of inertia of the ball $= \frac{2}{5}mR^2$
R radius of the ball.

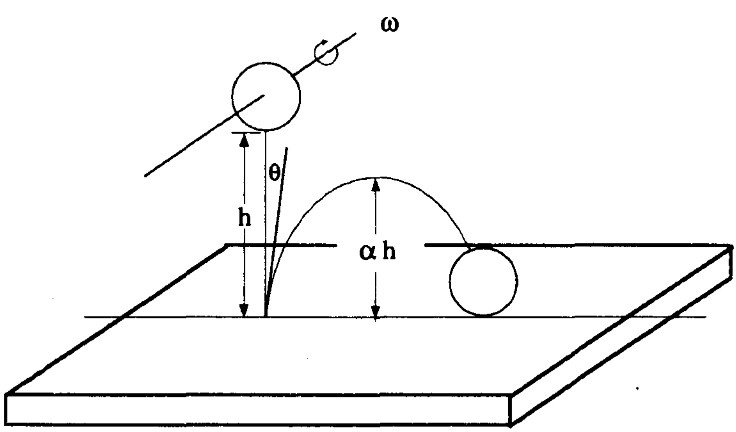



Fig. 22.1

Answer the following quesions:

1.1 If the ball is in scraping motion all the time while in contact with the floor before rebounding , determine

a. tangent of the reflecting angle $\theta$

b. horizontal distance covered by the centre of mass of the ball over the interval between the ball's first and second contacts with the floor.

c. The minimum value of $\omega_0$ which makes the motion described above possible.

1.2 If the scraping motion terminates and turns into pure rolling motion before rebound ing, answer questions a and b in 1.1 with the given condition described under 1.2.

1.3 Sketch a graph of $\tan\theta$ as a function of $\omega_0$ for cases of 1.1 and1.2.

# Solution

1.1 a. Calculation of the speed of the ball just before its contact with the floor $v_B$ ( velocity in the vertical direction before contact with the floor ).

From the principle of the conservation of mechanical energy

$$\frac{mv_B^2}{2} = mgh$$

$$v_B = \sqrt{2gh} \tag{1}$$

Calculation of the speed of the ball just after it leaves the floor $v_A$ ( velocity in the vertical direction after the contact with the floor ).

$$\frac{mv_A^2}{2} = mg\alpha h$$

$$v_A = \sqrt{2g\alpha h} \tag{2}$$

Vertical Impulse

$$F\Delta t = m\sqrt{2gh} - \left(-m\sqrt{2g\alpha h}\right)$$

$$= m\sqrt{2gh}(1+\sqrt{\alpha})$$

where F is the normal force of the floor acting on the ball

$$F = \frac{m}{\Delta t}\sqrt{2gh}\left(1+\sqrt{\alpha}\right) \tag{3}$$

The maximum value of static friction is given by

$$f = \frac{\mu_k m}{\Delta t}\sqrt{2gh}\left(1+\sqrt{\alpha}\right) \tag{4}$$

Let $\omega_f$ represent the angular speed of the ball just after the rebound.

The equation of motion of the ball is

$$I\left(\frac{\omega_0-\omega_f}{\Delta t}\right)=f.R \tag{5}$$

where I is the motment of inertia of the balll about the axis through the centre.
Substitute f from (4) in (5) to obtain

$$I\left(\frac{\omega_0-\omega_f}{\Delta t}\right)=\frac{\mu_k m}{\Delta t}\sqrt{2gh}\left(1+\sqrt{\alpha}\right)R$$

$$\omega_0-\omega_f=\frac{\mu_k m}{I}\sqrt{2gh}\left(1+\sqrt{\alpha}\right)R$$

$$\omega_0=\omega_f+\frac{\mu_k m}{I}\sqrt{2gh}\left(1+\sqrt{\alpha}\right)R \tag{6}$$

Let $v_x$ be the value of the velocity of the ball along the horizontal direction.
The hoizontal impulse on the ball is equal to change in the horizontal component of the ball.
i.e. $f\Delta t = mv_x$

$$\mu_k m\sqrt{2gh}\left(1+\sqrt{\alpha}\right)=mv_x$$

$$v_x=\mu_k\sqrt{2gh}\left(1+\sqrt{\alpha}\right) \tag{7}$$

$$\tan\theta=\frac{v_x}{v_A}$$

$$=\frac{\mu_k\left(1+\sqrt{\alpha}\right)}{\sqrt{\alpha}}$$

$$=\mu_k\left(1+\frac{1}{\sqrt{\alpha}}\right)$$

$$\theta=\tan^{-1}\mu_k\left(1+\frac{1}{\sqrt{\alpha}}\right)$$

The equation above indicates that $\theta$ does not depend on $\omega_0$. **Ans**

b. The horizontal distance covered by the ball over the interval between the first and second contacts of the ball with the floor is given by

$$x = v \times t$$

t is determined from $s = ut + a\,t^2$

where $u = \sqrt{2gh\alpha}$

$s = 0$

Substitute $\sqrt{2gh\alpha}-g\frac{t^2}{2}=0$

$$t=2\sqrt{2gh\alpha}\ \text{t}$$

$$x=\mu_k\sqrt{2gh}(1+\sqrt{\alpha})2\sqrt{2gh\alpha}$$

$$x = 4gh\mu_k\alpha\left(1+\frac{1}{\sqrt{\alpha}}\right)\textbf{Ans}$$

c. For the ball to be in the scraping motion, the velocity of the circumference of the ball at the contact point $v_x \leq w_f R$.

From (6) $$\omega_f = \omega_0 - \mu_k m\sqrt{2gh}\left(\frac{1+\sqrt{\alpha}}{I}\right)$$

and $$\omega_f \geq \frac{v_x}{R}\text{m.}$$

$$\omega_0 - \mu_k m\sqrt{2gh}\left(\frac{1+\sqrt{\alpha}}{\frac{2}{5}mR^2}\right) \geq \frac{v_x}{R}$$

Substitute $v_x$ from (7) in the above to arrive at

$$\omega_0 - \frac{5\mu_k\sqrt{2gh}\left(1+\sqrt{\alpha}\right)}{2R} \geq \mu_k\sqrt{2gh}\left(\frac{1+\sqrt{\alpha}}{R}\right)$$

$$\omega_0 \geq \frac{7}{2}\mu_k\sqrt{2gh}\left(\frac{1+\sqrt{\alpha}}{R}\right)$$

Value of minimum $\omega_0$ or $(\omega_0)_{MIN} = \frac{7}{2}\mu_k\sqrt{2gh}\left(\frac{1+\sqrt{\alpha}}{R}\right)$ **Ans**

1.2. The scraping motion turning into pure rolling motion takes place when the centre of mass of the ball has linear velocity $v_x$ and angular momentum about the point of contact = $mv_xR$

Initial angular momentum about the poin of contact = Angular momentum about the point of contact

$$\left(I + mR^2\right)\omega_0 = mv_xR$$

$$\left(\frac{2}{5}mR^2 + mR^2\right)\omega_0 = mv_xR$$

$$v_x = \frac{2}{7}R\omega_0$$

a. Determination of $\tan\theta$ for 1.2

$$\tan\theta = \frac{v_x}{v_A}$$

$$= \frac{2}{7}\frac{R\omega_0}{\sqrt{2gh\alpha}} \qquad \textbf{Ans}$$

b. Determination of horizontal distance covered by the ball between the first and second contacts with the ground

$$x = v_x t$$

$$= \frac{4}{7} R\omega_0 \sqrt{2gh\alpha} \qquad \textbf{Ans}$$

c. A sketch of $\tan\theta$ as a function of $\omega_0$

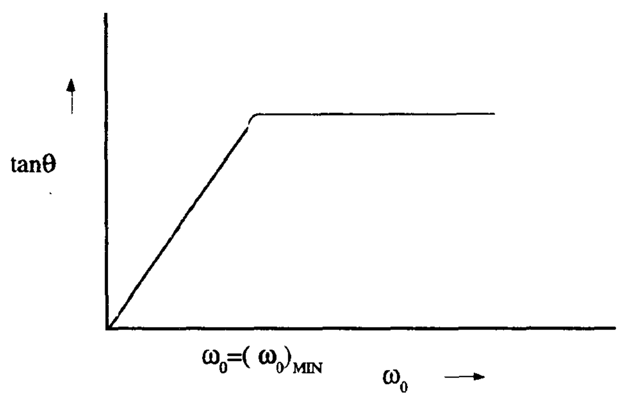



Fig. 22.2

Under Case 1a $\qquad \tan\theta = \mu_k\left(1+\frac{1}{\sqrt{\alpha}}\right)$ for $\omega \geq (\omega_0)_{MIN}$

$\tan\theta$ does not depend on $\omega_0$ but is defined only in the region $\omega \geq (\omega_0)_{MIN}$

Under Case 1b

$$\tan\theta = \frac{2}{7}\frac{R\omega_0}{\sqrt{2gh\alpha}} \text{ for } 0 \leq \omega \leq (\omega_0)_{MIN}$$

$\tan\theta$ depends linearly on $\omega_0$ in the region $0 \leq \omega \leq (\omega_0)_{MIN}$ **Ans**

## Problem 2

A square-shaped loop with length L on each side has a large number of charged beads moving along the sides of the loop in the same sense.

For an observer at rest relative to the loop, the beads are at distance a from their respective neighbours, and travel with velocity u along the side of the square. Given L is much larger than a i.e. L >>>> a. (See Fig. 22.3) The loop itself is made from insulator and carries uniformly distributed charge opposite in sign but equal in net magnitude o f all the beads.

If the loop itself moves with velocity **v** in the direction parallelto side AB in a uniform electric field **E** normal to **v** and makes angle $\theta$ with the plane of the loop (See Fig. 22.3)

Apply the principle of special theory of relativity to calculate the following qunatities for the frame in which the observer at rest sees the loop moving with velocity **v** along the direction parallel to AB.

2.1 Distance between two adjacent beads on the four sides of the sqare i.e. $a_{AB}$ $a_{BC}$ $a_{CD}$ and $a_{DA}$

2.2 Resultant or net charge in the material of the loop and the beads on each side of the square i.e $Q_{AB}$ $Q_{BC}$ $Q_{CD}$ and $Q_{DA}$.

2.3 Magnitude of torque $\tau$ which the electric field exerts on the loop.

2.4 Energy of interaction W between the system of loop and beads and the electric field E.

All answers must be expressed in terms of the variables given in the problem.

**Hints** The magnitude or value of charge does not depend on whi ch frame of reference. The diagram provided serves to illustrate the direction of the vectors only. Effects as a result of elctromagnetic radiation are to be ignored.

Useful formulas from Theory of Special Relativity are given below:

1. Frame of referece $S'$ moves with velocity v relative to frame of reference S in the direction parallell to x-axes of both systems. At $t' = t = 0$ the origins of both system coincide and v points in the direction parallel to positive x-axes of sytems S and $S'$ if a particle travels with velocity $u'$ relative to $S'$ the velocity of the particle as observed by an observer stays at rest in system S is given by

$$u = \frac{(u' + v)}{\left(1 + \frac{u'v}{2}\right)}$$ directed along +ve x axis

2. An object with length $L_0$ is observed by an observer at rest relative to the object. For another observer moving with velocity v relative to the objecct in the direction paralell to length $L_0$, the length of the same object is given by $L = L_0\sqrt{1-\frac{v^2}{c^2}}$

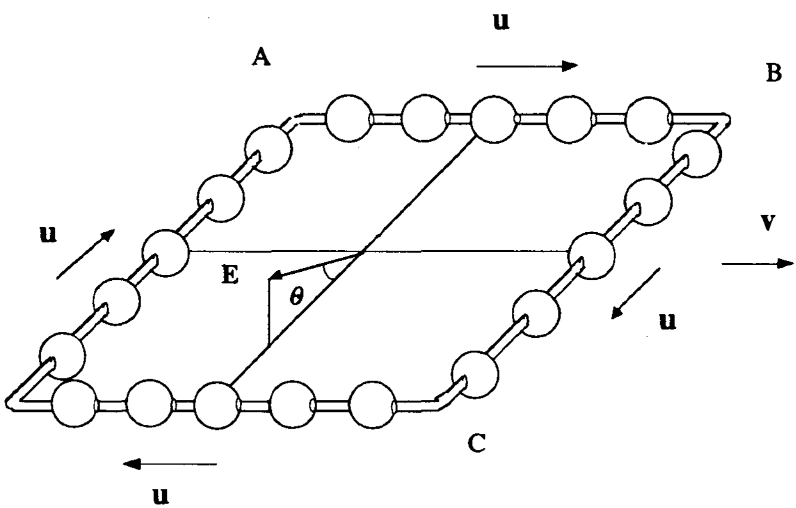



Fig. 22.3

## Solution

Let S be the frame of reference in which an observer is at rest

$S'$ be the frame of reference which moves with velocity v relative to S along AB. (The observer who moves with $S'$ observes the loop to be at rest relative to him.)

$S''$ be the frame which moves with speed u along AB or BC ( or CD or BA as the case may be) relative to $S'$.

1. One of the most convenient ways of solving this problem is to find required answers as seen by the observer at rest in $S''$ and relate the answers to the values as observed by the observer at rest in $S''$ to the values as observed by the observer at rest in S.
Let $a_{AB}''$ $a_{BC}''$ $a_{CD}''$ $a_{DA}''$ represent the distance between two adjacent beads along AB, BC, CD and DA respectively. It is to be noted too that these are the values as observed by the observer at rest in $S''$.

From the information provided in the problem $a_{AB}' = a_{BC}' = a_{CD}' = a_{DA}' = a$ are length of AB, BC, CD, and DA sides of the loop as observed by the observer at rest in $S'$.

A rod travels with velocity v in a direction along the length of the rod, an observer who is at rest observes the length of the rod measured along the direction of motion as L given by:

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

where $L_0$ is the length of the rod as observed by the observer at rest with the rod.
( the length becomes shorter - Lorentz' contraction)

The observer at rest in $S''$ , $a''$ is longer than $a'$. length of the square observed by the observer at rest in $S'$.

i.e. $$a''\sqrt{1-\frac{u^2}{c^2}} = a' = a$$

and $$a_{AB}'' = a_{BC}'' = a_{CD}'' = a_{DA}'' = \frac{a}{\sqrt{1-\frac{u^2}{c^2}}} \tag{1}$$

where u is velocity of the bead relative to the observer at rest in $S'$.
$v_s''$ velocity of the bead relative to the observer at rest in $S''$

Apply the formula provided in the problem, velocity $v_s''$ of frame $S''$ relative to S is:

$$v_s'' = \frac{u+v}{\left(1+\frac{uv}{c^2}\right)} \tag{2}$$

From (1) $$a_{AB} = a_{AB}'' = a_{AB}''\sqrt{1-\left(\frac{v_S''}{c}\right)^2} \tag{3}$$

Substitute $a_{AB}''$ from (1) and $v_s''$ from (2) in (3) to obtain

$$a_{AB} = \frac{a}{\sqrt{1-\frac{u^2}{c^2}}}\sqrt{1-\left(\frac{u+v}{c\left(1+\frac{uv}{c^2}\right)}\right)^2}$$

$$\sqrt{1-\left(\frac{u+v}{c\left(1+\frac{uv}{c^2}\right)}\right)^2} = \sqrt{1-\left(\frac{u+v}{\left(c+\frac{uv}{c}\right)}\right)^2}$$

$$= \sqrt{\frac{c^2 + 2uv + \left(\frac{uv}{c}\right)^2 - v^2 - u^2 - 2uv}{\left(c + \frac{uv}{c}\right)^2}}$$

$$= \sqrt{\frac{c^2 - v^2 - u^2\left(1 - \frac{v^2}{c^2}\right)}{\left(c + \frac{uv}{c}\right)^2}}$$

$$= \sqrt{\frac{c^2\left(1 - \frac{v^2}{c^2}\right) - u^2\left(1 - \frac{v^2}{c^2}\right)}{c^2\left(1 + \frac{uv}{c^2}\right)^2}}$$

$$a_{AB} = a\frac{\sqrt{1 - \frac{v^2}{c^2}}}{\left(1 + \frac{uv}{c^2}\right)} \qquad \textbf{Ans}$$

In the same spirit of the argument

$$a_{CD} = a_{CD}'' \sqrt{1 - \left(\frac{v_S''}{c}\right)^2}$$

In this case $\quad v_S'' = \dfrac{v - u}{\left(1 - \frac{uv}{c^2}\right)}$

Likewise the calculation gives

$$a_{CD} = a\frac{\sqrt{1 - \frac{v^2}{c^2}}}{\left(1 - \frac{uv}{c^2}\right)} \qquad \textbf{Ans}$$

The distance between two adjacent beads along BC as observed or measured by an observer at rest in frame S is $a_{BC}$. In this case observer at rest in S frame is on par with observer at rest in S′.

$$a_{BC} = a'_{BC} = a \qquad \textbf{Ans}$$

and $\quad a_{DA} = a'_{DA} = a \qquad$ **Ans**

2.2 Consider what the observer at rest in S sees when he measures charge on each side of the loop.

Let $Q_{AB}$ $Q_{BC}$ $Q_{CD}$ and $Q_{DA}$ be charge on the sides of the loop i.e. AB BC CD and DA respectively.
The quantity of charge on each side is obtained in the following way:

Take the length of the side under consideration divided by the distance between two adjacent bead on that side of the square to obtain the number of beads on that side ,and the number of the beads times charge of one single bead (q) gives the net ampount of charge of that side.
Hence the amount of charge of AB

$$Q_{AB} = \frac{\sqrt{1-\frac{v^2}{c^2}}.Lq}{a_{AB}}$$

Substitute $a_{AB}$ to obtain $$Q_{AB} = \frac{\sqrt{1-\frac{v^2}{c^2}}.Lq.\left(1+\frac{uv}{c^2}\right)}{a\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{Lq}{a}\left(1+\frac{uv}{c^2}\right)$$

In the same spirit of the argument

$$Q_{AB} = \frac{Lq}{a}\left(1-\frac{uv}{c^2}\right)$$

and $$Q_{CD} = \frac{Lq}{a}$$

$$Q_{DA} = \frac{Lq}{a}$$

Excess charge on AB $= Q^*_{AB} = Q_{AB} - \frac{Lq}{a} = \frac{Lq}{a}\frac{uv}{c^2}$ **Ans**

Likewise

Excess charge on CD $= Q^*_{CD} = Q_{CD} - \frac{Lq}{a} = -\frac{Lq}{a}\frac{uv}{c^2}$ **Ans**

Excess charge on BC and CD are $Q^*_{BC}$ and $Q^*_{CD}$ respectively, and

$$Q^*_{BC} = Q^*_{CD} = \frac{Lq}{a} - \frac{Lq}{a} = 0$$ **Ans**

2.3 Electric Field E exerts pushing force on net charge $\frac{Lq}{a}\frac{uv}{c^2}$ on AB

and exerts pulling force on net charge $-\frac{Lq}{a}\frac{uv}{c^2}$ on CD

giving rise to torque $|\tau| = \frac{Lq}{a}\frac{uv}{c^2}E.L\sin\theta$ **Ans**

2.4 Energy of interaction W = - L cosθ $\frac{Lq}{a}\frac{uv}{c^2}$

$$= \frac{L^2\cos\theta}{a}.\frac{q.uv.E}{c^2}$$ **Ans**

## **Problem 3** Laser Cooling of Atoms

In order to study properties of atoms with ever-increasing accuracies, we need to make the atoms to be at rest in an extremely confined region. Recently, a new technique known as " Laser Cooling " has been invented to achieve this particular purpose.

In a vacuum container, a narrow beam of $Na^{23}$ (from the evaporation of the sample at $10^3$ K )is sent in the direction of head-on collision with high-intensity laser beam.(See Fig.à 22.4) A proper frequency of the laser beam allows the beam to be absorbed by the $Na^{23}$ atom travelling with velocity $v_0$, causing the atom to be in the first excited state E . The width of this energy level is ΔE . As a result of the absorption, the magnitude of the velocity of the atom changes by $\Delta v = v_1 - v_0$ (See Fig.22.5) and the direction of the velocity by angle $\phi$.

The asorption and emission of photons take place in a continuous manner until the change in the magnitude of the velocity of the atom reaches the value of Δv, and the atom can no longer absorb the laser beam with frequency ν any more. It is therefore necessary to change the frequency of the laser beam in order to permit absorption of the beam by the atom again. As the process continues, the atoms slow down and the velocity of some of the atoms is close to zero.

Ignore interaction between atoms and assume that the intensity of the laser beam is so large that the duration which each atom is in the excited and ground states is close to zero.

### Information

| | | | |
|---|---|---|---|
| | $E$ | $= 3.36\times10^{-19}$ | J |
| | $\Delta E$ | $= 7.0\times10^{-27}$ | J |
| Speed of light | $c$ | $= 3.0\times10^{8}$ | m/s |
| mass of proton | $m_p$ | $= 1.67\times10^{-27}$ | kg |
| Planck's constant | $h$ | $= 6.62\times10^{-34}$ | J.s |
| Boltzmann's constant | $k$ | $= 1.38\times10^{-34}$ | $JK^{-1}$ |

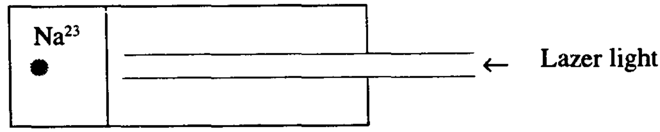



Fig. 22.4

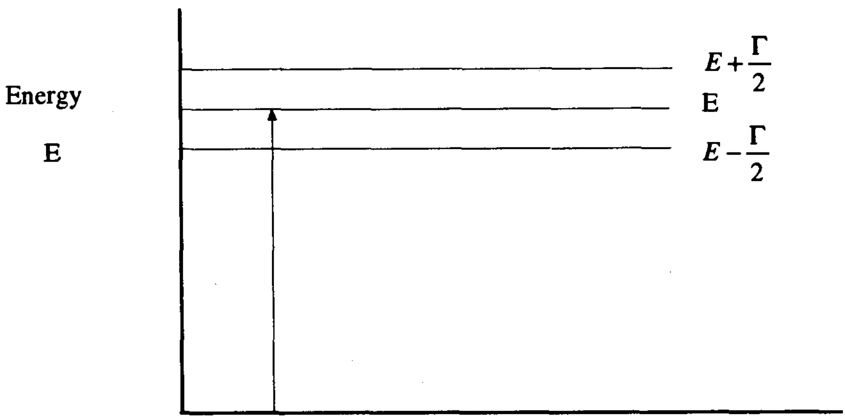



Fig. 22.5

## Questions

3.1 For each atom with the average value of kinetic energy in the vacuum container leaving the aperture, what is the frequency of photons absorbed by the Na atom? .(See Fig. 22.5). Determine momentum change $\Delta v_1$ after the first absorption of photon.

3.2 Determine range of velocity $\Delta v_0$ of the atom which enables the atom to absorb laser light energy calculated under 3.1.

3.3 For each emission of photon, the atom is deflected from its original direction of motion. Determine the largest angle of deflection $\phi_{MAX.}$

3.4 Calculate the number of the process of absorption and then emission of photons N which results in the velocity $v_0$ in the direction described under 3.1 reduced to almost 0.

3.5 Determine time duration $\Delta t$ required for the reduction of the initial velocity to zero value and also distace $\Delta s$ covered by the atom during that interval.

# Solution

## 3.1 Calculation of the velocity of Na atom.

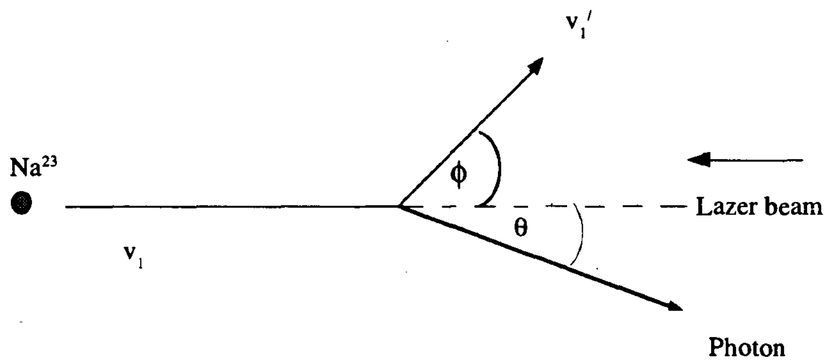



Fig 22.7

From Kinetic Theory

$$\frac{1}{2}mv_0^2 = \frac{1}{2}kT$$

$$v_0 = \sqrt{\frac{kT}{m}}$$

Substitute $\quad m = 23 \times 1.67 \times 10^{-27}$ kg

$k = 1.38 \times 10^{-23}$ $JK^{-1}$

$T = 10^3$ K

$$v_0 = \sqrt{\frac{3.1 \times 1.38 \times 10^{-23} \times 10^3}{23 \times 10^{-27}}} = 1.04 \times 10^3 \quad \text{m/s}$$

The calculation of $\nu$ and $\Delta v_1$ can be done in two different ways :

**Method 1** Apply the equations of conservation of energy and momentum, solve the equations to obtain $\nu$.

**Method 2** Apply the principle of Doppler's effect.

Here only the application of classical Doppler's effect will be considered.

When Na atom travels with velocity $v_0$ towards the photon source (i.e. laser source), the observer moving with the atom (i.e. in the frame in which the sodium atom is at rest) observes the frequency of the laser source as $\nu'$ given by

$$v' = v\left(1+\frac{v_0}{c}\right)$$

and energy of the photon frequency $v'$ given by

$$E = hv\left(1+\frac{v_0}{c}\right)$$

Substitute $hv' = E = 3.36\times10^{-19}$ J

$v_0 = 10^3$ m/s

$c = 3\times10^8$ m/s

$h = 6.62\times10^{-34}$ J.s

$$v = \frac{3.36\times10^{-19}}{6.62\times10^{-34}\left(1+\frac{10^3}{3\times10^8}\right)}$$

$$= 5.0\times10^{14} \quad \text{Hz}$$

Change in the momentum of Na atom is equal to the momentum ofphoton as observed by the observer at rest ( i.e. not moving along with Na atom).

$$m\Delta v_1 = \frac{hv}{c}$$

or

$$\Delta v_1 = \frac{E}{mc\left(1+\frac{v_0}{c}\right)}$$

Substitute $m = 23m_p = 23\times1.67\times10^{-27}$ kg

$E = 3.36\times10^{-19}$ J

to obtain $\Delta v_1 = 3.0\times10^{-2}$ m/s as change in the value of the velocity of Na from its original value. **Ans**

3.2 Calcualtion of $\Delta v_0$ associated with energy change of $\Delta E = \Gamma = 7.0\times10^{-27}$ J

From (1)

$$\left(1+\frac{v_0}{c}\right) = \frac{E}{hv}$$

$$v_0 = c\left(\frac{E}{hv}-1\right)$$

$$\Delta v_0 = \frac{c\Delta E}{hv}$$

where $v = 5.0\times10^{14}$ Hz

$\Delta E = \Gamma = 7.0\times10^{-27}$ J

$$\Delta v_0 = \frac{3\times10^8\times7.0\times10^{-27}}{6.67\times10^{-34}\times6.0\times10^{14}}$$

$$= 6.25 \quad \text{m/s}$$ **Ans**

**3.3 Determination of the angle of deflection of Na atom after emitting a photon while in flight.**

From Fig. 22.7, $\phi$ is maximum when the component of the momentum in the direction perpendicular to the line of flight is maximum. This is the situation in which a photon is released in the direction perpendicular to the line of motion.

In such a case, the component of the momentum of the Na after emitting a photon will be $\frac{h\nu}{c}$ in the direction perpendicular to the line of original motion.

The final momentum of the Na in the direction of original motion is $mv_0 - \Delta v_1$.

$$\tan\phi_{MAX} = \frac{h\nu}{c(mv_0 - \Delta v_1)}$$

$$\approx \frac{h\nu}{cmv_0}$$

It will be noted that Doppler's effect does not come into play ,as the photon is observed in the direction perpenducular to the direction of motion of the atom.

Substitute h, $\nu$, c, m and $v_0$ to obtain,

$$\phi_{MAX} = 2.8 \times 10^{-5} \text{ rad}$$ **Ans**

3.4 From 3.2 the range of speed of the Na atom from $v_0$ to thevalue which permits absorption of photons at frequency $\nu$ is $= \frac{6.52}{2} = 3.12$ m/s **Ans**

3.5 For each absorption and emission of photon, the speed of the atom is reduced by $\Delta v_1$.

$$\Delta v_1 = 3 \times 10^{-2} \text{ m/s}$$

The number of the encountering with photons followed by its emission (N times) and resulting in velocity of Na being reduced to almost 0 is :

$$N = \frac{v_0}{\Delta v_1}$$

$$= \frac{1.04 \times 10^3}{3 \times 10^{-2}}$$

$$= 35{,}000$$ **Ans**

3.6 From Heisenberg's principle of uncertainty

$$\Delta E \Delta t \sim h$$

$$\Delta t \sim \frac{h}{\Gamma}$$

Time interval for bringing Na atom to rest,

$$t \sim N\frac{h}{\Gamma}$$

Substitute $\Gamma = 7.0 \times 10^{-27}$ J

$$t \sim 3.5 \times 10^4 \times \frac{6.62 \times 10^{-34}}{7.0 \times 10^{-27}} \text{ s}$$

$$\sim 3.37 \times 10^{-3} \text{ s}$$

Na atom spends about $3.37 \times 10^{-3}$ s before its speed is reduced to almost zero. **Ans**

Assume that Na atom slows down uniformly, the distance covered by the atom before coming to rest is

$$= \frac{(0 + v_0)t}{2}$$

$$= \frac{1.04 \times 10^3 \times 3.37 \times 10^{-3}}{2}$$

$$= 1.75 \text{ m}$$ **Ans**

# Experiment

## Problem 1

Given a "black box" which may not be open, and on the covering lid there are three terminals marked by letters A B and C respectively. The contents in the box consist of three pieces of electronic equipment of different kinds such as battery,resistor (R > 100 W ), capacitor(C > 1 μF) or diode. Experiment will be conducted in order to answer the following questions:

1.1 What kinds of elctronic equipment are in the box? How are they connected to terminals A B and C?

Give your answer in the form of a diagram and make sure thatthey are correctly connected to terminals marked A B and C, complete with explanation . Also draw diagrams of other circuits which have similar properties to the actual circuit but are rejected. Give the reasons for rejection.

1.2 If there is a battery in the box, find the value of its e.m.f., and draw the circuit used.

1.3 If there is a resisstor in box, find the value of the resistance and draw the circuit used.

1.4 If there is a capacitor in the box, find the value of its capacity and draw the circuit used.

1.5 If there is a diode in the box, find $V_0$ $V_r$

where $V_0$ minimum potential when the diode begins to conduct current in a forward-bias connection.
$V_r$ potential which causes strong current in a reversed-bias connection.

1.6 . Estimate ± error of each measured value.

The following items of equipment and accesories may be used:

| | | |
|---|---|---|
| Black box with A B and C terminals | 1 | unit |
| Variable DC voltage source | 1 | set |
| 1 W multimeter | 2 | units |
| connecting wire | 10 | units |
| circit connecting boards | 2 | units |
| resistor 100kΩ 5% | 1 | unit |
| resistor 1 kΩ 5% | 1 | unit |
| capacitor 100 μF 20% | 1 | unit |
| timer | 1 | unit |

| | | |
|---|---|---|
| paper | 2 | sheets |
| rectangular ruler | 1 | unit |
| switch | 1 | unit |

| Internal Resistance of Voltmeter | | |
|---|---|---|
| Scale | Resisrtance(kΩ) | Accuracy(%) |
| 0-1 V | 3.2 | 1 |
| 0-3 V | 10 | 1 |
| 0-10 V | 32 | 1 |
| 0-20 V | 64 | 1 |
| 0-60 V | 200 | 1 |

| Internal Resistance of Ammeter | | |
|---|---|---|
| Scale | Internal resistance(Ω) | Accuracy(%) |
| 0-0.3 mA | 1,000 | 1 |
| 0-1 mA | 263 | 1 |
| 0-3 mA | 94 | 1 |
| 0-10 mA | 30.4 | 1 |
| 0-30 mA | 9.84 | 1 |
| 0-100 mA | 3.09 | 1 |
| 0-300 mA | 0.99 | 1 |
| 0-1 A | 0.31 | 1 |

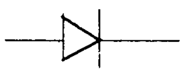
## Warnings

Using multimeter to measure resistance is strictly forbidden..
Use resistor to prevent large current in circuit. In any case the current used should be no more than 20 mA.

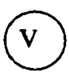
Present experimental results in the form of a table and/or a graph
When drawing a circuit diagram use the following symbols


for DC voltage source( battery )

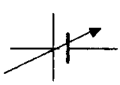

capacitor

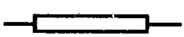

semiconductor diode

ammeter



voltmeter


adjustable DC voltage source


resistor

# Solution

**Step 1** Test for a batteryby meaasuring voltage across AB BC and CA,no voltaage is detected. It is concluded lthat there is no voltage source in the box.

**Step 2** Connect a test circuit and calculate resistance,using 1 kΩ .as depicted in the figure below.

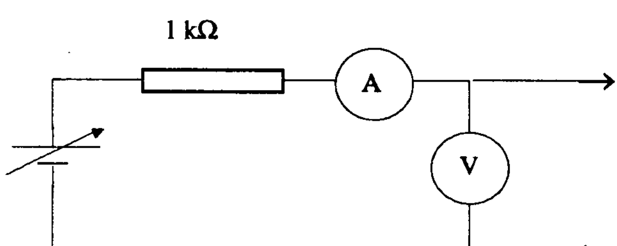



Fig .22.8

**Step. 3** Connect the probes across terminals AB BC and CA to take readings of voltage and current before switching polarity.

3.1 No current is recorded inspite of the detection of voltage across the terminals. It is inferred that their is a capacitor in the circuit.

3.2 It is discovered that for one set of terminals reversing polarity gives large current by a larage factor,indicating the presence of semiconductor diode in series with a resistor in the circuit.

3.3 Apply the procedure used in 3.1 to test the presence of a capacitor across one set of the terminals.

From the results of Step 3 , a most likely connection in the box is shown in Fig. 22.9

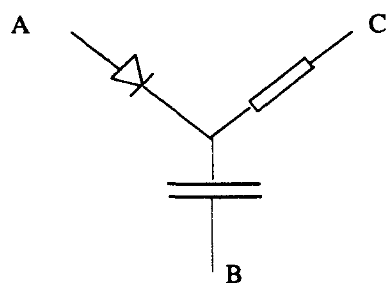



Fig. 22.9

**Step 4** To make sure that C is really connected to a resistor, a test on charging and discharging characteristic of RC circuit is carried out in the following manner:

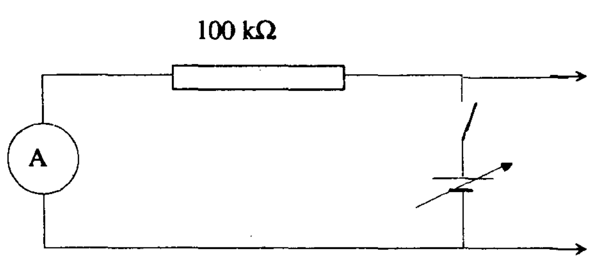



Fig. 22.10

4.1 Connect a test circuit as shown in Fig. 22.10

4.2 Connect the probes across AB, turn on the switch. After a time interval of some few seconds, the switch is turned off. Observe the pointer. Reverse the polarity of the probe and carry out a similar test. If the pointer of the ammeter does not kick gradually dies down, it is concluded that ther are a semiconductor diode and a resistor in series. If the pointer kicks and gradually dies off, it is to be concluded that there are a capacitor and a resistor in series across the terminals. In the diagram above terminals AB consist of a diode and a resistor.

4.3 To make sure that terminals BC consist of a capacitor and a resistor in series, the same test is conducted across BC terminals. The pointer kicks and then slowly returns to the zero value, a confirmation of conclusion of 4. 2 and a positive identificartion of capacitor and resistor.

## Step 5 **Determination of resistance of the resistor**

Use the test circuit shown in Fig. 22.8 and conduct the following tests.
Connect the probes across BC in a forwar-biased connection.
Record voltage and current readings (V, I).
Adjust the value of the current reading due to the presence of internal resistance in the ammeter to obtain the adjusted value (I*).
Connect the probes across BC in reverse-bias connection.
Adjust V until curerent reading changes from 0 negative values.
Record current and voltage readings.
Adjust the value of the current readings due to internal resistance of the ammeter to obtain I*.
Plot a graph of I*. as a function of V.
Calculte R from the slope of the curve in linear part of the forward bias connection.
Read $V_0$ and $V_R$ from the graph.

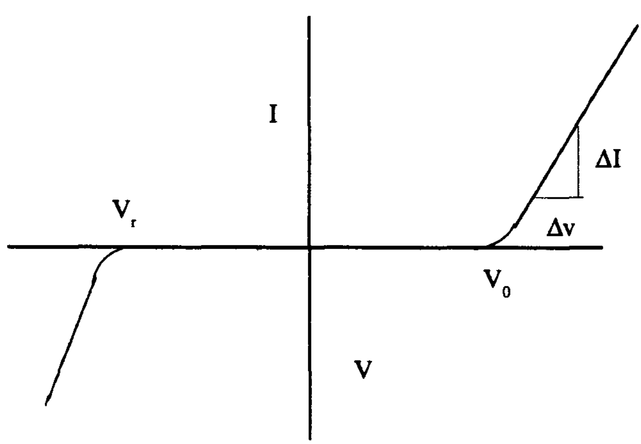



Fig. 22.11

# International Physics Olympiad XXIII

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## Theory

### Problem 1. Rotating Satellite

A satellite is in an orbit around the earth in the equatorialplane of the earth. The satellite consists of 4 daughter sattelites each of which is represented by B with masss m, circling around the fifth daughter satellite P which can betreated as mssless. Each of satellites B is attached to P by means of a light rigid cable of length r. In order to prevent the B satellites from becoming unstable, each B satellite is linked to its neighbour on either side by light cable so that the angles between the radial cables are always at 90°. (See Fig.23.1)

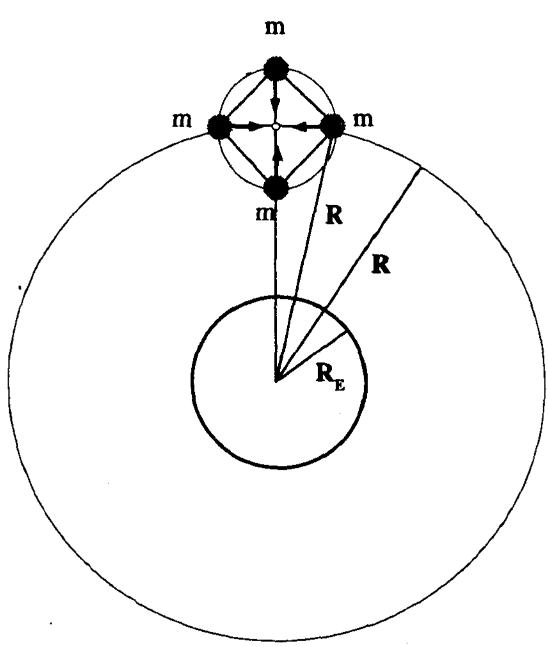


Fig. 23.1

Being secured by the cables described, The system of B satellites circle around P with the same angular speed $\omega$.The whole setup behaves as a rigid body.

The problem is to be analyzed to arrive at answers in the general form taking into consideration of every possible situation. For questions 1.1 and 1.2, the answers must be given in the quantitative form.

Necessary data appear in the annexed part to this problem.

## Questions

1.1 Determine tension in the radial cable pulling mass B, when the position vector **R** of P (measured from the centre of the earth)

a. points in the same direction with vector position r (measured from the P) of mass B.

b. points in the opposite direction with vector position r.

c. points in the direction perpendicular to vector position r.

1.2 In each daughter satellite B, there is an identical engine powered by solar energy, pulling satellite B towards P when the tension in the cable calculated in 1.1 is maximum and releasing the same away from P when the tesion in the cable calculated in 1.1 is minimum. In this way,the length of the cable is decreased or extended by 1 % of the average length of the cable, and over long and continous period of time, the average length of the cable is unchanged.

Determine the average power of each engine averaged over the period of one complete rotation of B around P.

Power in here is understood to be work performed by the engine when pulling in the cable subtracted by work performed by the cable when releasing the cable from the engine divided by the period of one complete rotation of the daughter satellites.

1.3 Discuss changes of the motion of the satellite system due to the working of the engine in the manner described above, and analyze all possible changes based on the conditions given in the table below.

Give the results of your analysis by filling in the blanks inthe table ( See instruction below)

## Table for Answers

Fiil in the blanks by wrting inequalities or equations asnecessary in a concise form. Use separate sheets if necessary.

| Quantities underlined | increase if | decrease if | unchange if | unchange in each and all situations |
|---|---|---|---|---|
| Velocity v in the orbit of the satellite | | | | |
| Radius R of the orbit of satellite | | | | |
| Angular velocity ω of the rotation around P. | | | | |
| Pottential Energy | | | | |

Can this satellite system move into higher orbits by the working of the engine?

Yes ( ) No ( )

Can the satellite system by the working of the engine described, move into any higher orbit we want or even escape from the gravitational field of the earth?

Yes ( ) No ( )

Give your aers in figures if the situations are as follows:

a. The radius of the orbit at the centre of the satellite system $R = R_E + 500$ km.
b. Average length of the cable $r = 100$ km and the diameter of the circular orbit of satellites B = 200 km.
c. mass m of each daughter satellite B = 1000 kg
d. At the start, all daughter satellites B( each with mass m ) move in a circular orbit about P with an angular speed of 10 revolutions per hour.
e. Mass of each stretch of the cable the mass and of P are negligible.

## Hints

Consider motion of satellites **B** in clockwise as well as anticlockwise directions for angular speed ω.
The question does not expect nor require anaysis in in detail, hence any answer with the accuracy of 5% or less is acceptable. In the calculation of this problem, the gravitational

effects of the sun and the moon are to be ignored.
Useful constants for the calculation in this problem

| | | | |
|---|---|---|---|
| Mass of the earth | $M_E$ | $= 5.97 \times 10^{24}$ | kg |
| Gravitational constant | G | $= 6.673 \times 10^{-11}$ | $m^3\ kg^{-1}\ s^{-2}$ |
| Radius of the earth at the equator | $R_E$ | $= 6378$ | km |

$$K = GM_E = 3.983 \times 10^{14} \quad m^3\ s^{-2}$$

# Solution

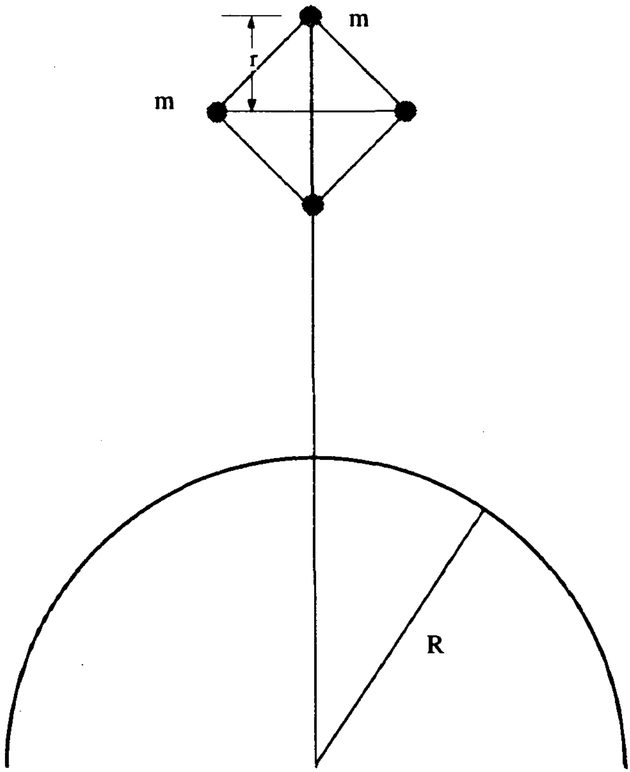


Fig.23.2

Let $\Omega$ be angular speed of satellite P around the earth.
$M_E$ mass of the earth
R radius of the orbit of satellite P around the earth
m mass of each daughter satellite B

Thus $$\frac{GM_E(4m)}{R^2} = 4m\,\Omega^2 R \qquad (1)$$

$$\Omega = \sqrt{\frac{GM_E}{R^3}}$$

$$= \sqrt{\frac{K}{R^3}}$$

Substitutition gives $$\Omega = \left[\frac{3.983 \times 10^{14}}{(6378+50) \times 10^3}\right]^{\frac{1}{2}}$$

$$= 1.244 \times 10^{-3} \quad \text{rad/s}$$

Period of satellite P is given by

$$T = \frac{2\pi}{\Omega} = \frac{2 \times 3.14}{1.244 \times 10^{-3}}$$

$$= 5150 \quad \text{s}$$

From information provided by the problem, satellite B rotates about P with the angular speed of 10 revolutions per hour, thus

$$\omega = \frac{2\pi \times 10}{60 \times 60} \quad \text{rad/s}$$

$$= 1.05 \times 10^{-2} \quad \text{rad/s}$$

1.1 To facilitate the determination of tension in the cable, we adopt the noninertial frame in which the observer is at rest relative to P

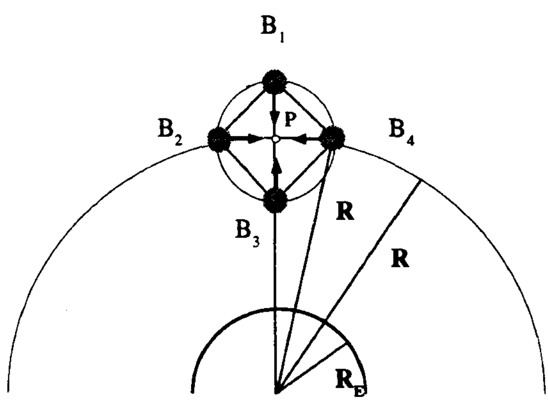


Fig.23.3

a. In this noninertial system, forces acting on $B_1$ are

Tension in the cable pulling towards P

Centrifugal force directed upward.

Gravitational pull pointing downward.

The resultant force causes $B_1$ to move in the circle with the angular speed ω about P.

$$mr\omega^2 = T + \frac{mM_E G}{(R+r)^2} - m(R+r)\Omega^2$$

$$= T + \frac{mM_E G}{R^2\left(1+\frac{r}{R}\right)^2} - m(R+r)\Omega^2$$

From (1) $\frac{GM_E}{R^2} = \Omega^2 R$

$$mr\omega^2 = T + mR\Omega^2\left(1-\frac{2r}{R}\right) - m(R+r) + \Omega^2$$

Maximum tension $\quad T = mr\omega^2 - m\Omega^2\left[R\left(1-\frac{2r}{R}\right) - R - r\right]$

$$T = mr\omega^2 + 3mr\Omega^2 \quad \textbf{Ans}$$

b. One may calculate tension in the cable similar to the procedure in a. as follows:

Forces acting on $B_2$ are

Tension T in the cable directed upward toward P.

Centrifugal force directed toward centre of satellite P.

Gravitational pull pointing downwardor away from P.

The resultant force causes $B_2$ to move in the orbit around P

Equation of motion is given by:

$$mr\omega^2 = T - \frac{mM_E G}{(R-r)^2} + m(R-r)\Omega^2$$

$$= T - \frac{mM_E G}{R^2\left(1-\frac{r}{R}\right)^2} + m(R-r)\Omega^2$$

$$= T - mR\Omega^2\left(1+\frac{2r}{R}\right) + m(R-r)\Omega^2$$

Maximum tension is $T = mr\omega^2 + 3mr\Omega^2$

This answer can be arrived at without formal calculation if weconsider that tensions in the same cable are equal in magnitude but opposite in sense. **Ans**

c. Tensions in the cables for $B_2$ and $B_4$ are the same in magnitude based on the argument given above.

Note that the cetrifugal force of the Non-inertial system does not come into play.

Tension in the cable $T = mr\omega^2$ **Ans**

d. Tension in the cable is the same as that in c. $T = mr\omega^2$ **Ans**

1.2 Calculation of power of the machine

$$P = \frac{\Delta E}{\Delta t}$$

$\Delta E$ is work performed or energy delivered by the engine to the satellite system.

$\Delta t$ the time duration for which work is performed .

$\Delta E$ can be derived from $\Delta E = \Delta F . \Delta R$

$\Delta F$ is the difference in magnitude of the forces pulling and releasing satellites B

$$= mr\left(\omega^2 + 3\Omega^2\right) - mr\omega^2$$

$$= 3mr\Omega^2$$

Distance through which the cable is pulled inward $= \frac{r}{100}$ m

Also distance by which the cable is extended $= \frac{r}{100}$ m

$$\Delta r = \frac{r}{50}$$

$$P = 3mr\Omega^2 \frac{r\omega}{50 \times 2\pi}$$

Substituting

$m = 1 \times 10^3$ kg
$r = 1 \times 10^5$ m
$\Omega = 1.23 \times 10^{-4}$ rad/s
$\omega = 1.0 \times 10^{-2}$ rad/s

$$P = 3 \times 10^3 \times 10^5 \times (1.23 \times 10^{-4})^2 \times \frac{10^5 \times 10^{-2}}{60 \times 2 \times 3.14}$$

$$= 14 \quad W$$ **Ans**

1.3 The net angular momentum (L) of the satellite system consists of two parts: i.e. angular momentum of satellites B about P, and the angular momentum of lthe centre of mass P about the earth.

The case of $\omega$ and $\Omega$ represent rotation in the same direction

$$L = 4m(\omega r^2 + \Omega R^2)$$

L is constant by virtue of the principle of conservation of angular momentum
r is also given as constant.

Substituting $\Omega = \sqrt{\frac{GM_E}{R^3}}$ to obtain

$$L = 4m\left(\omega r^2 + \sqrt{GM_E R}\right)$$

$$\Delta L = 4m\left(\Delta\omega . r^2 + \frac{1}{2}\sqrt{\frac{GM_E}{R}} . \Delta R\right) = 0$$

$$\Delta\omega r^2 = -\frac{1}{2}\sqrt{\frac{GM_E}{R}}\Delta R \qquad (1)$$

Total energy of the satellite system E also consists of 2 parts: i.e. rotational kinetic energy of satellites B about P,and rotational energy of the centre of mass at P about the earth.

$$E = \frac{4m}{2}\left(\omega^2 r^2 + \Omega^2 R^2\right) - \frac{4mGM_E}{R}$$

Substituting $\frac{GM_E}{R} = \Omega^2 R^2$

$$E = 2m\omega^2 r^2 + \frac{2mGM_E}{R} - \frac{4mGM_E}{R}$$

$$= 2m\omega^2 r^2 - \frac{2mGM_E}{R}$$

$$\Delta E = 2m\left[2\omega\Delta\omega r^2 + \frac{GM_E}{R^2}\Delta R\right] \qquad (2$$

Substitute $\Delta\omega$ from (1) in (2) to obtain

$$\Delta E = 2m\left[-\omega\sqrt{\frac{GM_E}{R}} + \frac{GM_E}{R^2}\right]\Delta R$$

$$= \frac{2mGM_E}{R^2}\left[1 - \omega R\sqrt{\frac{R}{GM_E}}\right]\Delta R$$

Substitute $\frac{GM_E}{R} = \Omega^2 R^2$ inside the bracket to obtain

$$\Delta E = \frac{2mGM_E}{R}\left[1 - \frac{\omega}{\Omega}\right]\Delta R \qquad (3)$$

## Analysis of possible types of motion

## When both ω and Ω are in the same sense (Note Ω is always from west to east)

If $\omega > \Omega$

In (3) $1-\frac{\omega}{\Omega}$ is negative i.e. $\Delta E = (-)\,\Delta R$

If E increases or ΔE is positive, ΔR is negative ie. the satellite altitude decreases.

The repercussions of decreaing R are:
Ω increases, due to the relationship

$$\Omega^2 = \frac{GM_E}{R^3} \quad \text{or} \quad \Delta\Omega = -\frac{3GM_E}{2R^4\Omega}\Delta R$$

$$\Delta\Omega = -\frac{3}{2R^2}\sqrt{\frac{GM_E}{R}}\Delta R \qquad (4)$$

and ω also increases.
This is concluded from (1)

or
$$\Delta\omega = -\frac{1}{2r^2}\sqrt{\frac{GM_E}{R}}\Delta R \qquad (5)$$

Equations (4) and (5) tell us that ω increases more rapidly than the increase in Ω.

**Conclusion** The orbit of the satellite system will become lower and lower while the angular speed of satellites B about P and the angular speed of the satellite system about lthe earth keep increasing.

If $\omega < \Omega$

In equation (3) $1-\frac{\omega}{\Omega}$ becomes positive i.e.

$$\Delta E = (+)\Delta R$$

In this case when E increases or ΔE is positiveΔR is positive,the satellite altitude increases.

The repercussions of increasing R are:
Ω decreases, conclusion follows from equation (4)
and ω also decreases, conclusion follows from equation (5).

Since ω decreases much more rapidly than Ω, ω reaches zero value before Ω. Under this situation the engine cannot perform as intended, and no change takes place.

If $\omega$ is oppossite to $\Omega$ in sense

In this case $(1-\frac{\omega}{\Omega})$ is always positive i.e.

$\Delta$ = (+)$\Delta$R instead of (-)$\Delta$R, as $\omega$ is negative

When E increase i.e. $\Delta$E is positive and R increases. Theconclusion follows from equation (2)

$$\Delta E = (+)\Delta R$$

Repercussions of increaing R are

$\Omega$ decreases. Conclusion follows from equation (4) $\Delta\Omega = (-)\Delta R$
and $\omega$ becomes more and more positive until $\omega = 0$. The performance of the engine produces no change.

If $\omega = \Omega$

The performance of the engine produces no change. The conclusion is base on equation(3)

$$\Delta E = \frac{2mGM_E}{R}\left[1-\frac{\omega}{\Omega}\right]\Delta R$$

The answersare summarized into the following table .

| | Increase | Decrease | No change | No change in all situations |
|---|---|---|---|---|
| Speed of satellite in the orbit around the earth v | If $\omega > \Omega$ and has the same sense with $\Omega$, since R decreases and v = $\Omega R$ = $GM_E/R$ | If $\omega < \Omega$ and has the same sense with $\Omega$. since R increases and v = GM E/R | If $\omega = 0$ | If $\omega = \Omega$ |
| Radius R of the orbit | 1.If $\omega < \Omega$ and has the same sensewith $\Omega$ 2. If $\omega$ is opposite to $\Omega$ in sense. | If $\omega > \Omega$ and has the same sense with $\Omega$. | If $\omega = 0$ | If $\omega = \Omega$ |
| Angular speed $\omega$ of satellites B | 1.If $\omega > \Omega$ andhas the same sense with $\Omega$. 2.If $\omega$ is opposite to $\Omega$ in sense | If $\omega < \Omega$ and has the same sense with $\Omega$. | If $\omega = 0$ | If $\omega = \Omega$ |
| Potential energy | If R increases | If R decreases | If $\omega = \Omega$. and has the same sense with $\Omega$. | If $\omega = \Omega$ |

The satellite system can move to higher orbits by the working of the engine.

However, the satellite system cannot be moved to into higher orbits without limit.
The satellite can be raised to higher orbits by the functioning of the engine when
1. $\omega$ is in thesame direction with $\Omega$ and $\omega < \Omega$. In such case, the satellite system gains altitude until $\omega$ is reduced to 0 value.
2. $\omega$ is in the opposite direction with $\Omega$. In such case, the satellite system gains altitude and the angular speed of daughter satellites turns more and more positive until $\omega = 0$ and the satellite cannot go any higher.

From this on there is no change in the satellite system by the performance of the engine. **Ans**

Given: $\omega = 1.0 \times 10^{-2}$ rad/s and in the same sense with $\Omega$.
$\Omega = 1.23 \times 10^{-4}$ rad/s

This is the case of $\omega > \Omega$, and the performance of the engine is accompanied by lowering of the orbit of the satellite. **Ans**
If $\omega$ is in the opposite sense with $\Omega$

and $\Omega = 1.23 \times 10^{-4}$ rad/s

Equation (1) becomes $\Delta\omega\, r^2 = \frac{1}{2}\Omega R \Delta R$

Substituting $\Delta\omega \sim 1 \times 10^{-2}$

$\sim 1 \times 10^{-2}\ \Omega$ rad/s

$\Omega = 1.23 \times 10^{-4}$ rad/s

$r = 100$ km

$R = 6.37 \times 10^{3}$ km

gives $$\Delta R = \frac{2 \times 1.0 \times 10^{-2} \times 10^{-4}}{1.23 \times 10^{-4} \times 6.37 \times 10^{3}}$$
$= 226$ km

The satellite system can be raised to a higher orbit with the maximum increase in the altitude of 226 km **Ans**

## Problem 2 Longitudinal Motion of Linear Molecular Chain

**Introduction.** This problem is concerned with the analysis of the linear motion in the molecule in the form of a straight chain and does not involve rotational motion nor motion of twisted or distorted chain.

Consider a molecule consisting of N atoms having mass $m_1\ m_2\ m_3\ m_4 \ldots m_N$ respectively. Each atom is linked to its neighbouring atom by means of a chemicl bond. Each of these bond may be treated as a massless spring which obeys Hookeûlaw, with the values of spring constant of $k_1\ k_2\ k_3\ k_4 \ldots\ldots k_N$ respectively as illustrated in Fig. 23.3.

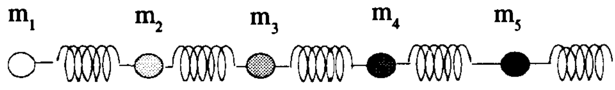


Fig 23.3

a. Find force $F_i$ acting on the ith atom as a function of $x_1$, $x_2$, $x_3$, $x_4$..... $x_N$ as the case may be

b. Determine relationship between $F_1$, $F_2$, $F_3$, $F_4$...... $F_N$

c. Apply the above relationship to determine the relationship between the magnitudes of the displacements $x_1$, $x_2$, $x_3$, $x_4$..... $x_N$ and explain its meanings based on appropriate concepts.

2.2 Analyze the motion of two atoms A,B (Fig 23.4) in a single molecule

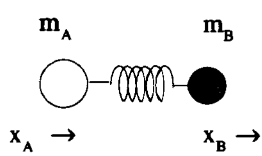


Fig. 23.4

k is the spring constant of the spring representing the chemical bond

a. Describe types of possible motion of the molecule.

b. Determine the frequencies of various types of motion, and explain in particular why atoms of different masses should vibrate with the same frequency.

2.3 Analyze the motion of a molecule consisting of three atoms and having a chemical formula in the form $A_2B$. (See Fig. 23.5)

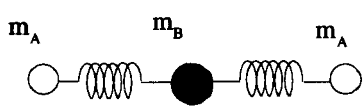


Fig. 23.5

In each case write the equation expressing the relationship between forces on individual atoms and the magnitude of displacement of associated atom. Determine all possible types of motion and associated frequencies

2.4 A molecule of $CO_2$ vibrates in the longitudinal direction of the molecule with frequencies $3.998 \times 10^3$ Hz and $7.024 \times 10^3$ Hz respectively. Use these values of the frequencies to calculate the spring constant of C-O bond. Comment on the validity of the simple spring model of molecule in explaining the motion of the actual molecule.

Given:

Atomic mass of carbon = 12
Atomic mass of oxygen =16
and one unt of atomic mass = $1.660 \times 10^{-27}$ kg

# Solution

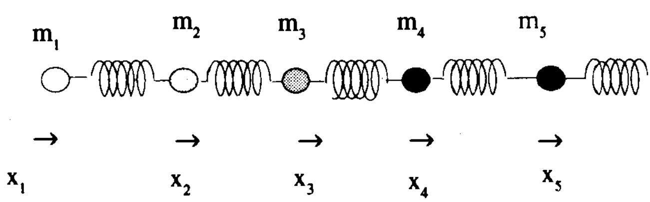


Fig.23.6

2.1 Let $x_1$ be the magnitude of the displacement of $m_1$
$x_2$ be the magnitude of the displacement of $m_2$
$x_3$ be the magnitude of the displacement of $m_3$
.......................................................................
.......................................................................
$x_N$ be the magnitude of the displacement of $m_N$

Equation of motion of $m_1$ is $F_1 = -\ k_1(x_1 - x_2)$ (1)
and that of $m_2$ $F_2 = -k_2(x_2 - x_3) - k_1(x_2 - x_1)$ (2)
and that of $m_3$ $F_3 = -k_3(x_3 - x_4) - k_2(x_3 - x_2)$ (3)
.............................................
.............................................
and that of $m_i$ $F_i = -k_i(x_i - x_{i+1}) - k_{i-1}(x_i - x_{i-1})$ (ith
and that of $m_N$ $F_N = -k_{N-1}(x_N - x_{N-1})$ (N)

(1)+(2)+(3)+.....(ith)...+(N) gives

$$\sum_{i=1}^{N} \mathbf{F}_i = \sum_{i=1}^{N} F_i = 0$$

$$\sum_{i=1}^{N} m\ddot{x}_i = 0$$

$$\sum_{i=1}^{N} m\dot{x}_i = constant$$

The aboe equation describes the motion of the whole molecule along the longitudinal direction wth constant velocity. Our problem involves the molecule at rest or velocity being zero.

$$\sum_{i=1}^{N} mx_i = 0$$

$m_1x_1 + m_2x_2 + m_3x_3 + ........M_Nx_N = 0$ **Ans**

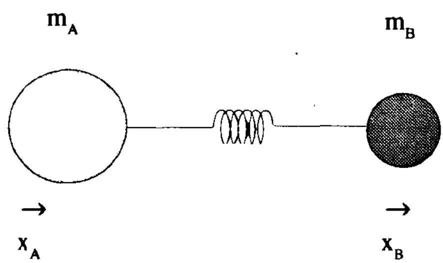


Fig. 23.7

2.2 Let $x_A$ and $x_B$ represent the magnitude of the displacement of $m_A$ and $m_B$ from the equilibrium position of A and B respecrtively, then

$$m_A\ddot{x}_A = -k(x_A - x_B) \quad (1)$$

$$m_B\ddot{x}_B = -k(x_B - x_A) \quad (2)$$

From 2.1 $$m_A\ddot{x}_A = -m_B\ddot{x}_B \quad (3)$$

$(1)\times m_B$ $$m_Bm_A\ddot{x}_A = -k(m_Bx_A - m_Bx_B) \quad (4)$$

$$m_Bm_A\ddot{x}_A = -k(m_A + m_B)x_A$$

$$\omega_A = \sqrt{\frac{k(m_A + m_B)}{m_Am_B}}$$

Likewise if we take $(2)\times m_A$ we have,

$$m_Am_B\ddot{x}_A = -k(m_Ax_B - m_Ax_A)$$

From 2.1 $$m_A\ddot{x}_A = -m_B\ddot{x}_B$$

$$m_Am_B\ddot{x}_B = -k(m_Ax_B + m_Bx_B)$$

$$m_Am_B\ddot{x}_B = -k(m_A + m_B)x_B$$

$$\omega_B = \sqrt{\frac{k(m_A + m_B)}{m_Am_B}}$$

The fact that two atoms of different masses vibrate with the same frequency is due to these atoms are not independent from each other but interact with each other through the force of chemical bond and no other force from the outside. The two atoms in this case vibrate with the same frequency in such a manner that the centre of mass of the system always remain at rest. **Ans**

2.3 In the case of 3 atoms in a molecule with the specified symmetry

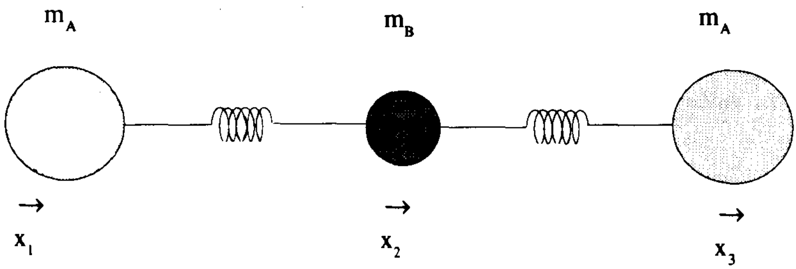


Fig. 23.8

From 2.1 $m_A x_1 + m_B x_2 + m_A x_3 = 0$

**Possible modes of motion**

**Case 1** From the symmetry of the problem, we deduce that one possible mode of motion is $x_2 = 0, \; x_1 = - x_3$

**Case 2** Mass $m_A$ on the left hand side and mass $m_B$ move together in the same direction opposite to that of mass mA on the right hand side i.e.
$m_A x_1 + m_B x_2 = - m_A x_3$

**Case3** Mass $m_A$ on one side( in this case on the left-hand side) remains at rest while mass $m_A$ on the other side moves in opposite direction of that of $m_B$.

In all cases the equation of motions are:

For $m_A$ on the left-hand side $m_A \ddot{x}_1 = -k(x_1 - x_2)$ (1)

and for $m_B$ $m_B \ddot{x}_2 = -k(x_2 - x_1) - k(x_2 - x_3)$ (2)

and $m_A$ on the right-hand side $m_A \ddot{x}_3 = -k(x_3 - x_2)$ (3)

**For Case 1** $x_2 = 0$

$x_1 = - x_3$

Substitute the above in (1) to obtain $m_A \ddot{x}_1 = -k x_1$

$$\omega_A = \sqrt{\frac{k}{m_A}}$$

Substitute the above in (1) to obtain $m_A \ddot{x}_3 = -k x_3$

$$\omega_A = \sqrt{\frac{k}{m_A}}$$

The value of $\omega_A$ obtained from two calculations based on (1) and (3) agree as they should. The conclusion is atom mA on the left-hand side and another atom ma on the right-hand side vibrate toward and away from each other (breathing mode) with the same frequency **Ans**

**For Case 2** $m_A(x_1 + x_3) = -m_B x_2$ (4)

(1) + (3) gives $m_A(\ddot{x}_1 + \ddot{x}_3) = -k(x_1 + x_3) + 2kx_2$ (5)

Substitute the value of $(x_1 + x_3)$ from (4) in (5)

$$-m_B\ddot{x}_2 = +\frac{km_B}{m_A}x_2 + 2kx_2$$

$$m_B\ddot{x}_2 = \frac{k(2m_A + m_B)}{m_A}x_2$$

$$\omega_B = \sqrt{\frac{k(2m_A + m_B)}{m_A m_B}} \quad \textbf{Ans}$$

From the discussion earlier,it follows

$$\omega_B = \sqrt{\frac{k(2m_A + m_B)}{m_A m_B}} \quad \textbf{Ans}$$

**For Case 3**

$$m_A x_1 + m_B x_2 = - m_A x_3$$

Substitute $m_A x_1 = -m_B x_2$ in (1) to obtain

$$-m_B\ddot{x}_2 = \frac{km_B}{m_A}x_2 + kx_2$$

$$\ddot{x}_2 = -k\left(\frac{1}{m_A} + \frac{1}{m_B}\right)x_2$$

$$\omega_A = \sqrt{\frac{k(m_A + m_B)}{m_A m_B}} \quad \textbf{Ans}$$

$$\omega_B = \omega_A = \sqrt{\frac{k(m_A + m_B)}{m_A m_B}} \quad \textbf{Ans}$$

2.4 Two experimentally measured values of the vibration frequency of $CO_2$ molecule are

$$(f_1)_{EXP} = 3.988 \times 10^{13} \text{ Hz}$$

$$(f_2)_{EXP} = 7.042 \times 10^{13} \text{ Hz}$$

$$\frac{(f_2)_{EXP}}{(f_1)_{EXP}} = \frac{7.042 \times 10^{13}}{3.988 \times 10^{13}}$$

$$= 1.9$$

The theoretical value of the frequency expressed in a general form cannot be used to calculate the absolute value of the frequency because the value of k is not known. What one can do is to calculate the ratio between the frequencies predicted by the theoretical consideration, and compare them to the experimental value.

**Theoretical values of f expressed in the nnits involving k**

| ω | $f = \frac{\omega}{2\pi}$ | f in the unit of $\frac{\sqrt{k}}{2\pi}$ |
|---|---|---|
| $\sqrt{\frac{k}{m_A}}$ | $\frac{1}{2\pi}\sqrt{\frac{k}{m_A}}$ | $f_1$=0.25 |
| $\sqrt{\frac{k(2m_A + m_B)}{m_A m_B}}$ | $\frac{1}{2\pi}\sqrt{\frac{k(2m_A + m_B)}{m_A m_B}}$ | $f_2$=0.48 |
| $\sqrt{\frac{k(m_A + m_B)}{m_A m_B}}$ | $\frac{1}{2\pi}\sqrt{\frac{k(m_A + m_B)}{m_A m_B}}$ | $f_3$=0.35 |

$m_A$ = 16 amu  $2m_A + m_B$ = 44 amu  $m_A m_B$ =192 amu
$m_B$ = 12 amu  $m_A + m_B$ = 28 amu

The value of $\frac{f_2}{f_1} = 1.9$ comes closest to the experimental value. We may infer that $CO_2$ molecule is likely to vibrate with the frequenccies $\frac{1}{2\pi}\sqrt{\frac{k}{m_A}}$ and $\frac{1}{2\pi}\sqrt{\frac{k(2m_A + m_B)}{m_A m_B}}$

.

**Calculation of k**

1 Set $f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m_A}} = (f_1)_{EXP}$

Substitution of known variable gives

$$\frac{k}{(2\pi)^2 \times 16 \times 1.660 \times 10^{-27}} = (3.998 \times 10^{13})^2$$

$$k = 1.598 \times 10^{27} \times 39.43 \times 2.66 \times 10^{-26}$$

$$= 1.66 \times 10^3 \quad N.m^{-1}$$

2.Set $\quad \frac{1}{2\pi}\sqrt{\frac{k(2m_A + m_B)}{m_A m_B}} = (f_2)_{EXP}.$

Substitution gives

$$\frac{k \times 44}{(2\pi)^2 \times 192 \times 1.660 \times 10^{-27}} = (7.042 \times 10^{13})^2$$

$$k = \frac{4.959 \times 10^{27} \times 39.43 \times 3.19 \times 10^{-25}}{44}$$

$$= 1.42 \times 10^3 \quad N.m^{-1}$$

Two values of k calculated from the spring model differ by about 16%. The results suggest that the simple spring model ot $CO_2$ molecule is still far from being satisfactory. **Ans**

# Problem 3 Satellite in the Sun

This problem is concerned with the determination of the temperature of the satellite exposed to solar radiation. The satellite under consideration is a spheroid of 1 m radius with all parts at the same temperature. The surface of the satellite is uniformly coated with the same substance. The satellite is in an orbit near the surface of the earth but not in the earth's shadow.

The surface temperature of the sun $T_{SUN}$ is that of a black body.

Given are:

Surface temperature of the sun $T_{SUN} = 6000$ K ,
Distance between the earth and the sun $R = 1.5 \times 10^{11}$ m
The solar radiation delivers heat energy to the satellite until the rate of energy intake from the sun is the same as the rate of heat lot by the satellite.

Assume that the radiation of energy of balck body follows Boltzman and Stefan's law i.e.

$$P = \sigma T^4$$

$\sigma$ is a constant = $5.67 \times 10^{-8}$ $Wm^{-1}K^{-4}$
In the first order aprroximation, both the sun and the satellite absorb energy of all frequencies.

3.1 Determine the expression for temperature T of the satellite and calculate the value of T fromthe expression obtained.

3.2 The spectrum of radiation distribution of a black body u(T,f) at temperature T follows Planck's law i.e.

$$u(T,f)df = \frac{8\pi k^4 T^4}{c^3 h^3} \cdot \frac{\eta^3}{e^{\eta} - 1} d\eta$$

where $u(T, f)df$ is density of electromagnetic radiation in the range between f and f+Δf

$$\eta = \frac{hf}{kT}$$

| | |
|---|---|
| Planck'constant | $h = 6.6 \times 10^{-34}$ J.s |
| Speed of light | $c = 3.0 \times 10^{10}$ ms$^{-1}$ |
| Boltzman's constant | $k = 1.4 \times 10^{23}$ |

Integration of the spectrum over the full frequency range gives lthe value of power per unit area, as provided by StefanBoltzamn's law

$$P = \sigma T^4$$

with $$\sigma = \frac{2\pi^5}{15} \cdot \frac{k^4}{c^2 h^3} \backslash$$

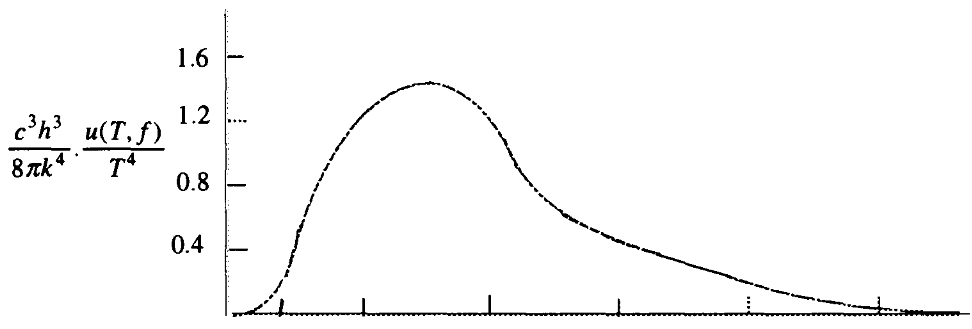


Fig 23.8

In applying this principle to the satellite, we need to lower the temperature of the satellite to lowest possible. Towards this end, space engineers invent coating substance which reflects radiation beyond a certain frequency known as "limited frequency." The satellites with this coating totally absorb radiation of frequencies lower lthan the limted frequency.

If the limited frequency corresponds to the temperature $T = \frac{hf}{k} = 1200$ K, apply the principle describedto determine the temperature of the satellite with coating.
Because of the complexity of the integration involved, one needs not to give an absoluterly correct value, and a good approximation of the answer is acceptable.
Given are the values of some useful integrations:

$$\int_0^\infty \frac{\eta^3}{e^\eta - 1} d\eta = \frac{\pi^4}{15}$$

The maximum value of $\frac{\eta}{e^\eta - 1}$ occurs when $\eta \approx 2.82$

and with low value of $\eta$ one may apply the formula $e^{\eta} \approx 1 + \eta$

3.3 If the satellite such as those used in the space research is equipped with a solar panel to generate electrical current and hence electrical power on board electron ic circuits . Heat given off by the electronic equipment in the satellite constitutes another source of heat energy. If heat is generated by the electronic circuit is at the rate of 1 kW, what is the temperature of the satellite described in 3.2?

3.4 An inventor claims that his "secret formula paint" can reflect more than 90% of the incident radiation both visible and invisiblei.e. infrared and ultraviolet and radiation of all frequencies"(both heat and light) lke a black body, hence can vastly reduce heat in the satellite. In effect the paint can lower the temperature of the satellite to any possible low values one wishes.
Does this paint exist ?

3.5 If one wishes to reduce the temperature to values lower tha that calculated under 2.3, what type of glazing material should one use ? Why ?

# Solution

3.1 Let $R_S$ be the radius of the sun
r the radius of the sprheroidal satellite
$T_S$ surface temperature of the sun
T temperature of the satellite

Within the time interval of 1 s The sun radiates energy into the spaceby the amount

$$4\pi R_S^2 \times \sigma T^4$$

At distance R measured from the centre of the sun, the energy density per unit aarea is

$$I = \frac{4\pi R_S^2 \sigma T^4}{4\pi R^2}$$

$$= \left(\frac{R_S}{R}\right)^2 \sigma T^4$$

The effective area which recieves energy from the sun is the satellite's surface facing the sun. This area is equal to the largest cross-section aarea of the shper-shaped satellite. The energy which the satellite recieves from the sun per unit time is

$$\left(\frac{R_S}{R}\right)^2 \sigma T^4 \times \pi r^2$$

The energy which the satellite radiates per unit time is $4\pi r^2 \sigma T^4$.

At equilibrium

$$\left(\frac{R_S}{R}\right)^2 \sigma T^4 \times \pi r^2 = 4\pi r^2 \sigma T^4$$

Substitute $R_S = 6.96 \times 10^8$ m
$R = 1.5 \times 10^{11}$ m
$T_S = 6000$ K

$$T = 6 \times 10^3 \sqrt{\frac{6.96 \times 10^8}{1.510^{11}}}$$
$$= 289 \quad \text{K} \quad \text{Ans}$$

3.2 Calculate η which corresponds to T = 1200 K by substituting $\frac{hf}{k} = 1200$ K and T = 6000 K in the formula

$$\eta = \frac{hf}{kT}$$
$$= \frac{1200}{6000}$$
$$= 0.2$$

The total amount of energy absorbed by the satellite is given by

$$E_{ab} = \int_0^{\eta = 0.2} \frac{\eta^3}{e^{\eta} - 1} d\eta$$

Ratio between $E_{ab}$ and $E_t$

$$\frac{E_{ab}}{E_t} = \frac{\int_0^{\eta=0.2} \frac{\eta^3}{e^{\eta} - 1} d\eta}{\int_0^{\infty} \frac{\eta^3}{e^{\eta} - 1} d\eta}$$

For small η $\int_0^{\eta=0.2} \frac{\eta^3}{e^{\eta} - 1} d\eta = \int_0^{\eta=0.2} \eta^2 d\eta$

$$= \left[\frac{\eta}{3}\right]_0^{\eta = 0.2}$$
$$= \frac{.008}{3}$$
$$\frac{E_{ab}}{E_t} = \frac{.008 \times 15}{3\pi^4}$$
$$= 4.1 \times 10^{-4}$$

Solar energy absorbed by the satellite $= \pi r^2 \left(\frac{R_S}{R}\right)^2 \sigma T^4 \times 4.1 \times 10^{-4}$

where T is temperature of the satellite at thermal equilibrium.

Hence $$4\pi r^2\sigma T^4 = \pi r^2\left(\frac{R_S}{R}\right)^2 \sigma T^4 \times 4.1\times10^{-4}$$

$$T = T_S\sqrt{\frac{R_S}{2R}} \times 4.1\times10^{-4}$$

$$= 6\times10^3 \times \sqrt{\frac{6.96\times10^{-3}}{2\times1.5}} \times 4.1\times10^{-4}$$

$$= 40 \text{ K}$$

3.3 consider power of 1 kW geerated by the generator in the satellite, heat given off inside the satellite per unit time is $1\times10^3$ J s$^{-1}$.
This amount of energy ( 1 kW or 103 Js$^{-1}$ )is quite large compared to

$\pi r^2\left(\frac{R_S}{R}\right)^2 \sigma T^4 \times 4.1\times10^{-4}$(solar energy ~ 40 Js$^{-1}$ per unit square meter

Hence at equilibrium

$$10^3 = 4\pi r^2\sigma T^4$$

$$T^4 = \frac{10^3}{4\pi\times0.5^2\times5.67\times10^{-8}}$$

$$T = 2.7\times10^2 = 274 \text{ K } \textbf{Ans}$$

3.4 The claim is false, forsuch paint does not existfor the following reasons:

a.The performance of the paint as claimed by its inventor is in contradiction with the second law of thermodynamics concerning heat transfer. The nature of heat is such that it always flows to the sink at lower temperature. If the sink has no means of getting rid of the heat recieved, heat energy accumulates until an equilibrium siuation is reached.

b. The claim amounts to saying that there is a machine by which heat is removed from the satellite, and this machine can perform without energy source. Again this contradicts the second law of thermodynamics.

c. If the paint can radiate like black body, it must also absorb heat as a black body i.e. it must absorb all wavelengths or all frequencies of radiation -instead of selective absorption as claimed.

3.5 A paint which may be used on the satellite to reduce heat from the sun shoud have similar properties to the environment around the satellite i.e. the paint which effectively absorbs high frequencies and at the same time radiates low frequencies with equal efficiency.

# Experiment

## Experiment 1 ( 2 hours) Investigation of breakdown of air molecules in the external electric field

The external electric field that causes breakdown of air molecules in this experiment is due to potential difference generated in piezo-electric material. The apparatus provided consists of an inclined plane and a hammer of mass m which will be allowed to slide along the inclined plane and collide with two piezo-electric cylinders pressed together. (See Fig.23.9) The collision causes charge to be developed at both ends of each piezo-electric cylinder. The potential difference is transmitted acoss the gap which can be adjusted to a maximum distance for an electrical spark to take place. If the gap is narrow, the spark which occurs readily can be easily observed. However if the potential difference is too low, no spark can take place. For a gap of a given width, the minimum potential difference which causes an elecrical spark is known as breakdown potential for air molecules.

Experiment is to be performed to determine breakdown potential as a function of the width of the gap. Estimation of errors and discussion of the validity of the experimental results in explaining the breakdown of air under any other conditions shall form part of the report of the experiment.

## Experimental Procedures

Explain in a concise manner how to overcome problems related to meaurements in this experment.
Record the coded numbers of the piezo-electric cylinders used in the experiment.

### Theory on Piezo-electric block

This experiment does not require an elaborate theory on piezo-electric materials, and the concpts given here should be sufficient. Piezo-electric materials are understood to function like springs and capacitors at the same time. the surface of each of the two ends of the cylinder serves as a plate of a capacitor. As we compress the the spring, we cause charge to flow from one plate to the other plate of the same capacitor. As a result, voltage is generated across the two plates. the amount of charge transferred depends on the the magnitude of the compression. This phenomenon is reversible, i.e if thecompresssion is removed, chrge returns to its initial location.

Example Consier events 1 2 and 3 of the piezo-electric cylinder

1. Compression is applied at both ends of the cylinder.
2. The two ends of the cylinder are shorted by connecting a wire to the two ends.
3. Remove the compression

In event 1 charge Q is tranferrred from one end to the other and voltage V is developed across the two ends

and $V = \frac{Q}{C}$

In event 2 voltage becomes zero i.e. $V = 0$

In event 3 Voltage assumes the value of - V across the piezo electric cylinder.

In this experiment when we compresses a piezo-electric cylinder which is free of charge and is not yet subjected to any compression, kinetic energy of the hammer is converted into electrical energy as the capacitor is charged.

Given:

$C_P$ capacity of the capacitor

Electrical energy = K $\times$ kinetic energy

The value of K depends on type of peizo-electric materialunder consideration The manufacturer of the piezo-electriccylinders used in the experiment gives the valuae of $K = 0.5$

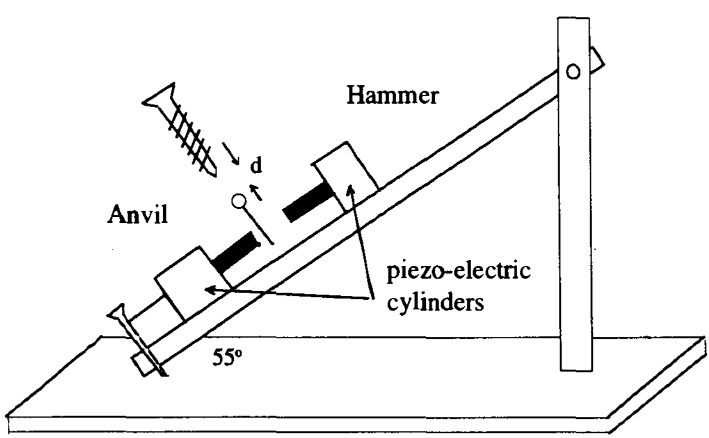


Fig.23.9

# Experiment

This experiment is designed to create compression which generates charge at the facing ends ( marked with+ sign) of two piezo-electric cylinders (See Fig 23.9) These two ends marked with + signs are electrically connected and at the same time are in electrical contact with a terminal in the form of a thin wire located between the two ends. This wire functions as one of the two terminals across which the sparks take place. The hammer is set to make an impact with the upper end of the upper cylinder. When this happens the upper cylinder is electrically connected to the metal rail on which the apparatus set is mounted.

The lower piezo-electric cylinder is fixed to the metal anvil which rests on the metal rail causing the whole ensemble to be electrically connected to the rail. . The foam pad at the end of the anvil dampens the impact between the anvil and the base supporting the rail and the ensemble. A copper wire connect the rail to a screw which may be turned to give the maximum distance through which the spark can take place.

A small safety bar on the rail prevents the hammer from sliding over a distance of about 13 cm. Shoud the contestant fail to produce a spark, contact a superviser.

## Observation of electrical sparks

**Method 1** Connect an electrical wire between the metal rail and the screw and observ visually the spark which lands on the screw.

**Method 2** Instead of using a connecting wire, put the index finger of the rail and the thumbe of the same hand on the screw. A weak electrical shock will be felt when there is a spark generated across the gap. The contestant may use any method or both.

## Necessary Data

Gravitational acceleration $g = 9.82 \ ms^{-2}$

Capacity of piezo electric cylinders $C_p = 20 \ pF \pm 2 \ pF$

Mass of hammer $m = 34.6 \ g$

Mass of hammer and piezo-electric cylinders $M = 87.5 \pm 0.5 \ g$

Hammer and piezo-electric cylinders are to be treated as figid bodies

The pitch of the screw which is used to vary the distance of the spark is 0.80 mm

Inclination angle between the rail and the horizon is $55^{\circ} \pm 1^{\circ}$

## Cautions

1. The capacitor has high resistance which effectively prevents charge leakage, the piezo-electric cylinder as a result can retain charge over a long period of time. In performing this experiment, take this fact into account.

2. Charge generated is extremely small and cannot cause any injury to the contestant. One may detect the spark or the flow of electricity in this experiment without any danger.

3. There is a possibiltity that a cylinder amy shatter due to repeated impacts. If this happens, contact a laboratoary supervisor for obtaining spare piezo electric cylinders.

To guard against the possibility of the breakage of the cylinders, carefully place the cylinders in their proper positions and make sure that the lower cylinder rests securely on the anvil before allowing the hammer to slide along the rail. This practice will help the hammer to slide smoothly and free from falling off the rail.

4. The capacity of the sparking gap is very small and can be ignored in this experiment.

# Solution

1. Set up the apparatus as shown in Fig 23.9

2. Set the hammer at about the distance of 4 cm from the cylinders and release the hammer to slide along the rail.

3. Adjust the distance to find the maximum distance over which the spark is possible.(d)

4. Vary the distance through which the hammer is allowed to slide to 6 cm 8cm 10 cm and 12 cm respectively and repeat the experiment in accordance of steps 1,2 and 3 descrived above. In each case record distance d.

**Hints.** Prior to each experiment, short-circuit two tearminals of the piezo-electric ensemble to ensure lthat there is no charge left in all piezo-electric cylinder.

## Calculation

Let kinetic energy transferred from the hammer to piezo-electric cylinders be given by

$$E = Mg\,L\sin 55^{\circ}$$

where $M = 87.5 \times 10^{-3}$ kg

$g = 9.82$ $ms^{-2}$

$L = 4,6,7,10,12, \ (\times 10^{-2})$ m

$\sin 55^{\circ} = .8192$

hence $$\frac{1}{2} C_P V^2 = K \times 87.5 \times 10^{-3} \times 9.82 \times 10\text{-}2 \times L \times 10^{-2} \times .8192$$

$K = 0.5$

$C = C_1 + C_2 = 40 \times 10^{-12}$ F

$$V^2 = \frac{2 \times 0.5 \times 87.5 \times 10^{-3} \times 9.82 \times 10^{-2} \times L \times 10^{-2} \times .8192}{40 \times 10^{-12}}$$

$$V = 0.132 \sqrt{L} \qquad \text{V}$$

Calculate V for each value of L from the above equation and fill the table as shown below:

| L(cm) | (L)1/2 | V(Volt) | d(m) |
|---|---|---|---|
| 4 | | | |
| 6 | | | |
| 8 | | | |
| 10 | | | |
| 12 | | | |

Plot a graph of V against d.The slope of the graph is the value of the electric field for the break down of air molecules.

The calculation is strictly valid for the terminals( between which the spark jumps) that are parallel surfaces. If one of the terminal is a sharp point or a small conducting sphere ( small raadius of curvature), the field can be very large even at a low potential V, and strong enough to cause the breakdown of the air molecules.

# Experiment 2 Grating and Light Filters

Given are:

- -a small flash light
- -reflecing graing mounted on a plasticine block.
- -small plasticine blocks which may be used as supports for apparatus pieces
- -optic films marke#1(red),#2(red),#3(blue),#4(pink),#5(purple),#6(grey), #7(white)
- -three sheets of graph paper
- -cardboard box which may be used for housing apparatus.

1. Determine distance between two adjacent slits of the grating to maximum possible accuracy. Also calulate error.

Describe the principle or theory on which the calculation is based, together with a diagram depicting experimentaal set up. Give a complete report of measurements calculation errors and final answer.

2. Optic films from #1 to #5 are filters. Determine the range which are absobed and transmitted by each filter. Identify item#6.

3. Item #7 is a fine wire mesh. Determine the width and length of each square in the mesh. Demonstrate the principle with a diagram.

Visible light consists of wavelengths from.40 $\mu$m to .70 $\mu$m

**Caution.** Batteries for the flash light eventually run down and most noticeable after some 40 minutes of operating time. Turn off the flash light when it is not in use.

# Solution

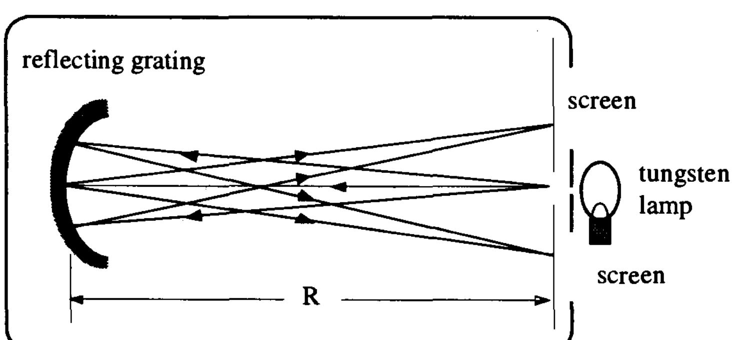


Fig.23.10

Since the reflecting grating functions as a grating and a concave morror at thee same time, the image or the diffraction pattern is formed and clearly observed at the image position only. We determine the radius of curvature of the concave mirror which is twice the focal distance by looking for a non- parallax position between the object such as the tip of a pencil and its image. This position of the object and the image is the centre of lcurvature of the mirror. Let R represent the radius of curvature of the concave mirror.

1. Set up the apparatus as shown in the diagram. The grating is at distanceR from the slit. The diffraction pattern is formed on the screen which is also at distance R from the grating.
The whole assembly is placed inside the card board box to reduce background or scattered light.

2. Observe the position of the first order bright band of blue, green, yellow ,orange and red light in the continuous spectrum.

If we take wavelength of the blue light to be .40 $\mu$m.
wavelength of the red light to be .70 $\mu$m
wavelength of the green light 0.47 $\mu$m
wavelength of the yellow light is thus 0.55 $\mu$m .
wavelength of the orange light 0.62 $\mu$m

Each of these wavelengths has the error of half bandwidth i.e $\pm 0.03$ $\mu$m

3. Calculation

Condition for the first bright band to be formed

$$d \sin\theta = n\lambda \qquad n = 1$$

or

$$x = \frac{\lambda L}{d}$$

where x is the position of the first order bright band measured from the middle of the screen.
L distance of the grating to the screen

Meaure distance x for blue,green, yellow,orange and red wavelengths.

4. Plot a graph of x against $\lambda$. Slope $m = \frac{L}{d}$

With known value of L, d can be calculated. The error of the result is due mainly to error in assumed wavelength which may be taken to be $\frac{3\times 2}{(40+70)} \times 100 \approx 6\%$

5. With known value of d, we can use the grating to find absorbed and transmitted wavelengths of all filters.

6. With chosen known value of wavelength, one can determine the width of the mesh in both directions.

# International Physics Olympiad XXIV

# 1993

# Williamsburgh Virginia, USA

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## Theory

### General Information

| Physical Quantities | Symbol | Value | Unit |
|---|---|---|---|
| average radius of earth | $R_E$ | $6.4 \times 10^{8}$ | m |
| gravitional acceleration | g | 9.8 | $ms^{-2}$ |
| gravitational constant | G | $6.67 \times 10^{-11}$ | $Nm^2kg^{-2}$ |
| Permittivity of free space | $e_0$ | $8.85 \times 10^{-12}$ | $C^2N^{-1}m^{-2}$ |
| Permeabilty of free space | $m_0$ | $4\pi \times 10^{-7}$ | $NA^{-2}$ |
| speed of light in vacuum | c | $3.00 \times 10^{8}$ | $ms^{-1}$ |
| elementary charge | e | $1.60 \times 10^{-19}$ | C |
| mass of electron | $m_e$ | $9.11 \times 10^{-31}$ | kg |
| mass of proton | $m_p$ | $1.67 \times 10^{-21}$ | kg |
| Planck's constant | h | $6.63 \times 10^{-34}$ | Js |
| Avogadro's number | $N_A$ | $6.02 \times 10^{23}$ | $mol^{-1}$ |
| Boltzmann's constant | k | $1.38 \times 10^{23}$ | $JK^{-1}$ |
| gas constant for 1 mol | R | 8.31 | $J.mol^{-1}K^{-}$ |

1

## Problem 1 Electricity in the Atmosphere

In electrostatics, earth's surface can be regarded as a good conductor with total charge Q and surface charge density $\sigma_0$.

1.1 Under a certain weather condition, the electric field of the earth points downward, and the value of the elctric field of the earth near the surface is 150 V/m , determine total amount of surface charge of the earth and associated surface charge density.

1.2 If the strength of the electric field which points downward decreases with increase in altitude measured from the surface of the earth and assumes the value of 150 V/m at a distance 100 m measured from the surface of lthe earth. Calculate average net charge in 1 cubic metre of the air between the surface of the earth at the altitude of 100 m measured from the ground.

1.3 The average volume charge density calculated in 1.2 is the resultant charge due to the presence of positive and negative ions( of single charge type) per unit volumei.e.$n_+$ and $n_-$ in almost the same quantity. Under fair weather condition $n_+ - n_-$ near the earth's surface ~ $6 \times 10^8$ $m^{-3}$. These positive and negative ions are in motion under the influence of the electric field pointing downward.

Assume the speed of of these ions are directly proportional to the electric field i.e. $v_0 = 1.5 \times 10^4$ $E_0$with the unit of $v_0$ in m/s and $E_0$ in V/ m.

Calculate the time for ion current in the atmosphere flowing into the earth's surface resulting in the total surface charge of the earth reduced to one half of its original value.

1.4 One method of determining the electric field in the atmosphere and $v_0$ employs the apparatus illustrated in Fig.24.1. In the diagram, the lower metal plate is in the form of two quadrants that are electrically connected.

Being secured to the electric insulator fixed to the axis, the lower plate is not grounded. The upper plate is cut to form two slots each of which in the form of a quadrant. (In the figure, the distance between the upper and lower plates has been exaggerate the form of two quadrants receives full electric flux on its surface twice and no more until the next rotation of the upper plate.

If T is the period of the rotation of the upper plate

$r_1$ and $r_2$ are inner and outer radii of the quadrants the lower plate

Determine the total of charge q(t) induced in the lower plate as a function of time from t = 0 to t = T/2 and sketch the corresponding graph. (In this situation, we need not be concerned with the effect of ionic flow in the atmosphere.)

1.5 If the apparatus described in 1.4 is connected to a signal amplifier which has the input circuit equivalent to a capacitor and resistor in parallel, (Capacity of metal plates is negligible when compared with capacity C of the capacitor in the signal amplifier.)

Skech a graph of potential difference V between M and N as a function of time t during one period of rotation of the upper plate of lthe apparatus.

Give separate answers for period T of different conditions

a. $T = T_a <<< CR$
b. $T = T_b >>> CR$

( In the calculation, the values of C are R remain constant, and only $T_a$ and $T_b$ of cases a and b vary.) Determine ratio $V_a/V_b$ if $V_a$ and $V_b$ are maximum vaues of V(t) of cases a and b respectively.

1.6 Given $E_0 = 150$ V/m $r_1 = 1$ cm $r_2 = 7$ cm $C = 0.01$ μF $R = 20$ M and the upper plate rotates with the angular speed of 50 Hz, estimate maximum potential during one period of rotation.

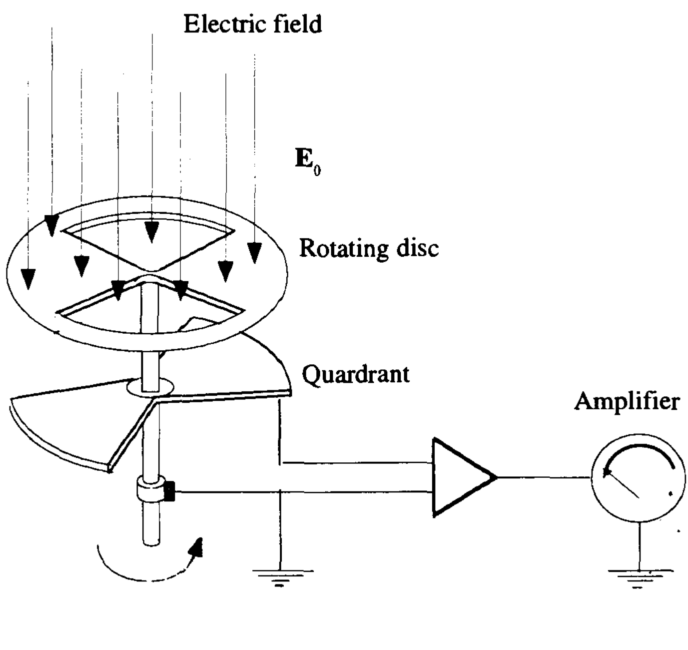



Fig.24.1

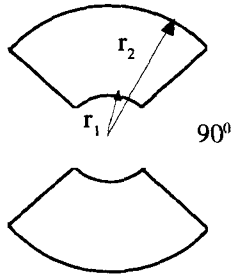



Fig. 24.2

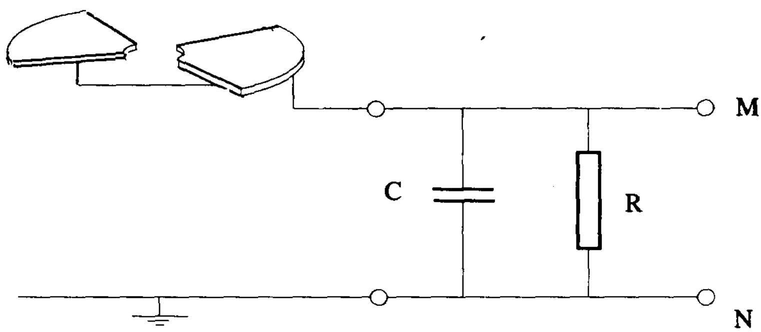



Fig 24.3

# Solution

1.1 Applying Gauss's law

$$\frac{4\pi R_E^2 \sigma}{\varepsilon_0} = E_0 4\pi R_E^2$$

Whereas $\varepsilon_0$ is permittivity of vacuum = $8.85 \times 10^{-2}$ $C^2N^{-1}m^{-2}$
$E_0$ electric field at the earth' sirface = 150 V/m
s surface charge density per 1 $m^2$
$R_E$ radius of the earth

Substitute $\sigma$ = $-8.85 \times 10^{-12} \times 150$ C/ $m^2$
= $-1.3 \times 10^{-9}$ C/ m2

Total charge on the surface of the earth

$$Q = 4\pi R_E^2 \times \left(-1.3 \times 10^{-9}\right)$$

$$= 4\pi \times \left(6.4 \times 10^6\right)^2 \times \left(-1.3 \times 10^{-9}\right) = -6.2 \times 10^5 \quad \text{C } \mathbf{Ans}$$

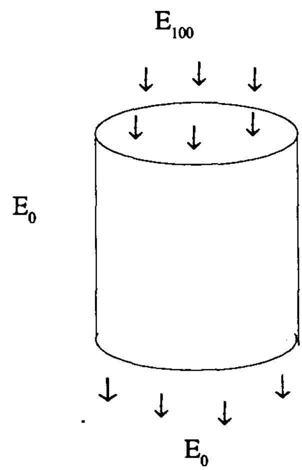



Fig. 24.5

1.2 Consider a cylindrical surface with the axis in the verticl direction and cross-sec tional area A, and .the upper surface is 100 m above the ground while the bottom surface on the ground.

Application of Gauss's law gives $(E_0 + E_{100})\,A = \dfrac{q}{\varepsilon_0}$

$$= \frac{\rho_{av} \times 100A}{\varepsilon_0}$$

whereas $\rho_{av}$ is average value of volume charge density per 1 $m^3$.

$$\rho_{av} = \frac{(150-100)\times 8.85\times 10^{-12}}{100} \quad C/m^3$$

$$= 4.4\times 10^{-12} \quad C/m^3$$

(area unit vector pointing out of the surface is positive and into the volume negative)

1.3 Let $n_+$ and $n_-$ be volume charge density per unit cubic meter of positive and negative charges respectively. and the magnitude of each charge is e .

As the direction of the electric field points towards the centre of the earth, positive charge moves towards the earth, while negative charge moves away from the earth into the atmosphere. The number of negative charges on the surface of the earth keeps decreasing.

Current due to positive charge carriers j is given by

$$j_+ = n_+ e\, v$$

whereas v is magitude of velocity of positive charge and equal to $1.5\times 10^{-4}\ E_0$

thus $$j_+ = 6\times10^8\times1.6\times10^{-19}\times1.5\times10^{-4}E_0$$

$$= -1.4\times10^{-14}\ E_0$$

Current $j_+$ is the rate of change of area charge density on the surface of the earth i.e.

$$j_+ = \frac{d\sigma}{dt}$$

Substitute $E_0 = \frac{\sigma}{\varepsilon_0}$ to get

$$\frac{d\sigma}{dt} = -1.44\times10^{-14}\times\frac{\sigma}{\varepsilon_0}$$

$$= \frac{-1.44\times10^{-14}}{8.85\times10^{-12}}\sigma$$

$$= -1.83\times10^{-3}\sigma$$

and $$\sigma = \sigma_0 e^{-\frac{t}{T}}$$

in which $$T = \frac{1}{1.83\times10^{-3}} = 600\ \text{s}$$

If the amount of charge is redued by half $\sigma = \frac{\sigma_0}{2}$ and

$$\frac{\sigma_0}{2} = \sigma_0 e^{-\frac{t}{600}}$$

$$-\frac{t}{600} = \ln\frac{1}{2}$$

$$t = 600\ln 2$$

$$= 600\times0.693 = 415\ \text{s}$$ **Ans**

1.4 Imagine a cylinder of small length placed in verticl position between the upper and lower plates of the apparatus. Also consider only the sections of the cylinder with its crossection area equal to that of two quardrant slots. See Fig.24.6 aand Fig.24.1)

Total area of the two quardrant slots $= \frac{\pi}{2}\left(r_2^2 - r_1^2\right)$

The upper surface of the sections of the cylinder under consideration are subject to elctric flux only when they are directly beneath the slots of the upper plate of the apparatus only. This happens $\frac{4}{T}$ times per second for each slot.

The area of the upper surace of the imaginary cylinder exposed to the electric field is therefore a function of time. During time interval t, the value of this area is given by,

$$A(t) = \frac{\pi\left(r_2^2 - r_1^2\right)}{2} \times \frac{4t}{T}$$

The application of Gauss's law gives

$$-E_0 A(t) = \frac{q(t)}{\varepsilon_0}$$

where q(t) is total charge on the surface under consideration.

$$q(t) = -\frac{2\pi(r_2^2 - r_1^2)t}{T}\varepsilon_0 E_0$$

During time interval $0 \le t \le \frac{T}{4}$ , $\quad q(t) = -\frac{2\pi(r_2^2 - r_1^2)t}{T}\varepsilon_0 E_0$

During time interval $\frac{T}{4} \le t \le \frac{T}{2}$ $\quad q(t) = -\pi(r_2^2 - r_1^2)\varepsilon_0 E_0\left(1 - \frac{2t}{T}\right)$

Maximum value of charge collected is

$$\left|q_{MAX}\right| = \frac{\pi}{2}\left(r_2^2 - r_1^2\right)\varepsilon_0 E_0$$

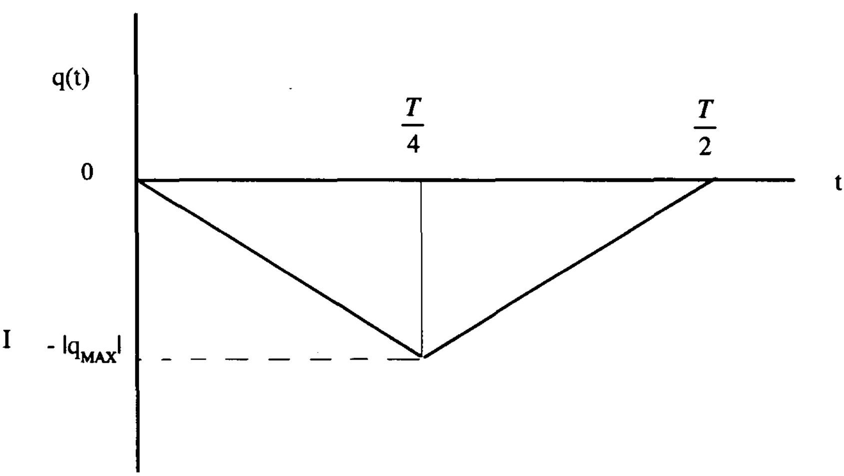



Fig. 24.6

1.5 In case a $T = T_a \lll CR$ i.e. periodic time is less than time constant, corresponding to large signal frequency. AC resistance of the capacitor is small, and most of the current goes through the capacitor. Output voltage $V_O$ is that of ,voltage of capacitor $V_C$. In this case the capcitor is rapidly charged to its maximum voltage and also discharges very rapidly to zeo voltage.

Maximumvoltage of the capacitor

$$V_{MAX} = \frac{q_{MAX}}{C}$$

and

$$|q_{MAX}| = \frac{\pi}{2}(r_2^2 - r_1^2)\varepsilon_0 E_0$$

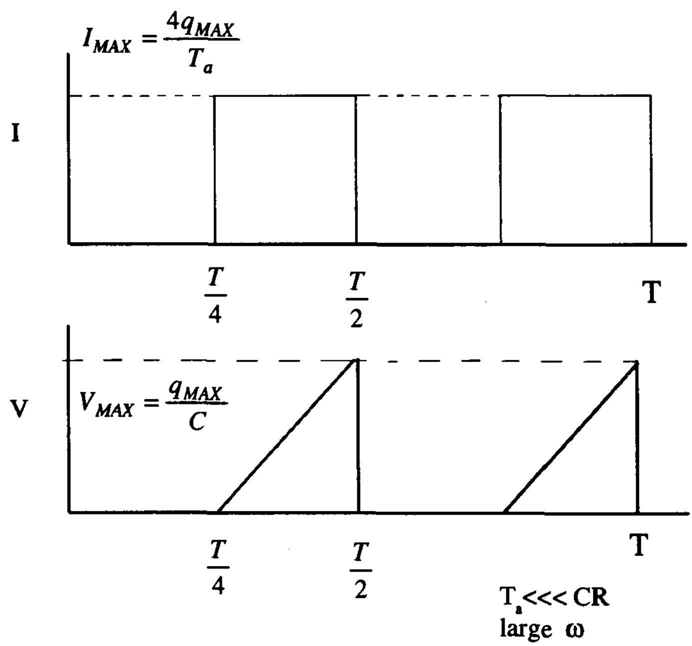
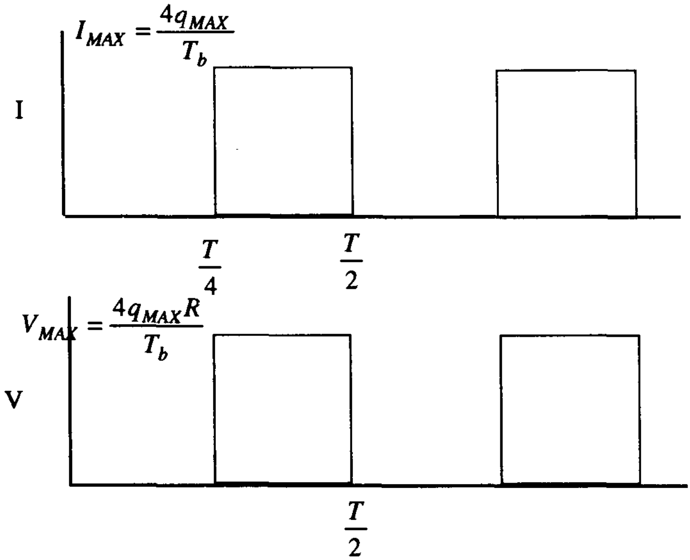



Fig. 24.7

Under case b $T = T_b \ggg CR$ periodic time of the rotation of the plate is much larger than time constant of the circuit, corresponding to very low signal frequency. In this situation, AC resistance of the capacitor is very large and most of the current goes through resistance R. Hence output voltage follows that of the resistor.

Current of input signal $\quad I_{MAX} = \frac{4q_{MAX}}{T_a}$

$$V_O = V_R = V_{MAX} = \frac{4q_{MAX}R}{T_b}$$




$T_b <<< CR$
small ω

Fig. 24.8

If $V_a$ and $V_b$ represent maximum voltage values of cases a and b respectively,

$$\frac{V_a}{V_b} = \frac{\frac{q_{MAX}}{C}}{\frac{4q_{MAX}R}{T_b}}$$

$$= \frac{T_b}{4RC}$$

1.6 If $C = .01 \times 10^{-6}$ F

$R = 20 \times 10^{6}$ Ω

$T = \frac{1}{50} = 0.02$ s

$CR = 10^{-8} \times 2 \times 10^{7} = 0.2$ s

which is the case of CR >>> T

Maximum potential $V_{MAX} = \frac{q_{MAX}}{C}$

Sustitution of known values gives

$$q_{MAX} = \frac{\pi}{2}\left(r_2^2 - r_1^2\right)\varepsilon_0 E_0$$

$$V_{MAX} = \frac{\pi}{2}\left(7^2 - 1^2\right)\frac{8.85\times 10^2 \times 150}{10^{-8}}$$

$$= 7.5\times 10^{-3} \times 1.33\times 10^{-1}$$

$$= \quad 1\times 10^{-3} \qquad \text{V} \quad \textbf{Ans}$$

## Problem 2 Force of laser beam on transparent bodies

A laser beam of sufficient intensity can exert force on transparent bodies by means of refraction. In order to demonstrate this phenomenon, we consider a small glass prismwith a triangular base, prism angle $A = \pi - 2\alpha$ length of the base of the triangle = 2h and width = W .The refractive index of the prisms is n, and density $\rho$.

The prism is set in the path of laser beam which is directed along the positive x-axis placed in the horizontal direction. (See Fig.24.9)

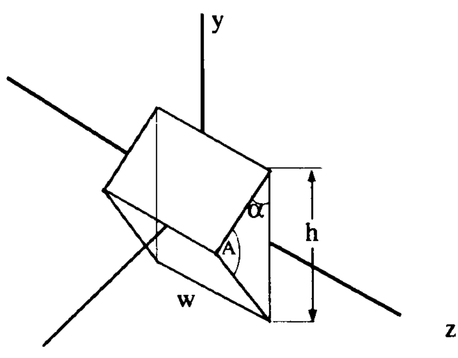



Fig. 24.9

In solving this problem,the prism is treated as being stable against rotation. This is to say, the prism angle always points opposite to the direction of the laser beam.and the trangular cross sectiona area of the prism always paralell to the xy plane while the base of the prism is kept parallel to the yz direction.

Refractive index of the air around the prism $n_a = 1$. All sides of the prism are coated by non-reflecting marerial., so no reflection can take place.

The laser beam used in the experiment has uniform intensity along the width of the prism(z axis) but decreases in a linear fashion with the distance measured along y axis (i.e.the disance of the laser beam measured from x axis. )

If y is a distance of the laser beam measured from x axis,while maximum intensity $I_0$ ,occurs at $y = 0$ and becomes 0 at $y = \pm 4h$ (See Fig. 24.9) (The intensity of the laser beam is power per unit area expressed in the unit of $W/m^2$ )

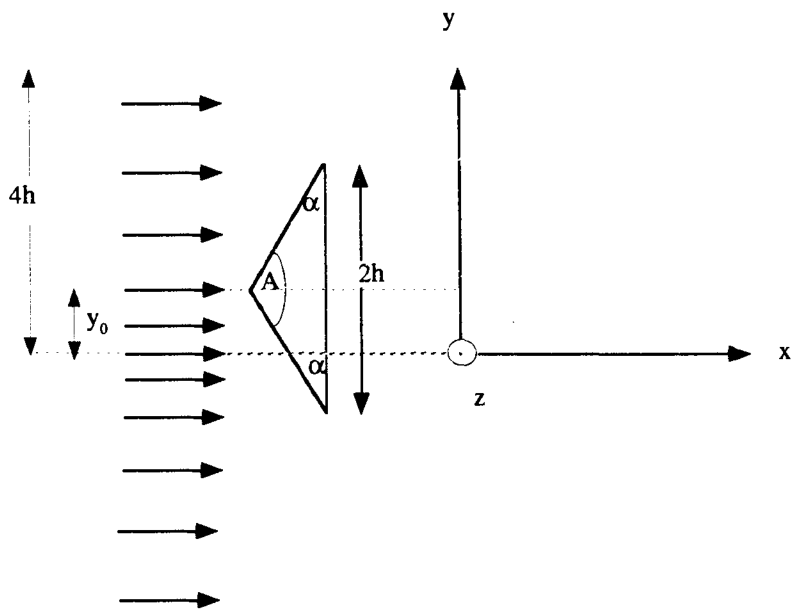



Fig. 24.10

2.1 Derive an equation giving θ (See Fig. 24.11) as a function of a and refractive index n, when the laser baem falls on the upper portion of the prism.

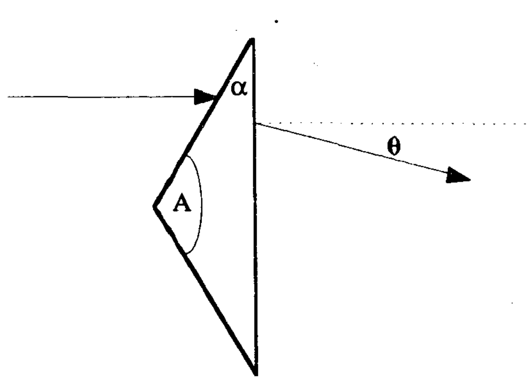



Fig. 24.11

2.2 Determine the components of the resultant force on the prism resolved along x and y axes respectively.

Express the answers in terms of intensity $I_0$, angle $\theta$, length h, width W, and distance $y_0$, when the tip of the prism is at distance $y_0$ from x axis, with $|y_0| \leq 3h$.

Sketch a graph of the horizontal and vertical components of the resultant force as a function of variable y.

2.3 If the laser beam has a frontal width of 1 mm measured along z axis and 80 mm wide measured along y axis, and given the angle of prism

$\alpha$ = 30°,
h = 10 $\mu$m ,
refractive index n = 1.5,
width W = 1 mm
density $\rho$ = 2.5 g/cm³,

and the top of the prism is at distance $y_0$ = -h/2 ( = - 5 $\mu$m ) below the mid- point of the intensity distribution of the laser beam,

how much power is required to keep the prism in equilibrium ( along negative y axis) with the pull of the gravitational force on the prism?

2.4 If the experiment is performed where no gravitational pull exists using the same prism ,with the same length and width of the laser beam employed in 2.3 , with the value of $I_0 = 10^8$ W/m²

Determine period time of the oscillation, if the prism angle is placed in an initial position at y = h/20 and released so that the prsim can move freely up and down.

# Solution

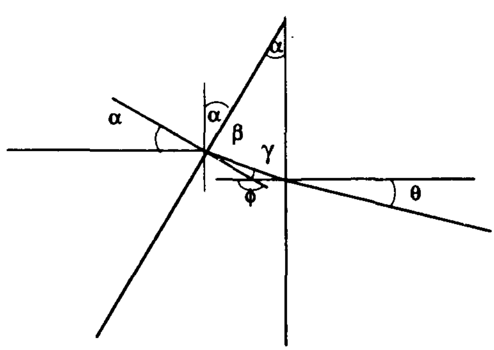



Fig. 24.12

From the above diagram, application of the law of refraction at the frontal surface of the prism gives,

$$1 \sin\alpha = n \sin\beta$$

where $\alpha$ is angle of incidence in the air
$\beta$ angle of refraction in the prism
n refractive index of prism relative to air

From the geometry of the diagram

$$\alpha + \phi = \pi$$

where $\phi$ is the angle between two normal lines (one of the frontal surface and other of the vertical surface.
Let $\gamma$ be the angle of incidence inside and at the second surface of the prism

hence $\gamma + \beta + \phi = \pi$

It follows $\gamma + \beta = \alpha$

$$\gamma = \alpha - \beta$$

Application of law of refraction at the second surface of the prism gives,

$$n \sin(\alpha - \beta) = 1 \sin\theta$$

$$\sin\theta = n[\sin\alpha \cos\beta - \cos\alpha \sin\beta]$$

$$\sin\theta = n[\sin\alpha\cos\beta - \cos\alpha\sin\beta]$$

$$\sin\theta = n\left[\sin\alpha\left\{1 - \frac{\sin^2\alpha}{n^2}\right\}^{\frac{1}{2}} - \frac{\cos\alpha\sin\alpha}{n}\right] \quad \textbf{Ans} \quad (1)$$

2.2 Photon which enters the prism in the direction parallel to the horizon suffers deflection angle $\theta$.

If $E_p$ is photon energy
and $\Delta p$ represents mometum transferred from phton to the prism,

Component of $\Delta p$ resolved alonmg x axis $= \frac{E}{c}(1 - \cos\alpha)$

Component of $\Delta p$ resolved along y axis $= \frac{E}{c}\sin\alpha$

If N photons enter the prism every second,
the rate of momentum transfer is force acting on the prism ( = F )

The ensuing calculation must take into account of the fact that the directions of force acting on the prism in the upper and lower parts are not the same.

Let $N_U$ be the number of photons entering the upper part of the prism
$N_D$ the number of photons entering the lower part of the prism.

Force acting on the prism in x direction is $= (N_U+N_D)\dfrac{E(1-\cos\theta)}{c}$

in y direction is $= (N_U-N_D)\dfrac{E\sin\theta}{c}$

From the information provided, the intensity of laser beam in the unit of W/m$^2$ is directly proportional to y

$$\text{i.e. } I(y) = I_0\left[1 - \frac{|y|}{4h}\right]$$

When the tip of the prism is at distance $y_0$ from x axis and $|y_0| \le 3h$, two situations are possible:

1. Prism is entirely in either the upper region or the lower region of intensity distribution of the laser beam. In either case the calculation is the same due to the symmetric property of the problem.

2. The lower part of the prism is partly located in the lower region of intensity distribution of the laser beam, or the upper part of the beam is partly in the lower region of intensity distribution of the laser beam. Again the analyses of the two situations are the same.)

Case 1 $h \le y_0 \le 3h$ (Whole prism is completely in the upper region of intensity distribution of laser beam.)

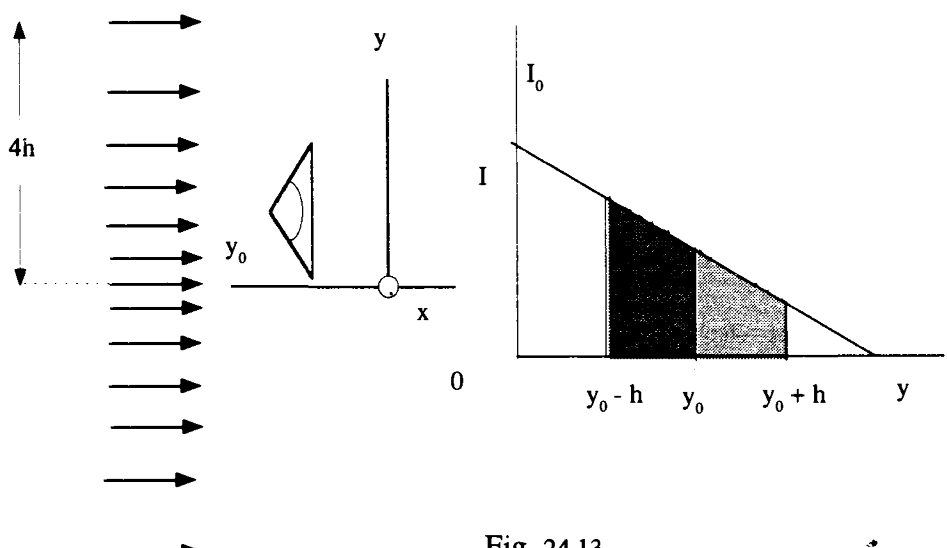



Fig. 24.13

Average values of the intensity of laser beam at mid-point of the upper and lower parts of front surface of the prism are given by

$$\bar{I}_P = I_0\left[1-\frac{y_0+\frac{h}{2}}{4h}\right] \quad \text{for the upper half of the prism}$$

$$\bar{I}_D = I_0\left[1-\frac{y_0-\frac{h}{2}}{4h}\right] \quad \text{for the lower half of the prism}$$

Hence

$$F_X = \frac{\bar{I}_P+\bar{I}_D}{E}\,\frac{E(1-\cos\theta)}{c} \qquad \text{N/m}^2$$

$$= \frac{2I_0}{c}\left(1-\frac{y_0}{4h}\right)(1-\cos\theta) \qquad \text{N/m}^2$$

$$F_Y = \frac{\bar{I}_P-\bar{I}_D}{E}\,\frac{E\sin\theta}{c} \qquad \text{N/m}^2$$

$$= \frac{I_0}{4c}\sin\theta \qquad \text{N/m}^2$$

Consider all laser beams entering the whole prism

x component of net force $= \frac{2I_0}{c}\left(1-\frac{y_0}{4h}\right)(1-\cos\theta)Wh$ N

y component of net force $= \frac{I_0}{4c}\sin\theta Wh$

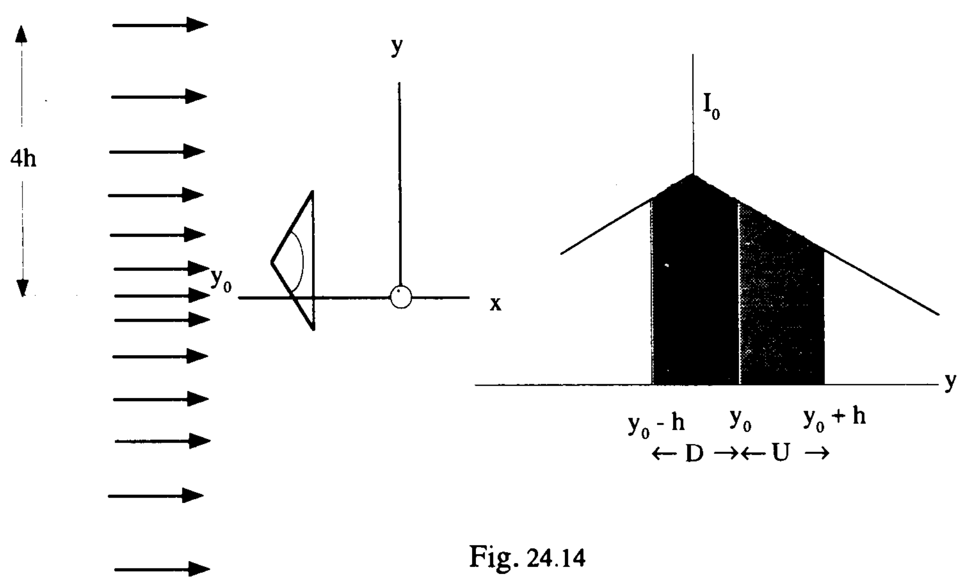



Fig. 24.14

Case 2 $0 \leq y_0 \leq h$ (the lower half of the prism is partly in the lower region of intensity distribution)
Average values of the intensity of laser beam at mid point of the surfaces of the upper and lower halfs of the prism are:

$$\bar{I}_U = I_0\left[1-\frac{y_0+\frac{h}{2}}{4h}\right]$$ for the upper part of the prsim

$$\bar{I}_{D(1)} = I_0\left[1-\frac{\frac{y_0}{2}}{4h}\right]$$ for the first or bottom part of the lower half of theprism.

$$\bar{I}_{D(2)} = I_0\left[1-\frac{\frac{h-y_0}{2}}{4h}\right]$$ for the second or upper part of the lower half of the prism

x component of resultant force

$$F_X = \frac{1}{c}\left[I_0\left\{1-\frac{y_0+\frac{h}{2}}{4h}\right\}Wh - I_0\left\{1-\frac{\frac{y_0}{2}}{4h}\right\}Wy_0 + I_0\left\{1-\frac{\frac{h-y_0}{2}}{4h}\right\}W(h-y_0)\right](1-\cos\theta)$$

$$= \frac{I_0}{c}Wh\left(\frac{7}{4}-\frac{y_0^2}{4h^2}\right)(1-\cos\theta)$$

and y component of resultant force

$$F_y = \frac{1}{c}\left[I_0\left\{1-\frac{y_0+\frac{h}{2}}{4h}\right\}Wh - I_0\left\{1-\frac{\frac{y_0}{2}}{4h}\right\}Wy_0 + I_0\left\{1-\frac{\frac{h-y_0}{2}}{4h}\right\}W(h-y_0\right]\sin\theta$$

$$= \frac{I_0}{c}W\left[\frac{y_0}{2}-\frac{y_0^2}{4h}\right]\sin\theta$$

$$= \frac{I_0}{c}Wh\left[\frac{y_0}{2h}-\frac{y_0^2}{4h^2}\right]\sin\theta$$

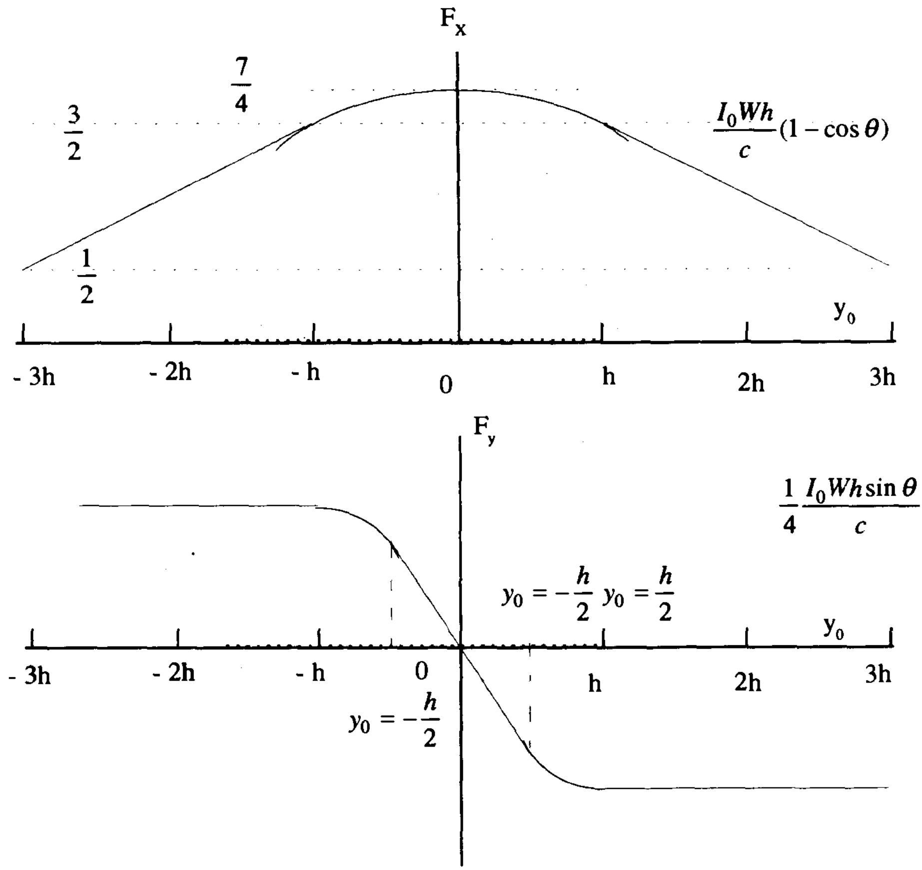



Fig.24.15 Graph of $F_x$ and $F_y$ as a function of $y_0$

2.3 It is clear from the graph that $F_Y$ has nonzero values in the region $y_0 < 0$. It impies that the prism can float in the air and in equilibrium with its own weight when

$$F_Y = \rho h^2 \tan\alpha W g$$

i.e

$$y_0 = -\frac{h}{2}\frac{I_0}{c}\left[\frac{y_0}{2h} - \frac{y_0^2}{4h^2}\right]\sin\theta = \rho h g \tan\alpha$$

From Equation (1) in Section 2.1 $\sin\theta = n\left[\sin\alpha\sqrt{1-\frac{\sin^2\alpha}{n^2}}\right]\sin\alpha\cos\alpha$

Substitute
n = 1.5
α = 30°
sinθ = 0.274
$y_0$ = $\frac{h}{2}$ m

$$h = 10^{-5} \quad m$$
$$c = 3\times10^{8} \quad m$$
$$\rho = \frac{2.5\times10^{-3}}{10^{-6}} = 2.5\times10^{3} \quad kg/m^3$$
$$g = 9.8 \quad m/s^2$$
$$\tan\alpha = \frac{1}{\sqrt{3}}$$

To obtain $$I_0 = \frac{2.5\times10^{3}\times9.81\times3\times10^{8}\times10^{-5}}{\sqrt{3}\times\left(1-\frac{1}{4}\right)\times0.274}$$

$$= 8.30\times10^{8} \quad W/m^2$$

$I_0$ is maxium laser intesity per second per square meter,
hence the minimum power of laser required for floating the prism

$$= \frac{I_0}{2} \times \text{area normal to incident laser beam}$$

$$= \frac{1}{2}\times8.30\times10^{8}\times10^{-3}\times80\times10^{-6} \quad W$$

$$= 25.2 \quad W \quad \textbf{Ans}$$

2.4 For $y_0 = \frac{h}{20}$, the situation comes under the category of $y_0$ <<<< h

$$F_Y = -\frac{I_0}{c}.y.\frac{W}{2}\sin\theta$$

$$m\ddot{y} = -\left(\frac{I_0 W\sin\theta}{2c}\right)y$$

Angular velocity $$\omega = \sqrt{\frac{I_0 W\sin\theta}{2mc}}$$

Substituting $$m = \rho h^2 \tan\alpha$$

to obtain $$T = \frac{2\pi}{\omega}$$

$$= 2\pi\sqrt{\frac{2\times3\times10^{8}\times2.5\times10^{3}\times10^{10}}{10^{8}\times0.274\times1.73}}$$

$$= 11.2\times10^{-3} \quad s \quad \textbf{Ans}$$

# Problem 3 Electron Beam in Electric Field

A parallel electron beam consists of electrons of large kinetic energy obtained from the application of accelerating voltage $V_0$. These elctrons are sent traveling in the direction normal to an infinitely long straight copper wire of radius $r_0$ as shown in Fig. 24.14. The wire carries uniform positive charge. The distance of the electronûs closest approach to the wire if uncharged is represented by b.

The electrons after passing the charged copper wire land on the screen located at distance L ( L >>>> b ) from the wire. (See Fig. 23.14)

At the beginning of the experiment, the electron beam is confined within the normal distance to the wire of b ( collision parameter) (See Fig. 24.14)

In answering this problem, the width of the electron beam measured along the wire, like the length of the copper wire is assumed to be infinite.

Some figure data are provide d however other necessary data are given in the table on the first page.

| | | | | |
|---|---|---|---|---|
| radius of copper wire | $r_0$ | = | $10^{-6}$ | m |
| maximum distance of approach | $b_{max}$ | = | $10^{-4}$ | m |
| linear charge desity per unit length | $q_\lambda$ | = | $4.4\times 10^{-11}$ | C/m |
| accelerating voltage | $V_0$ | = | $2\times 10^{4}$ | V |
| distance beween the wire and the screen | L | = | 0.3 | cm |

**Hints** In answering questions from 3.1 to 3.4, use any reasonable approximation to arrive at useful formula and answers in figures.

3.1 Determine electric field E due to the charged copper wire and sketch a graph of electric field E as a function of distance measured from the axis of the wire to inside as well as outside the wire.

3.2 Use Classical Physics to calculate angle of deflection of electron beam which has the collision parameter b and the electron does not collide with the wire.

If $\theta_f$ is a small angle between the direction of the original velocity of the electron and the direction of final velocity of the electron arriving at the screen, calculate $\theta_f$

3.3 Use the results obtained from the application of classical physics, sketch a diagram illustrating the deflection of of the electron beam and of electron distribution i.e. the intensity of electrons collected by the screen as a function of distance on or along the screen.

3.4 Use quantum or modern physics to show that two plane waves representing deflected beams in the upper and lower parts give interference pattern on the screen.
Calculate the number of bright bands in the interfence pattern.
Sketch a graph depicting the distribution of electrons as afunction of distance on or along the screen.

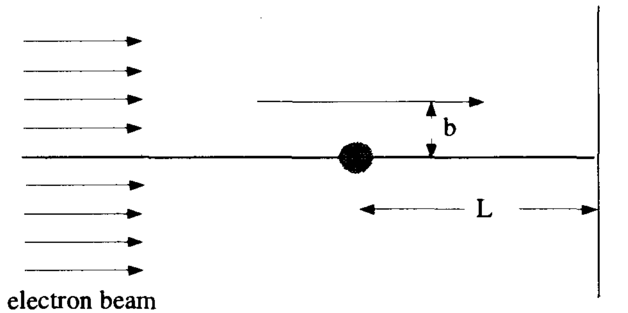



Fig. 23.14

## Solution

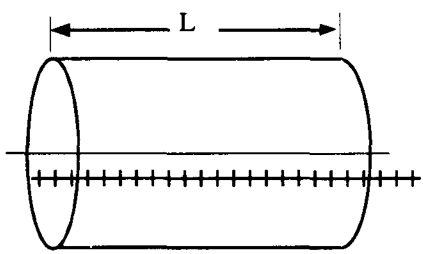



Fig. 20.15

3.1 Let $q_\lambda$ be linear charge density per 1 m length of wire.

Consider a cylindricl surface of radius $r_>$ ( $r_>$ > $r_0$ radius of wire) and length L and is concentric with the wire.

Apply Gauss's law

$$2\pi r_> . L.E_{r>} = \frac{q_\lambda L}{\varepsilon_0}$$

$$E_{r>} = \frac{q_\lambda L}{2\pi\varepsilon_0 r_>}$$

Whereas $r_>$ is the value of r agreater than the radius of the wire ($r_0$).
The direction of electric field $E_{r>}$ points in the direction of increasing r.

If $r_<$ is less than $r_0$, apply Gauss's law to obtain

$$2\pi r_< L E_{r<} = 0$$

(Charge resides on the surface of the conductor only.)

$$E_{r<} = 0$$

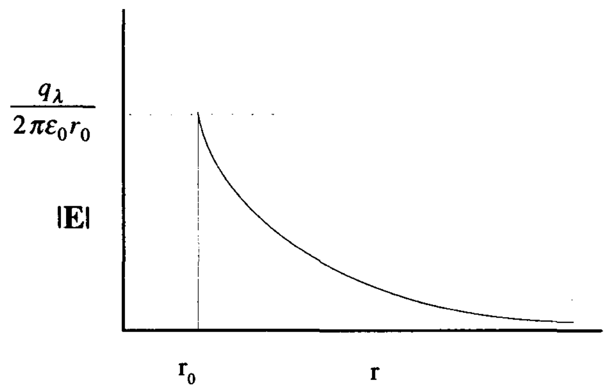



Fig. 24.16

1.2 Calculation of $\theta_f$

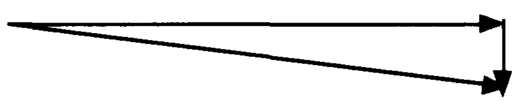


Fig. 24.17

## Method 1

Impuls **F**Δt causes momentum change of Δ**p** (in the direction normal to **p**)

Hence $\mathbf{F}\Delta t = \Delta\mathbf{p}$

$$-\frac{eq_\lambda}{2\pi\varepsilon_0 r}\mathbf{1}_r = \Delta\mathbf{p}$$

Whereas $\mathbf{1}_r$ is a unit vector along direction of vector **r**.

Force **F** have greatest effect on the change of momentum when $\mathbf{1}_r$ points along b i.e when the electrón is at the minimum distance of approach from the wire.

When this is the case, $\Delta t$ represents the duration for which $\Delta \mathbf{b}$ and $\Delta \mathbf{p}$ are effected

Or
$$\Delta t = \frac{\Delta b}{\Delta \mathbf{v}} = \frac{\Delta b}{\Delta \mathbf{v}}$$

From the principle of conservation of momentum

$$m\mathbf{b} \times \mathbf{v} = \text{constant}$$

$$\text{mbv} \sin \phi = \text{constant}$$

Interaction between the charged wire and the elctron is
strongest when $\phi \approx 90^{\circ}$

$$(\Delta b)\, v + b(\Delta v) = 0$$

( The negative sign indicates that an increases in b produces a decrease in v )

Hence
$$\left| \frac{eq_\lambda \mathbf{1}_r}{2\pi\varepsilon_0 b} \frac{b}{v} \right| = |\Delta \mathbf{p}|$$

$$\theta \approx \frac{|\Delta \mathbf{p}|}{p}$$

$$\approx \frac{eq_\lambda}{2\pi\varepsilon_0 m v^2}$$

$$\frac{1}{2} m v^2 = eV_0$$

$$m v^2 = 2eV_0$$

$$\theta_f \approx \frac{eq_\lambda}{2\pi\varepsilon_0 2eV_0}$$

$$\approx \frac{q_\lambda}{4\pi\varepsilon_0 V_0} \qquad \textbf{Ans}$$

## Method 2

At $r = \infty$ $E_r$ and electric potential vanish.
Elecric potential at r is given by

$$V_r = -\int_{\infty}^{r} \frac{q_\lambda}{2\pi\varepsilon_0 r} \mathbf{1}_r . d\mathbf{r}$$

$$V_r = -\int_{\infty}^{r} \frac{q_\lambda}{2\pi\varepsilon_0 r} dr$$

$$\approx -\frac{q_\lambda}{2\pi\varepsilon_0} \ln\frac{r}{r_\infty}$$

Whereas is distance $r_\infty$ is distance at $r \to \infty$

$$\ln\frac{r}{r_\infty} \approx \frac{r}{r_\infty} - 1 \approx -1$$

$$V_f = \frac{q_\lambda}{2\pi\varepsilon_0}$$

From the priciple of the conservation of mechanical eneergy

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 - \frac{eq_\lambda}{2\pi\varepsilon_0}$$

Whereas $v'$ is the magnitude of the velocity of the elctron striking the screen.

$$\frac{1}{2}m\left(v'^2 - v^2\right) = \frac{eq_\lambda}{2\pi\varepsilon_0}$$

$$\left(v' + v\right)\left(v' - v\right) = \frac{eq_\lambda}{\pi m\varepsilon_0}$$

$$2v\Delta v \approx \frac{eq_\lambda}{\pi m\varepsilon_0}$$

$$\theta_f \approx \frac{\Delta v}{v}$$

$$\frac{\Delta v}{v} \approx \frac{eq_\lambda}{2\pi\varepsilon_0 mv^2}$$

Substituting $\frac{1}{2}mv^2$ to get $\theta_f \approx \frac{eq_\lambda}{4\pi\varepsilon_0 eV_0} \approx \frac{q_\lambda}{4\pi\varepsilon_0 V_0}$

Also substituting

| | | | |
|---|---|---|---|
| $q_\lambda$ | = | $4.4 \times 10^{-11}$ | C/m |
| e | = | $8.85 \times 10^{-12}$ | $C^2N^{-1}m^{-2}$ |
| $V_0$ | = | $2 \times 10^4$ | V |

$$\theta_f \approx \frac{4.44 \times 10^{11}}{4 \times 3.1 \times 8.86 \times 10^{-11} \times 2 \times 10^4}$$

$\sim$ $6.21 \times 10^{-5}$ radian **Ans**

3.3 Application of classical physics to determine $\theta_f$ tells us that angle of deflection does not depend on the value of b.

For $b_{max} \geq b > r_0$ elctron beams are deflected towards the principal axis by distance $\theta_f$ L measured along the screen.

If b = $r_0$ elctrons are absorbed by the wire.

$$\theta_f L = 6.21 \times 10^{-5} \times 0.3$$
$$= 1.86 \quad m$$

which is equal to 19 $r_0$

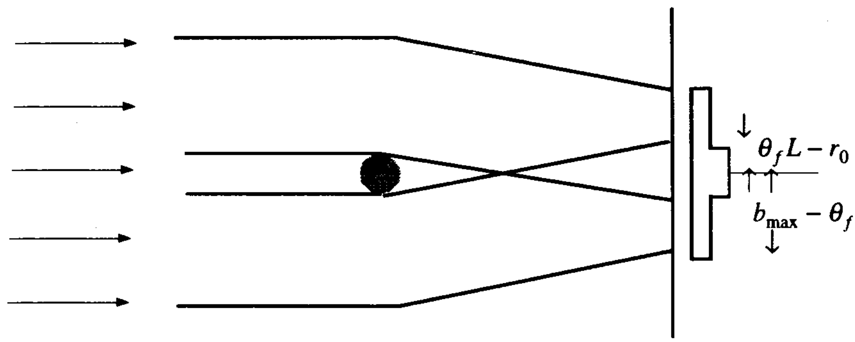



Fig. 24.18

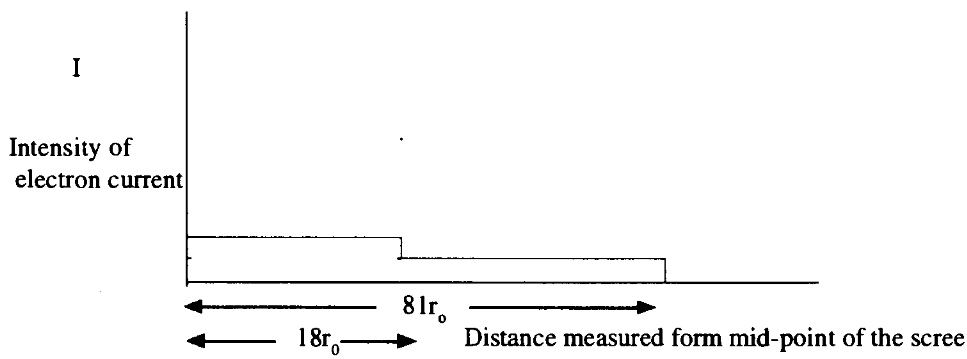



Fig 24.19 Graph of number of electrons per unit area on the screen as a function of distance form midpoint

3.4 De Broglie's wavelength of electron beam is

$$\lambda = \frac{h}{mv}$$

$$= \frac{h}{\sqrt{2meV_0}}$$

$$= \frac{6.36 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.60 \times 10^{-19} \times 2 \times 10^4}}$$

$$= 8.86 \times 10^{-12} \quad m$$

Due to the deflection of electron beam in the upper and lower parts of the apparatus layout, two plane waves with wavefronts making an angle of $2\theta_f$ with each other give an interfernce pattern of alternating " bright and dark bands".as depicted in Fig 24.20

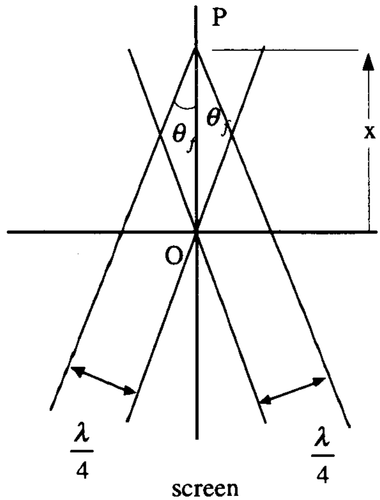



Fig. 24.20

In the diagram above, the zeroth order of bright band is at O.

At P, the differerence in the light path is $\frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$ .

A dark band is therefore produced at P.

Also from the diagram,

$$x \sin \theta_f = \frac{\lambda}{4}$$

or $$2x \sin \theta_f = \frac{\lambda}{2}$$

In the general form:

Condition for the formation of a bright band $2x \sin \theta_f = 2n\frac{\lambda}{2}$

and for a dark band $2x' \sin \theta_f = (2n+1)\frac{\lambda}{2}$

where x and x/ are distances of the bright and dark bands measured from O along the screen.

$\lambda$ De Broglie' s wavelength

n = 0,1,2,3,........

Distance between two consecutive bright bands

$$\Delta x = 2(x' - x)$$
$$= \frac{\lambda}{2 \sin \theta_f}$$
$$= \frac{8.6 \times 10^{-2}}{2 \times 6.21 \times 10^{-5}}$$
$$= 7.0 \times 10^{-8} \quad \text{m}$$

The region in which interference pattern is formed covers the distance of

$$(2 \times 19r_0 - 2r_0) = 36r_0$$

The number of bright bands fomed $= \dfrac{36 \times 10^{-6}}{7.0 \times 10^{-8}} = 500$ **Ans**

# Experiment

## Problem 1 Heat of Vapourization of Liquid Nitrogen

The purpose of this experiment is to determine the heat of vaporization of liquid nitrogen (L) by two different methods.

In the first method, an aluminum block provided is dropped into liquid nitrogen. At the same time, the weight of liquid nitrogen is recorded as a function of time in order to determine the quantiy of liquid nitrogen lost to evaporization. In the meantime,the temperature of the aluminum block keeps falling until it reaches the temperature of liquid nitrogen.

In the second method, a known quantity of heat is supplieded to liquid nitrogen while the weight of liquid nitrogen is being recorded also as a function of time in order to determine the quantity of liquid nitrogen lost to evapourization.

Liquid nitrogen to be used in the experiment is stored in a storage tank. Some of the liquid nitrogen may be transferred from the tank and poured in to a container on a balance. Under normal condition, liquid nitrogen continuously undergoes evaporization even before the aluminum block is dropped into the liquid or heat from the electric coil is given to liquid nitrogen. The reading of the weight of liquid nitrogen keeps falling. The decrease in weight (and thus mass) of liquid nitrogen is attributed to:

1. The dontainer on the balance is not a perfect heat insulator.
2. Liquid nitrogen takes in heat from the aluminum.
3. Liquid nitrogen takes in heat given off by the heating coil.(Second method).

**Equipment at disposal**

Multimeter which may be used to measure potentil V, current Iand resistance R.
Stopwatch
Manual guide on the use of the multimeter and the stopwatch.

**Warnings**

1. Liquid nitrogen is very cold, care must be taken to guard against exposing skin or clothing or letting objects other than those described in the instruction to come into contact with liquid nitrogen.

2. Do not dicard or put any object other than those requied by the experiment into liquid nitrogen. The experimenter, while performing the experiment needs to wear protective eyeglasses to protect their eyes from possible danger.

3. Slowly and gently drop the aluminum block into the liquidnitrogen, otherwise the liquid will start boiling violently

4. The heating coil will become very hot when not kept in the liquid nitrogen, and the current must be switched on when the heating coil is immersed in the liquid nitrogen only.

**Information for Method 1**

The specific heat of aluminum (c) varies greatly between room temperature and the temerature of liquid nitrogen (77 K) at 1 atmosperic pressure. A graph depicting variation of c with temperature T is shown in Fig 24.21.

An experiment is to be carried out to determine the quantity of liquid nitrogen evaporating while the temperature of the aluminum block is falling.

From data on the quantity of liquid nitrogen undergoing vaporization, and the graph of specific heat of aluminum as a function of temperature, determine the heat of vaporization of liquid nitrogen. (Assuming room temperature of $21^0 \pm 2^0$ Cs ) and estimate the errror of the value calculated.

**Information for Method 2**

In this experiment the rate of evaporization of liquid nitrogen when electric current is through the heating coil immersed in the liquid nitrogen,using the DC voltage source provided with the indicator pointing at number 8 on the scale. The capacitor connected across the terminals of the DC voltage source must not be removed.

Use information obtained from the experiment to determine the heat of vaporization of liquid nitrogen and also estimate the error of the answer.

**Additonal Instruction**

1. Sketches, diagrams and tables of values must be accompanied by symbols representing appropriate units etc. in order to allow the examiner to assess that the experiment is carried out in an appopriate and logical manner

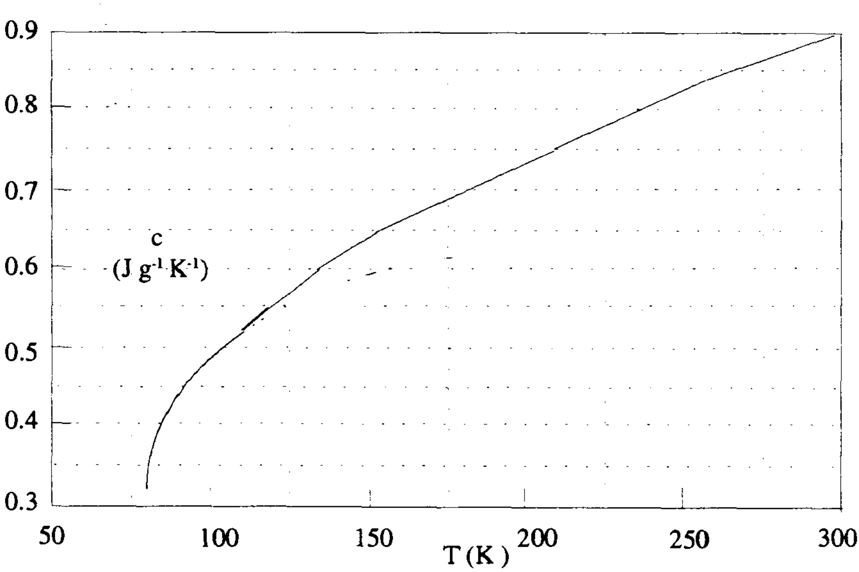



Fig 24.21Graph of specific heat of aluminum as a function of temperature

2. In case of any item of equipment or apparatus does not function properly, obtain help from the laboratory supervisor.

# Solution

### Method I

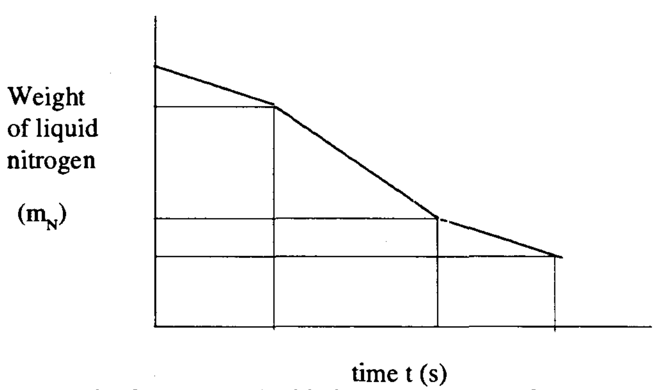
1. Set up the apparatus in accordance with the instruction.

2. Weigh the aluminum block ( $m_A$ ) and the container to be filled with liquid hydroden. ($m_C$)

3. Take readings of weights of liquid hydrogen ($m_N$) plus container ($m_C$) at regular intervals, such as every 5 s before slowly dropping the aluminum block into the container filled with liquid nitrogen.

4. Slowly dip the aluminum block into liquid nitrogen. The operation should take about 5 seconds. Record weights of liquid nitrogen ($m_N$), container ($m_C$) plus aluminum block ($m_A$) at every interval of 5 s or any other similar suitable time interval chosen by the performer of the experiment.

5. Plot a graph of weight of liquid nitrogen ($m_N$) as a function of time (t ). The curve obtained should look like the one shown in the Fig. 24.21.




Graph of weight of liquid nitrogen ($m_N$) as a functionof time (t).

Fig. 24.21

Let $t_1$ be time when the aluminum block is dropped into liquid nitrogen

$t_2$ be time when temperature of aluminum block just assumes the same value as that of liquid nitrogen.

The curve consists of three parts: ie. parts I and III representing the evaporation of liquid nitrogen due to taking heat from the ambient environment and part II evaporation of liquid nitrogen due to heat from the ambientenviroment and the aluminum block.

Let $\alpha_1$ be rate of nitrogen loss during parts I and III .
$\alpha_2$ rate of nitrogen loss during parts II
$T_2$ temperature of aluminum block before being dipped into liquid nitrogen
$T_1$ temperature of liquid nitrogen
c average value of specific heat of aluminumblock taken over the temperature interval $T_2$ and $T_Â$.(Thisis tobe graphically evaluated from the graph of specific heat of aluminum versus temperature)

The amount of heat transferred from aluminum block to liquid nitrogen = $m_A c(T_2 - T_1)$

Quantity of liquid nitrogen undergoing evaporation due to heat transferred from the two entities

$$= \frac{m_A c(T_2 - T_1)}{L}$$

The quantity of evaporated nirogen asmeasured from the experiment = $(\alpha_2 - \alpha_1)(t_2 - t_1)$

Hence $$\frac{m_A c(T_2 - T_1)}{L} = (\alpha_2 - \alpha_1)(t_2 - t_1) \quad (1)$$

in which L represents latent heat of evaporation

$$L = \frac{m_A c(T_2 - T_1)}{(\alpha_2 - \alpha_1)(t_2 - t_1)} \quad (2)$$

The chief souce of error of L is due ot error in $(t_2 - t_1)$

Small inaccuracy in $(t_2 - t_1)$ occurs when $m_A$ is large and time of heat transfer $(t_2 - t_1)$ between aluminum and liquid nitrogen is long enough to be measured with good accuracy.

**Method II**

Carry out the experiment similar to Method I, the only differenc is heat is delivered to liquid nitrogen by means of electric heater immersed in the liquid nitrogen.

Take readings of voltage across the heating coil, current and time during which the current is applied.

Record mass of liquid nitrogen ($m_N$) (and that of the heating coil) at regularly recorded intervals.
Plot a graph of mass of liquid nitrogen ($m_N$) as a function of time (t). Curve obtained should look like that depicted in Fig. 24.21. One major differnce is time interval beween

$t_1$ and $t_2$ can be measured with good accuracy.

Equation (1) of method I will be replaced by,

$$\frac{V.I.(t_2 - t_1)}{L} = (\alpha_2 - \alpha_1)(t_2 - t_1) \quad (3)$$

$$L = \frac{V.I}{(\alpha_2 - \alpha_1)} \quad (4)$$

In this method the accuracy of L will depend on the acurracies of the readings of V and I taken from the voltmeter and ammeter.

## Problem 2. Magnetic Moments and Fields

This experiment has two parts;

2.1 To determine the absolute magnitude $m_X$ of the magnetic moment of a small cylindr ical permanent magnet, contained in the envelop marked "X" ( A similar magnet, also needed for the experiment, is contained in the envelop marked "A".)

2.2 To investigate the magnetic field of a given axial distribution of magnets, contaained in the envelop marked "B".

The experiments in this problem make use of the following principles:

a. The magnetic fields **B** produced by a dipole magnet at a point along its axis at distance x from its centre is paralell to that axis and of strength given by:

$$|\mathbf{B}(x)| = 2\mu \frac{K}{|x|^3}$$

where **B** is magnetic field expressed in Tesla [ $NA^{-1}m^{-1}$ ]
K a constant = $10^{-7}$ Tesla m $A^{-1}$
m permeability in $Am^2$

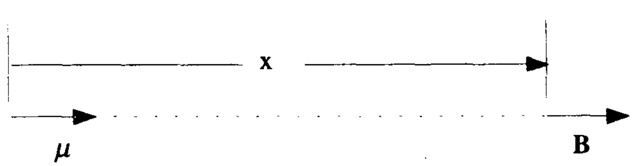



Fig. 24.23

b. The period of small torsional (angular) oscillations of a freely oscillating magnet suspended in the horizontal direction in an external magnetic field , such as a compass needle in the earth's magnetic field is given by

$$T = 2\pi\sqrt{\frac{I}{\mu B_H}}$$

where B is the horizontal component of the net field at the magnet,
and I is the moment of inertia of the magnet about avertical axis through its centre.

For a homogeneous cylinder,

$$I = m\left[\frac{L^2}{12} + \frac{d^2}{16}\right]$$

## Apparatus

The apparatus is illustrated in Fig.24.24 .

In the figure, a thin thread is suspended from the upper shelf on the wooden stand. A magnet marked "X" or "A" as the case may be can be secured to the lower end of the thread. A copper plate can be placed on the lower wooden shelf just directly below the suspended magnet to dampen out motion of the magnet if desired.
Two auxilliary wooden stands are provided. One of these serves as a holder for either magnet "A" or "X" in Part 1; the other holds the magnet system B (used in Part 2).

Distance between suspended magent and magnet mounted on the auxiliary sand can be measured with a ruler mounted on on one these auxiary stands.

A stop watch is provided . However the appararus set doses not include a protractor. This is deliberate.

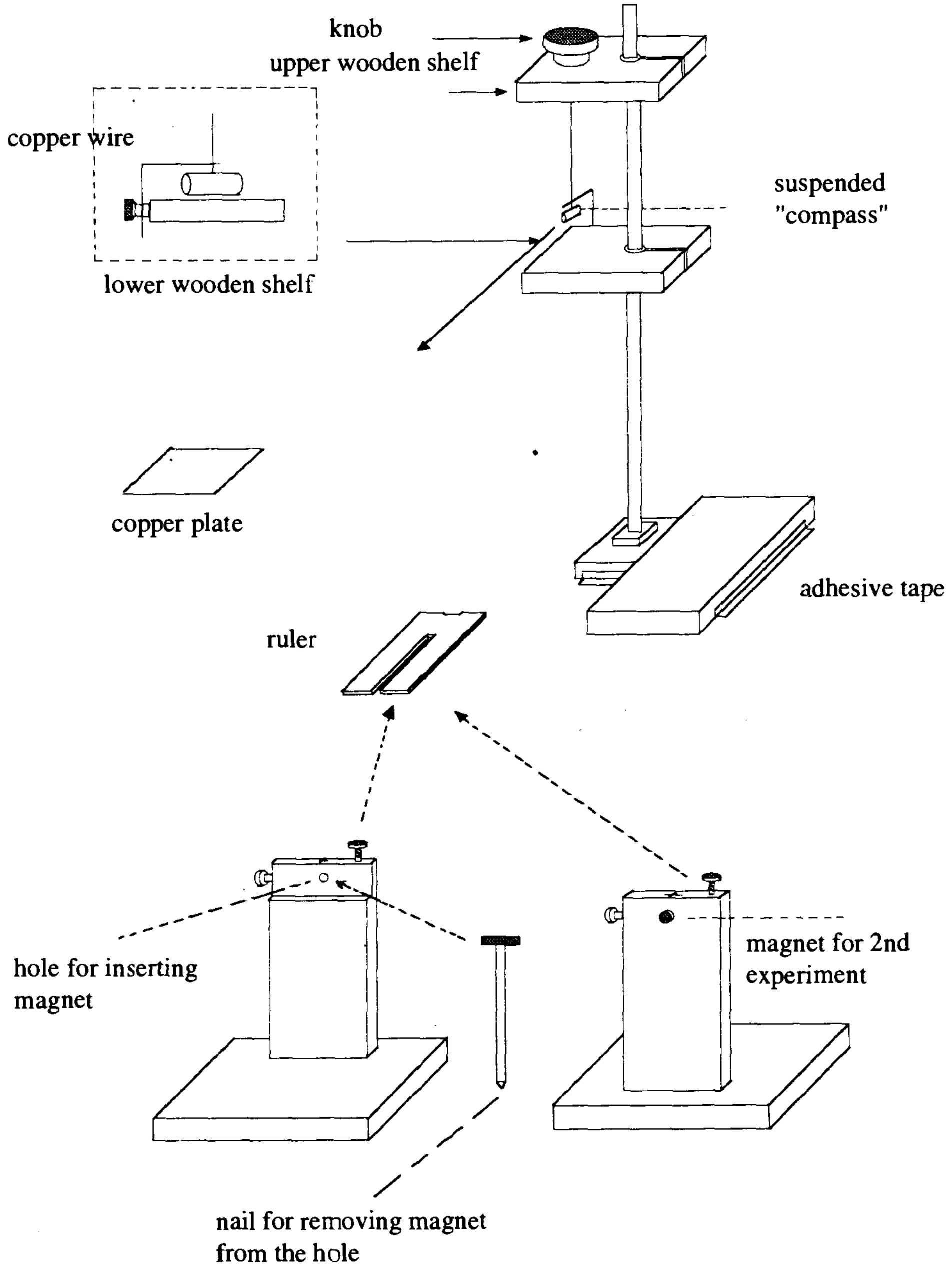

knob
upper wooden shelf
copper wire
suspended
"compass"
lower wooden shelf
copper plate
adhesive tape
ruler
hole for inserting
magnet
magnet for 2nd
experiment
nail for removing magnet
from the hole

Fig. 24.24

## Experiment 2.1

The magnetic moment to be determined $\mu_X$ is that of the pair of magnets in envelop X, labelled at the lends with a letter-number cmbination. Always keep this pair together. The moment of inertia of this pair has lbeen calculated and written on envelop X. Envelop A contains another pair of magnets with north and south poles marked respectively with black and rd spots. This pair is similar to the pair from envelop X, though its magnetic moment $\mu_A$ cannot be assumed equal to $\mu_X$.

A given pair of magnets can be "splitted" and placed around the bronz disk attached to the thread, forming a "compass" whose torsional ascillation period may be measured. (The valaue of I given on envelop X includes the effects of the
bronz disk.)

One magnet pair, centred in the hole in the wooden holder, can be used to influence the "compass" pair, possibly affecting its period and its angular equilibrium position. The langular position is best studied by placing the copper plate a few millimeters below the "compass" so as to provide electromagnetic damping. Do not write or mark on the copper plate.

More than one arrangement of the magnets may be employed. Draw clearly labelled diagrams showing each experimental setup used. Also write equations to show the manner in which diffierent observations may be combined to obtain the value $\mu_X$.

Keep all magnets in the same horizontal plane. Note for the main stand that the top knot can be rotated, and the thread length adjusted. The position of each shelf can also be adjusted.

## Practicl Details (Important )

1. Compass essembly and use: Hold one magnet from a given pair between the thumb and forefinger of one hand. Centre thebronz disk over one end. Then, carefully and without pulling on the thread, slowly bring in the second lmagnet. This forms the compass pair("X"or"A"). Also avoid pulling on the thread in taking the compass apart..

## Warning:

Rapid snapping of magnets or maagnetic pairs together can break the thread or chip the magnets. The tiny loop can be threaded again if thread breakage occurs.

2. Study the torsional mode of oscillation. To prevent excitation of the "pendulum" mode, a small assembly made of copper wire in L-shape is mounted on the lower shelf of the mainstand. Rotate this assembly so that the horizontal piece is up against the thread at a point about 2mm above where the threadis tied. With a slight additional rotation in the

same direction, move the wire a few mm further.

## Warning:

If this is not done, the two modes can be "coupled" causing a periodic variation in the amplitude of the torsional oscillations, and effecting their period.
Use the nail (See Fig. 24.24) to start the torsional oscillations in a controlled way.

3. Keep magnetic or magnetizable objects not required in the experiment stationary, and as far as possible from the experimental area. Conduct test for magnetic properties of particular objects such as nails, wristwatches,pens, supporting tables etc. The same precaution must be taken when the apparatus is shifted or set in a new area.

## Hints

1. The constant of torsional motion is small and can be neglected. ( This assumption is valid when the thread is long i.e. about 15 cm and more.)

2. The compass assembled for uses in this experiment is not exactly oreinted in the horizontal direction, as the earth's magnetic field consists of vertical as well as horizontal components. However the effects from this consideration is negligible and can be ignored. In our calculation, it will be asssumed that the compass lies along the horizontal direction.

3. Pospone estimation of errors in the first experiment, until the performance and calculation in the second experiment are concluded.

4. Any assumption about the value of the earth's magnetic field for the purpose of answering this problem is not allowed.

## Experiment 2.2

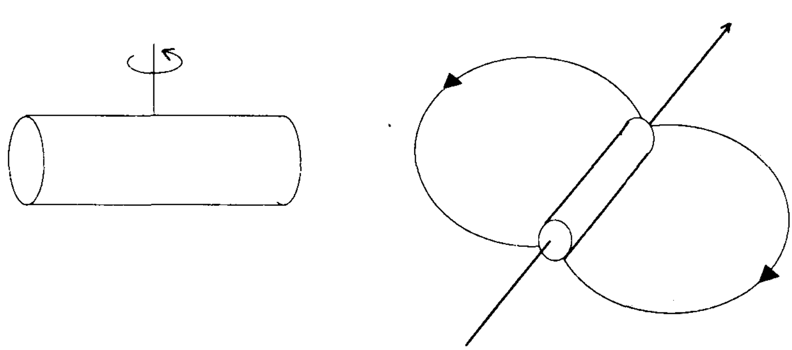


Fig. 24.25 Magnetic field of a cylindrical magnet.

An aluminum cylinder(in envelop B) consists of magnets placed along the line of the axis of cylidrical symmetry. Determine magnetic field $B_x$ along the axis of symmetry as a function of distance x measured from the mid-point of the aluminum cylinder.The range of x should be as wide as possible. The region for which the magnetic field is to be determined in indicated in Fig. 24.25, corresponding to the area next to the pole marked by a small black dot.

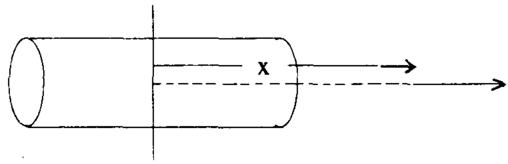



Fig. 24.26

In answering this problem, the experimenter is free to use the method by which the compass is allowed to be at rest in the equilibrium position or to oscillate and its periodic time is measured. In the first case, a copper plate provided should be used to provide damping and induce the compass to settle down in its equilibrium in a short time.

Plot a graph to demonstrate experimental results complete with bars to indicate erros on log-log graph provide.Compare the experimental results with the relation:

$$B(x) = C x^p$$

Determine p and give an estimation of its error.

## Warnings

In order to arrive at proper values of the physical quantities under consideration, more than one experimental method should be adopted.

Diagrams and experimental setup must be clearly indicated in the report.

$B_x$ must be expressd in terms of the variables measred in the experiment.

The curves in log-log graph paper must be accompanied by appropriate signs to indicate values measured by different methods.

# Solution

## Experimental 2.1

1. Suspend magnet A in the earth's magnetic field $B_H$. ( Follow the instruction in a careful manner)

Measure periodic time of the oscillation of magnet A in the earth's magnetic field. (This can be done by taking time reading from 20 to 50 oscillations and calculate periodic time. The experiment must be repeated several times for the purpose of obtaining averaged periodic time.)

From $$T_A = 2\pi\sqrt{\frac{I_A}{\mu_B \mu_H}}$$

where $T_A$ is period time of the oscillation of magnet A in the earth's magnetic field

$I_A$ is moment of inertia of magnet A.

$\mu_A$ magnetic moment of magnet A

$B_H$ the earth's magnetic field

Thus $$\mu_A B_H = 4\pi^2 \frac{I_A}{T_A^2} \quad (1)$$

2. Suspend magnet X in the earth's magnetic field $B_H$. Measure periodic time of the oscillation. Take the average of the readings.

Likewise $$\mu_X B_H = 4\pi^2 \frac{I_X}{T_X^2} \quad (2)$$

where $T_A$ is periodic time of the oscillation of magnet X in the earth's magnetic field $B_H$

$I_X$ moment of inertia of magnet X

$\mu_X$ magnetic moment of magnet X

$B_H$ the earth's magnetic field

3. Set up the apparatus as shown in Fig. 24.27

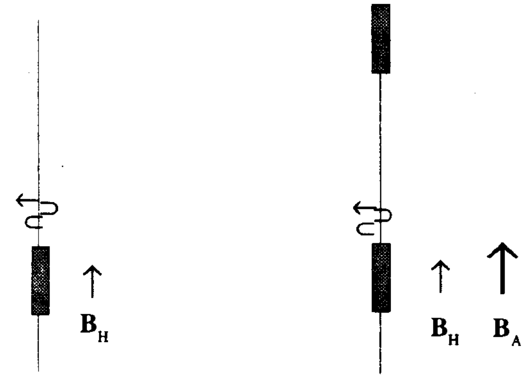



Fig. 24.27

Measure periodic time of the oscillation of magnet X in thecombined magnetic field of magnet A ($B_A$ and the earth ( $B_H$), with the directions of both fields point in the same direction.

Let distance measured from the midpoint of magnet A to the midpoint of magnet X along their axes be x.

Record periodic time T as a funtion of x

From
$$T = 2\pi\sqrt{\frac{I_X}{\mu_X(B_A + B_H)}}$$

where $B_A$ is the magnetic field of magnet A at distance x measured from the midpoint of magnet along its axis.

Thus
$$T^2 = 4\pi^2 \frac{I_X}{\mu_X\left(\frac{2\mu_A K}{x^3} + B_H\right)}$$

$$\frac{2\mu_A\mu_X K}{x^3} + \mu_X B_H = 4\pi^2 \frac{I_X}{T^2}$$

$$\frac{1}{T^2} = \left(\frac{2\mu_A\mu_X K}{4\pi^2 I_X}\right)\left(\frac{1}{x^3}\right) + \frac{\mu_X B_H}{4\pi^2 I_X}$$

Record the results in the table.

| x | $1/x^3$ | T | $T^2$ | $1/T^2$ |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

Plot a graph of $\frac{1}{T^2}$ as a function of $\frac{1}{x^3}$ .

The curve obtained is linear,with slope m given by,

$$m = \frac{2\mu_A \mu_X K}{4\pi^2 I_X} \tag{3}$$

From (1) $$\mu_A B_H = 4\pi^2 \frac{I_A}{T_A^2} \tag{4}$$

and from (2) $$\mu_X B_H = 4\pi^2 \frac{I_X}{T_X^2} \tag{5}$$

and from (3) $$\mu_A \mu_X = 4\pi^2 \frac{m I_X}{2K} \tag{6}$$

(4) + (5) $$\frac{\mu_A}{\mu_X} = \frac{T_X^2}{T_A^2} \tag{7}$$

( Note that $I_A = I_X = I$ )

(6) $\times$(7) $$\mu_A^2 = \frac{2\pi^2 m I}{K} \frac{T_X^2}{T_A^2}$$

$$\mu_A = \pi \sqrt{\frac{2mI}{K}} \frac{T_X}{T_A} \tag{8}$$

(6) + (7) $$\mu_X = \pi \sqrt{\frac{2mI}{K}} \frac{T_A}{T_X} \tag{9}$$

$T_A$ $T_X$ and m are known from the experiment while the value of K is provided by the problem.

Substitute the known values in (8) and calculate $\mu_X$.

## Experiment 2.2

Set up the apparatus as shown in Fig. 24.27

Place maganet B on the stand in the easrt-west direction.

Suspend magnet X in a position that its midpoint is along the axis of magnet B.

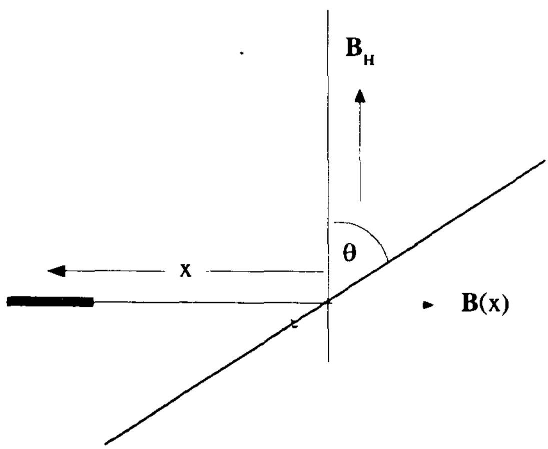



Fig. 24.28

In the eqilibrium position, magnet X makes angle θ with magnetic north-south direction.

and $$\frac{B(x)}{B_H} = \tan\theta$$

From the information provided by the problem

$$B(x) = c\,x^p$$

Hence $$\frac{cx^p}{B_H} = \tan\theta$$

$$p \log x + \log \frac{c}{B_H} = \log \tan\theta$$

For each value of x , calculate tan q from the values of the length of the right angle triangle. (This is done by inserting the copper plate underneath magnet X)

Record the results in the table below.

| tanθ | x |
|---|---|
| | |
| | |
| | |
| | |
| | |

Plot a graph of log tanθ as a function of log x on log-log graph paper.

Calculate the value of he slope to obtain p.

Estimation of errors of the anser in Experiment 2.1

The main source of error of $\mu_X$ is a systematic kind due to the error in the value of $B_A$ exerted on magnet X. The value of $B_A$ used is calculated at the midpoint of magnet X instead of at the respective ends of the magnet

$$\frac{\Delta\mu_X}{\mu_X} = \frac{\Delta\mu_A}{\mu_A} = \frac{3\Delta x}{x}$$

$\Delta x$ is the systematic error of x which is of the order of $\frac{l}{2}$ where l is the length of magnet X.

$$\frac{\Delta\mu_X}{\mu_X} = \frac{3}{2}\frac{l}{x}$$

Error in $\mu_X$ is small when x is large and vice versa.

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# 1993
# Beijing, People's Republic of China

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## Theory

### Problem 1. Relativistic Particle

In the theory of special relativity the relation between energy E and momentum p of a free particle with rest mass $m_0$ is

$$E = \sqrt{p^2c^2 + m_0^2c^4} = mc^2$$

when such a particle is subject to a conservative force, the total energy of the particle , which is the sum of E and the potential energy, is conserved. If the energy of particle is very high, the rest mass energy of the particle can be ignored (such a particle is called ultra-relativistic particle).

1.1 Consider the one dimensional motion of a very high energy particle(rest mass energy can be neglected) subject to an attractive central force of constant magnitude f.

Suppose the particle is located at the centre of force with initial momentum $p_0$ at time t = 0. Describe the motion of the particle by separately plotting momentum p against space coordinates x and x against time, for at least one period of motion; specify the coordinates of the turning points in terms of given parameters $p_0$ and f and indicate with arrows the direction of the progress of the motion in (p, x) diagram.

1.2. A meson is a particle made up of two quarks. The rest mass M of the meson is equal to the total energy of the two-quark system divided by $c^2$.

Consider a one-dimensional model for a rest meson, in which the two quarks are assumed to move along the x-axis and attract each othe with a force of constant magnitude f; it is assumed they can pass through each other freely. For analysis of the high energy motion of the quarks the rest mass of the quarks can be neglected.

At time t = 0 the two quarks are both at x = 0. Show separately the motion of the two quarks graphically by (x,t) diagram and (p,x) diagram, specify the coordinates of the turning points in terms of M and f, indicate the direction of the process in (p,x) diagram and determine the maximum distance between the two quarks.

1.3 The reference frame used in Part 1.2 will be referred to as frame S ; the Laboratory frame referred to as S′ which moves in the negative x direction with velocity v = 0.6 c. The coordinates in the two reference frames are so chosen that the point x = 0 in S coincides with the point x′ = 0 at time t = t′ = 0.

Plot the motion of the two quarks graphically in a (x′, t′) diagram, specify the coordinates the turning points in terms of M, f and c, and determine the maximum distance between the two quarks observed in Laboratory frame S′.

1.4 For a meson with rest-mass energy $Mc^2$ =140 MeV and velocity 0.60c relative to the Laboratory frame S′, determine its energy E′ in the Laboratory frame S′.

# Solution

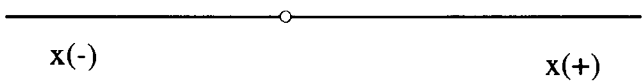



**Fig.** 25.1

1.1 Consider motion of the particle to the right along x-axis. Since f is a conservative force with spherical symmetry, potential energy U(x) is a function of x only.

$$-f = -\frac{du}{dx} \quad \text{when x is +, and } U(x) = +fx$$

$$+f = -\frac{du}{dx} \quad \text{when x is − , and } U(x) = -fx$$

From the symmetrical property of the potential field, potential value on the left and on the right with same magnitude of x ,are the same only if U is a function of $|x|$ only.
(For +values of x , x = $|x|$, and -values of x , -x = $|x|$ )

Total energy $$E = f|x| + \sqrt{p^2c^2 + m_0^2c^4} = \text{constant}$$

At the beginning, the particle has momentum $p_0$ and $p_0$ >>> $m_0c^2$

Hence $$f|x| + \sqrt{p^2c^2} = p_0c_0$$

$$f|p| + |p|c = p_0c_0$$

Consider the following cases:

| | | | | | |
|---|---|---|---|---|---|
| $x > 0$ | $p > 0$ | $x = \lvert x\rvert$, | $p = \lvert p\rvert$ | $fx + pc = p_0c$ | (1) |
| $x > 0$ | $p < 0$ | $x = \lvert x\rvert$, | $-p = \lvert p\rvert$ | $fx - pc = p_0c$ | (2) |
| $x < 0$ | $p < 0$ | $-x = \lvert x\rvert$, | $-p = \lvert p\rvert$ | $-fx - pc = p_0c$ | (3) |
| $x > 0$ | $p < 0$ | $x = \lvert x\rvert$, | $-p = \lvert p\rvert$ | $-fx + pc = p_0c$ | (4) |

Let L represent the magnitude of maximum displacement measured from O.

From (1) $\quad fL + \sqrt{0 + m_0^2c^4} = p_0c$

$$fL = p_0c - m_oc^2$$

Since $p_0c >> m_oc^2$ $\qquad L = \frac{p_0c}{f}$

Let T be time taken by the particle to move over distance L.

$$-\int_0^T fdt = -p_0$$

$$fT = p_0$$

$$T = \frac{p_0}{f}$$

Average magnitude of the velocity of the particle $= \frac{L}{T} = \frac{p_0c}{f} \Big/ \frac{p_0}{f} = c$

This impies that the particle travels with the velocity of light almost all the way and suddenly braked. This situation can be realized when the rest-mass is almost zero.

Coordinates of the turning point on the right is $x = \frac{p_0c}{f}$, $p = 0$ and $t = \frac{p_0}{f}$

From Newton 's second law of motion

$$\frac{dp}{dt} = \begin{cases} -f & x > 0 \\ +f & x < 0 \end{cases}$$

$$p = \begin{cases} -ft + k_1 & x > 0 \\ +ft + k_2 & x < 0 \end{cases}$$

$p = 0$ when $t = 0$.
Sustitute these values in equation $p = -ft + k_1$
to obtain $k_1 = p_0$ and $\qquad p = -ft + p_0$ when $x > 0$

Substitute $p = - ft + p_0$

in (1) to obtain $fx + ( - ft + p_0)c = p_0c$ , $x = ct$

in (2) $fx - ( -ft + p_0)c = p_0c$ , $x = \frac{2pc_0}{f} - ct = 2L - ct$

When $x < 0$, substitute $p = -p_0$ , $t = 2T$ in equation $p = +ft + k_2$

to get $-p_0 = 2fT + k_2$, $k_2 = -p_0 - 2fT$

and arrive at $p = f.(t-2T) - p_0$

Substitute $p = f(t-2T) - p_0$

in (3 )to obtain $- fx - f(t- 2T)c + p_0c = p_0c$, $x = -ct + 2Tc = - ct + 2L$

in (4) $- fx + f(t- 2T)c - p_0c = p_0c$, $x = ct - 2Tc - \frac{2pc_0}{f} = - ct - 4L$

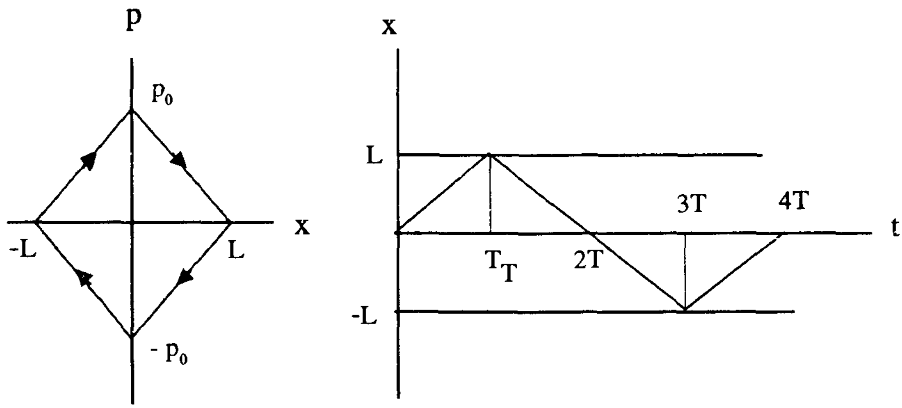



**Fig.** 25.2

1.2 Total energy and potential energy of the system of two quarks is

$$\sqrt{p_1^2c^2 + m_0^2c^4} + \sqrt{p_2^2c^2 + m_0^2c^4} + f|x_1 - x_2| = Mc^2$$

where $p_1$ and $x_1$ are momentum and displacement of the first quark

$p_2$ and $x_2$ momentum and displacement of the second quark

$m_0$ rest mass of each quark

M rest mass of meson

Hence $p_1c + p_2c + f|x_1 - x_2| = Mc^2$

In Centre- of-Mass Frame

$$p_1 = -p_2 \text{ and } x_1 = -x_2$$

The two quarks have maximum momenta when they pass each other at $x = 0$.

**Let** $p_0$ be maximum value of momenta

$x_0$ maximum displacement of each quark

$$|p_1|_{MAX} = |p_2|_{MAX} = p_0$$
$$|x_1|_{MAX} = |x_2|_{MAX} = x_0$$

Substitute these values in (1) $\quad 2p_0c = Mc^2$ and $\quad p_0 = \dfrac{Mc}{2}$

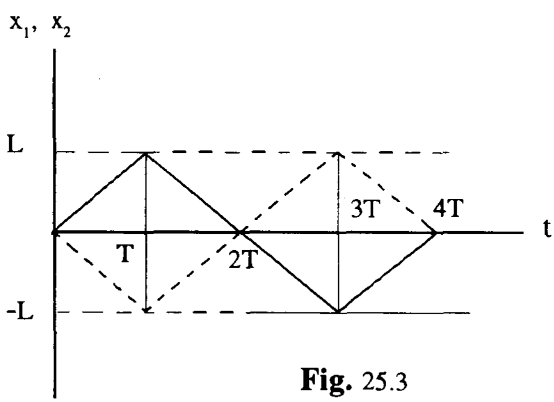
From the discussion in 1.1, maximum displacement of each quark is

$$L = \frac{p_0 c}{f} = \frac{Mc^2}{2f}$$

and maximum distance that the two quarks are from each other is 2L

$$2L = \frac{2p_0 c}{f} = \frac{Mc^2}{f}$$

Time taken by each quark to travel distance L is $\quad T = \dfrac{Mc}{2f}$



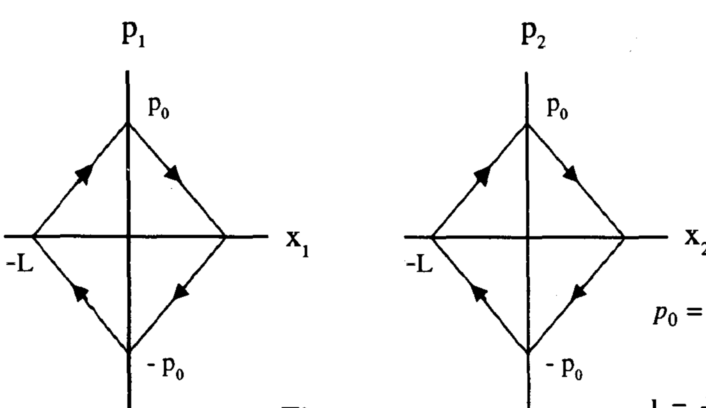

$x_1$ continuous line

$x_2$ broken line

$$T = \frac{Mc}{2f}$$

$$L = \frac{Mc^2}{2f}$$

**Fig.** 25.3




$$p_0 = \frac{Mc}{2}$$

$$L = \frac{Mc^2}{2f}$$

**Fig.**25.4

1.3 $v = 0.6c, \quad \beta = \frac{3}{5}$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{5}{4}$$

Lorentz transformation gives $x' = \frac{5}{4}(x+0.6ct) = \frac{5}{4}x + \frac{3}{4}ct$

$$t' = \frac{5}{4}\left(t + 0.6\frac{x}{c}\right) = \frac{5}{4}t + \frac{3}{4}\frac{x}{c}$$

For the first quark:

| S | | S′ | |
|---|---|---|---|
| $x_1$ | $t_1$ | $x_1'$ | $t_1'$ |
| 0 | 0 | 0 | 0 |
| L | $T = \frac{L}{c}$ | $\frac{5}{4}L + \frac{3}{4}c \times \frac{L}{c} = 2L,$ | $\frac{5}{4}\frac{L}{c} + \frac{3}{4}\frac{L}{c} = \frac{2L}{c} = 2T$ |
| 0 | $2T = \frac{2L}{c}$ | $\frac{3}{4} \times \frac{2L}{c} = \frac{3}{2}L,$ | $\frac{5}{4} \times \frac{2L}{c} = \frac{5}{2}\frac{L}{c} = \frac{5}{2}T$ |
| -L | $3T = \frac{3L}{c}$ | $-\frac{5}{4}L + \frac{3}{4}c \times \frac{3L}{c} = L$ | $\frac{5}{4} \times \frac{3L}{c} + \frac{3}{4}\left(-\frac{L}{c}\right) = \frac{3L}{c} = 3T$ |

For the second quark:

| S | | S′ | |
|---|---|---|---|
| $x_2$ | $t_2$ | $x_2'$ | $t_2'$ |
| 0 | 0 | 0 | 0 |
| -L | $T = \frac{L}{c}$ | $-\frac{5L}{4} + \frac{3L}{4} = -\frac{L}{2},$ | $\frac{5}{4} \times \frac{L}{c} - \frac{3}{4} \times \frac{L}{c} = \frac{L}{2c} = \frac{T}{2}$ |
| 0 | $2T = \frac{2L}{c}$ | $0 + \frac{3}{4} \times 2L = \frac{3L}{2}$ | $\frac{5}{4} \times \frac{2L}{c} = \frac{5}{2}\frac{L}{c} = \frac{5}{2}T$ |
| L | $3T = \frac{3L}{c}$ | $\frac{5L}{4} + \frac{3}{4} \times \frac{3L}{c} = \frac{7L}{2}$ | $\frac{5}{4} \times \frac{3L}{c} + \frac{3}{4}\left(-\frac{L}{c}\right) = \frac{3L}{c} = 3T$ |

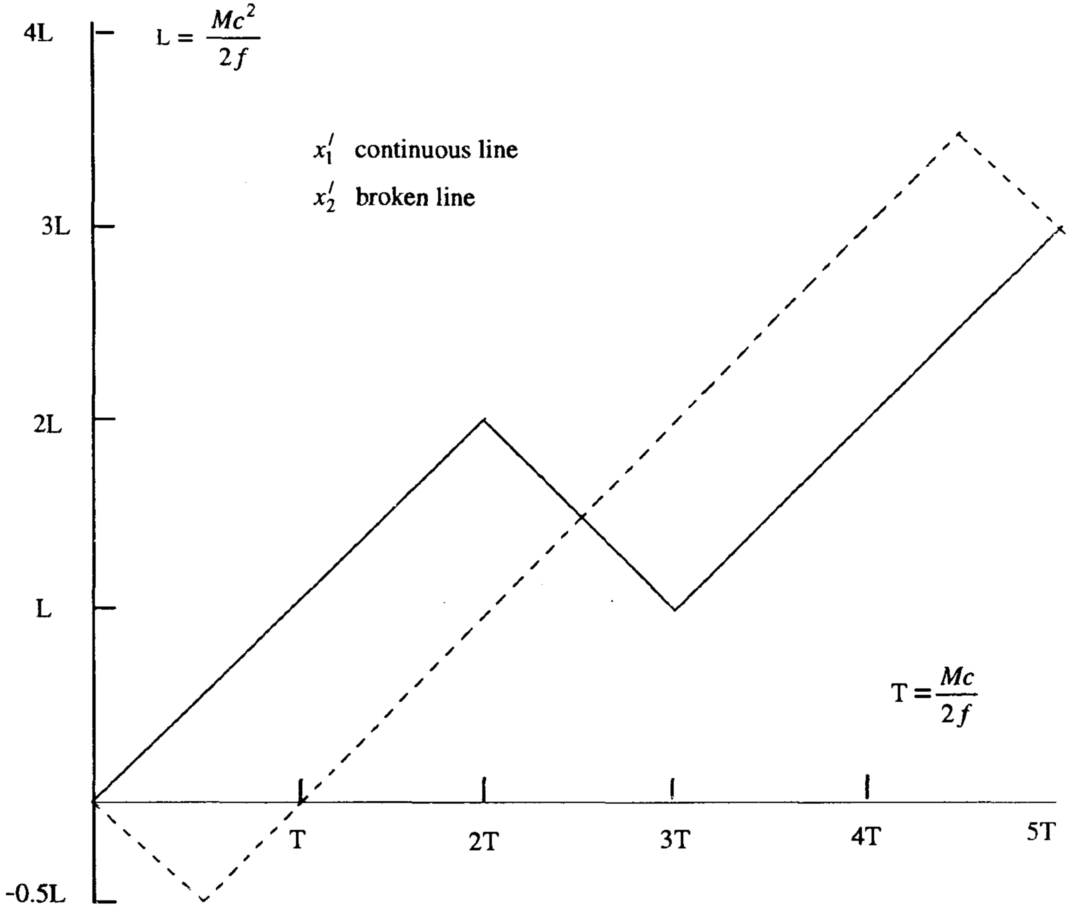



**Fig.**25.5

1.4 Meson travels with velocity 0.64c measured in the Laboratory frame of reference, hence total energy is

$$E = \sqrt{p^2c^2 + M^2c^4}$$

$$= \sqrt{\frac{M^2v^2c^2}{1 - v^2/c^2} + M^2c^4}$$

$$= \sqrt{\frac{0.36}{0.64} + 1}\, Mc^2$$

$$= \frac{5}{4} \times 140 = 175 \text{ MeV} \quad \textbf{Ans}$$

## Problem 2. Superconducting Magnet

Superconducting magnets are widely used in laboratories. The most common form of superconducting magnet is a solenoid of superconducting wire. The wonderful thing about superconducting magnet is that it produces high magnetic fields without any energy dissipation due to Joule heating, since the electric resistance of the superconducting wire becomes zero when the magnet is immersed in liquid helium at a temperature of 4.2 K Usually, th magnet is provided with a specially designed superconducting switch, as shown in Fig 25.6.

The resistance r of the switch can be controlled: either $r = 0$ in the superconducting state, or $r = r_n$ in the normal state. When the resistance is in the superconducting state, the magnet may be operated in the persistent statewith a current circulatingthrough the magnet and superconducting switch indefinitely. The persistent mode allows a very steady magnetic field to be maintained for long periods with external source cut off.

The details of the superconducting switch are not given in Fig 25.6 . It is usually wrapped with a heater wire lsusitably thermally insulated from the liquid lhelium bath. On being lheated, the temperature of the superconducting wire increases and it reverts to the resistive normal state. The typical value of $r_n$ is a few ohms, here say 5 Ω. The inductance of a supermagnet depends on its size, let it be 10 H for the magnet in Fig 25.6. The total current I can be changed by adjusting the resistance R.

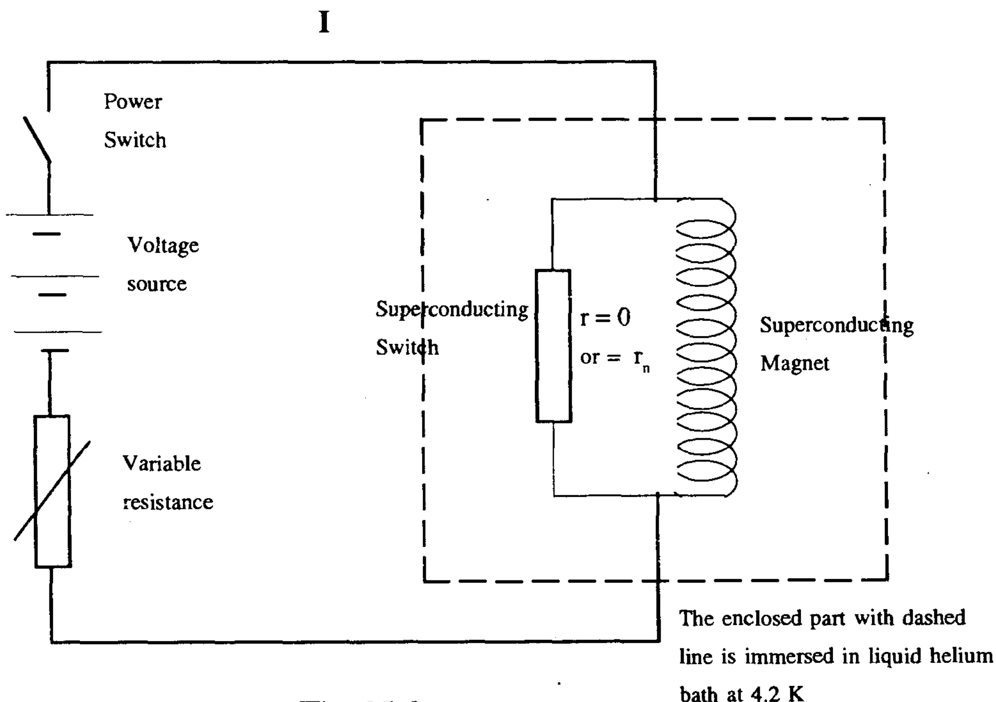



Fig. 25.6

2.1 The total current I and resistance r of the superconducting switch are controlled to vary with time in the way shown in Fig. 25.7a and b respectively, and suppose the current $I_1$ and $I_2$ flowing through the magnet and switch respectively are equal at the beginning (Fig. 25.7 c and d), how do they vary with time from $t_1$ to $t_4$? Plot the answers in Fig. c and d.

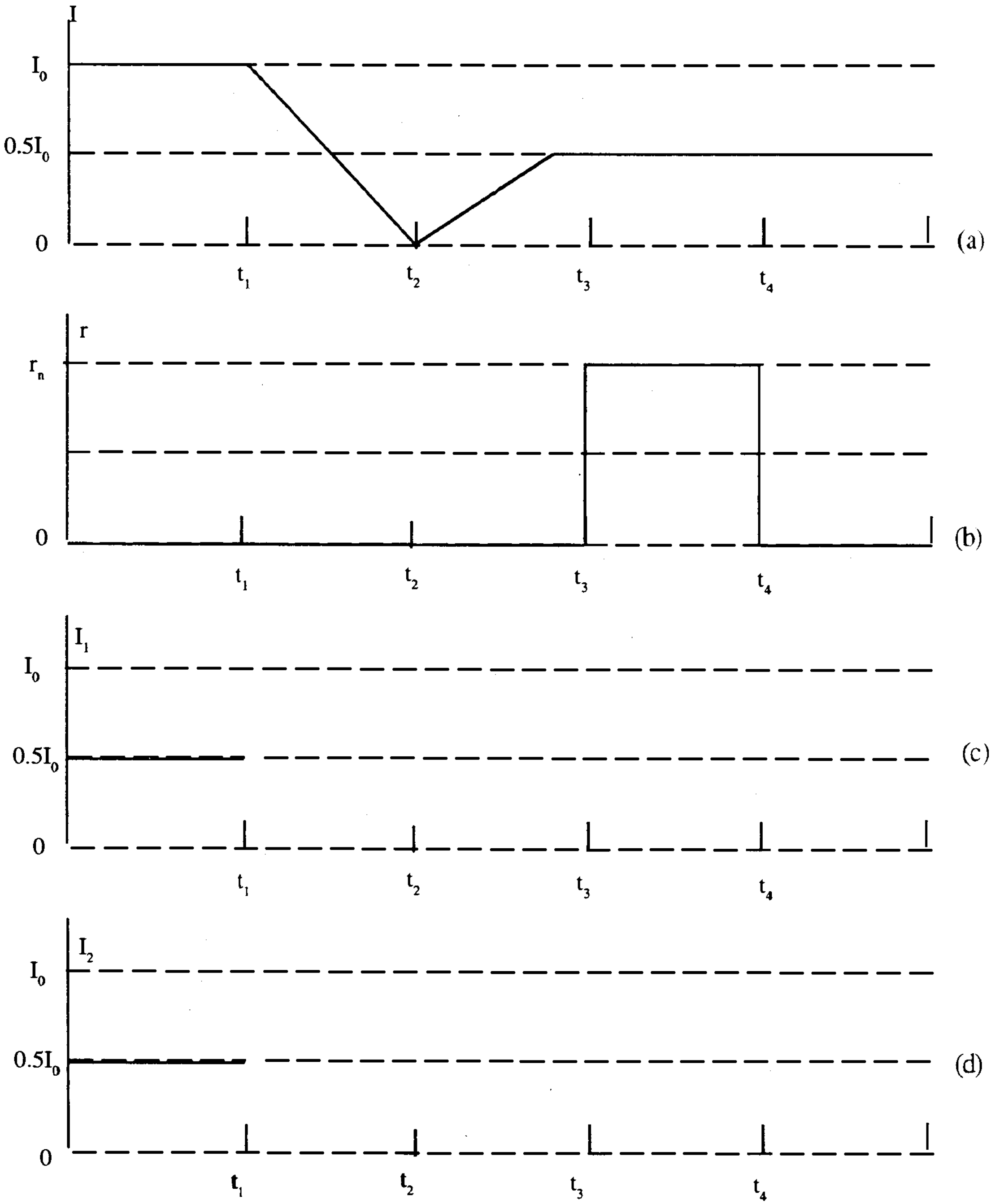



**Fig 25.7**

2.2 Suppose the power switch is turned on at time $t = 0$ when $r = 0$, $I_1 = 0$ and $R = 7.5$ W, and the total current I is 0.5 A. With K kept closed, the resistance r of the superconducting switch is varied in the way shown in Fig. 25.8. Plot the corresponding time dependence of I, $I_1$, and $I_2$ in Fig 25.8 c and d respectively.

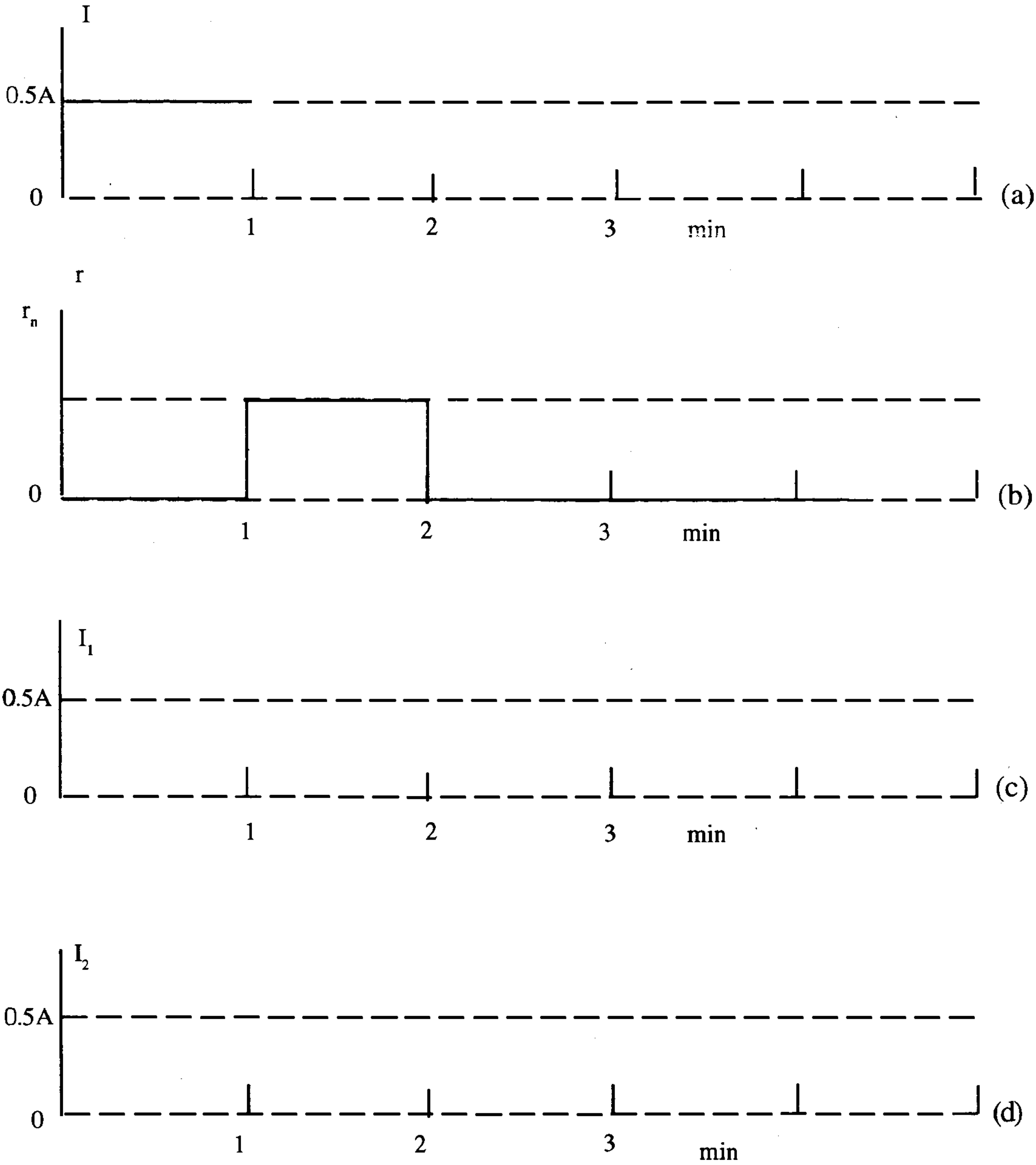



**Fig.** 25.8

2.3 Only small currents, less than 0.5 A are allowed to flow through the superconducting switch when it is in the normal state. With currents larger than 0.5 A, the superconducting switch will be burned out. Suppose the superconducting magnet is operating in the persistent mode i.e. $I = 0$ and $I_1 = i_1 = 20$ A, $I_2 = -i_1$ from $t = 0$ to $t = 3$ minutes as shown in Fig 25.9. If we want to stop the experiment and make the current through the superconducting magnet to become zero, how would you do it ? This has to be done in several steps. Answer by plotting the corresponding changes of I, r, $I_1$ and $I_2$ in Fig. 25.9.

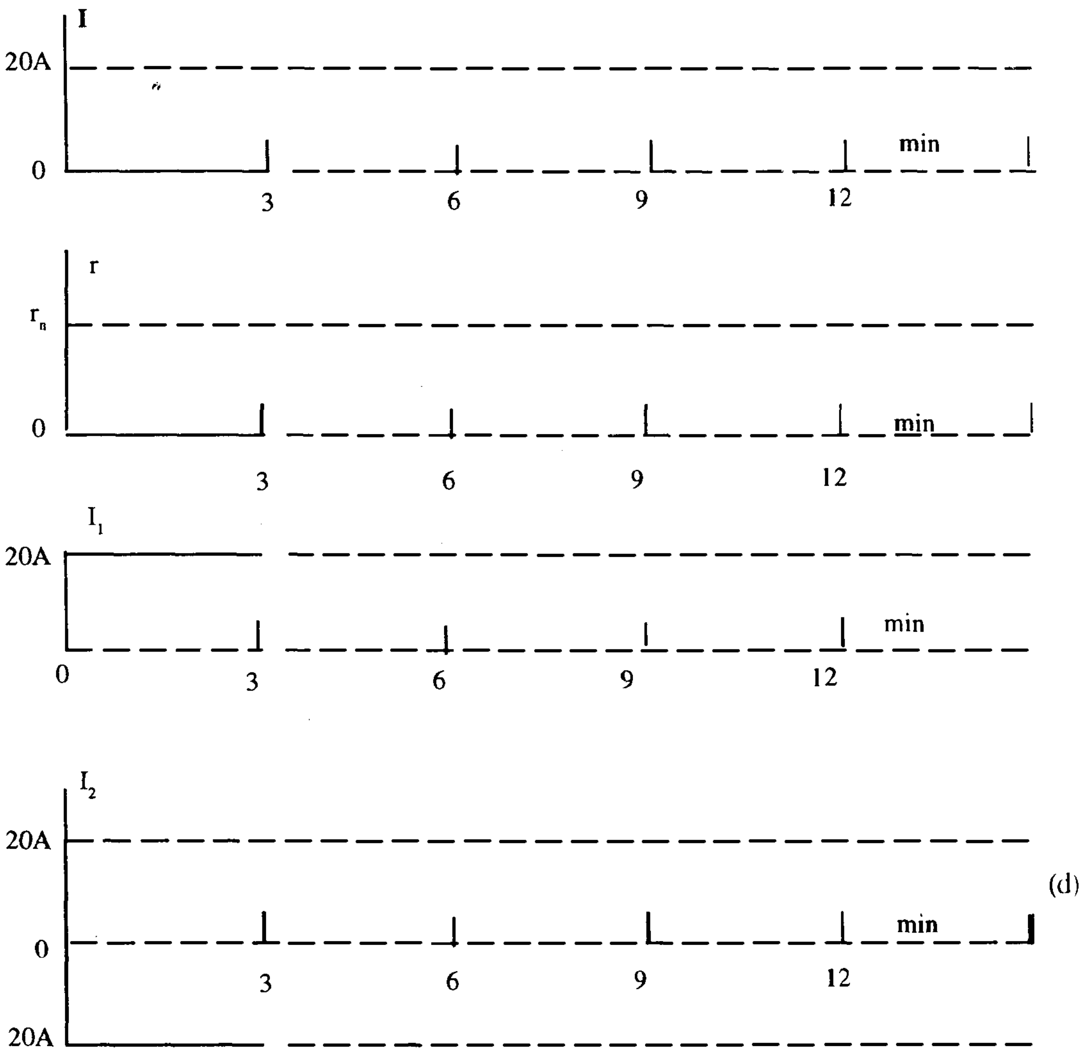



**Fig.** 25.9

2.4 Suppose the magnet is operated in a persistent mode with a persistent current of 20 A ( t= 0 t = 3 min. See Fig 25.10). How would you change it to a persistent mode with a current of 30 A. Plot your answer in Fig. 25.10

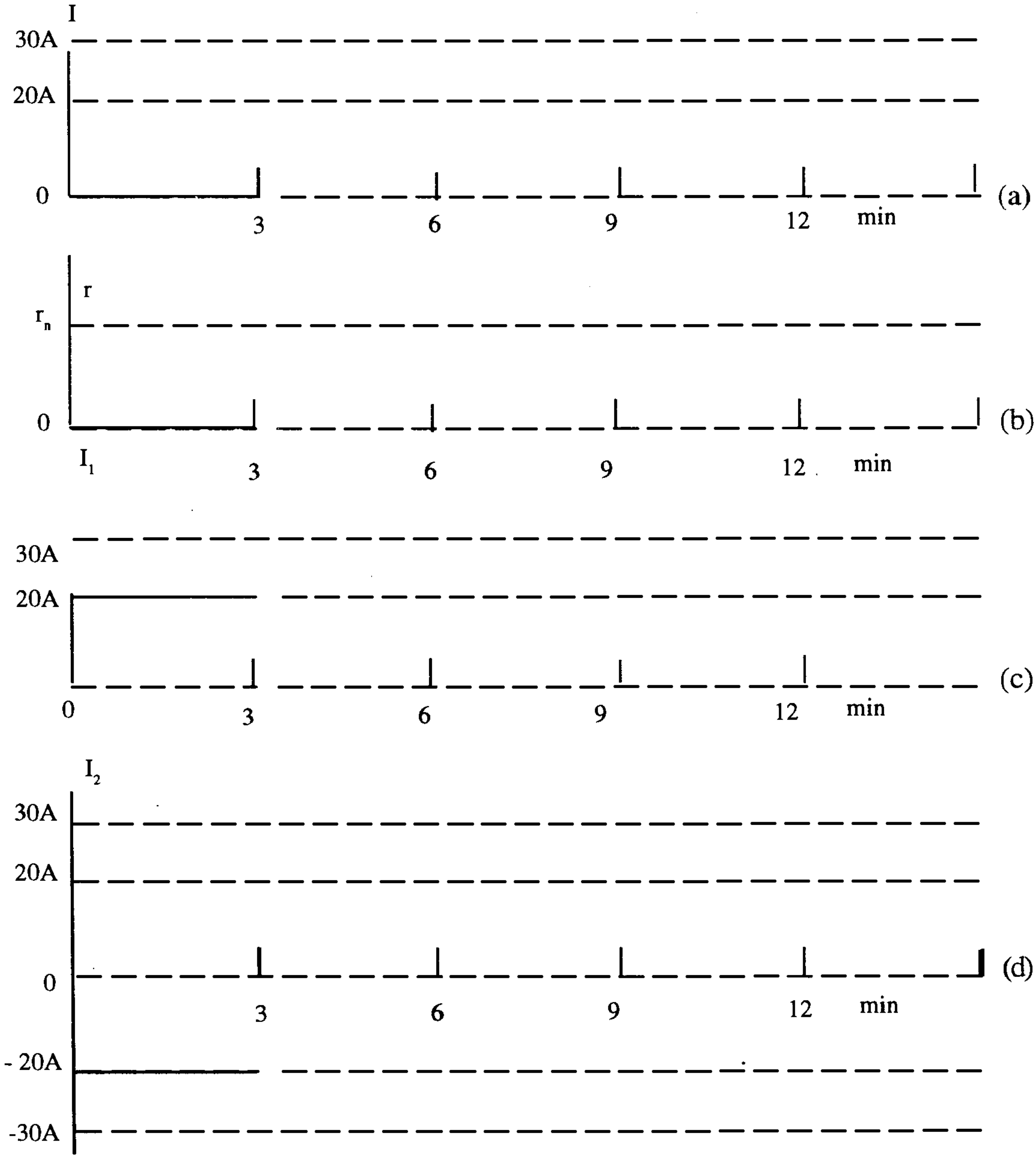



**Fig** .25.10

## Solution

2.1 From $t = t_1$ to $t = t_3$

Apply Kirchhoff's law in the circuit loop immersed in liquid helium:

$$L\frac{dI_1}{dt} + 0 = 0$$

$$I_1 = I_1(t_1) = \frac{1}{2}I_0;$$

$$I_2 = I - I_1 = I - \frac{1}{2}I_0;$$

And from $t = t_3$ to $t = t_4$

Since $I_2 = 0$ at $t = t_3$. and I is maintained at constant value of $\frac{1}{2}I_0$ after $t_3$, hence voltage across the superconducting magnet $V_M = I_2 r_n = 0$, and $I_1$ as well as $I_2$ do not change.

$$I_1 = \frac{1}{2}I_0 \text{ and } I_2 = 0$$

These changes are demonstrated in Fig 25.11

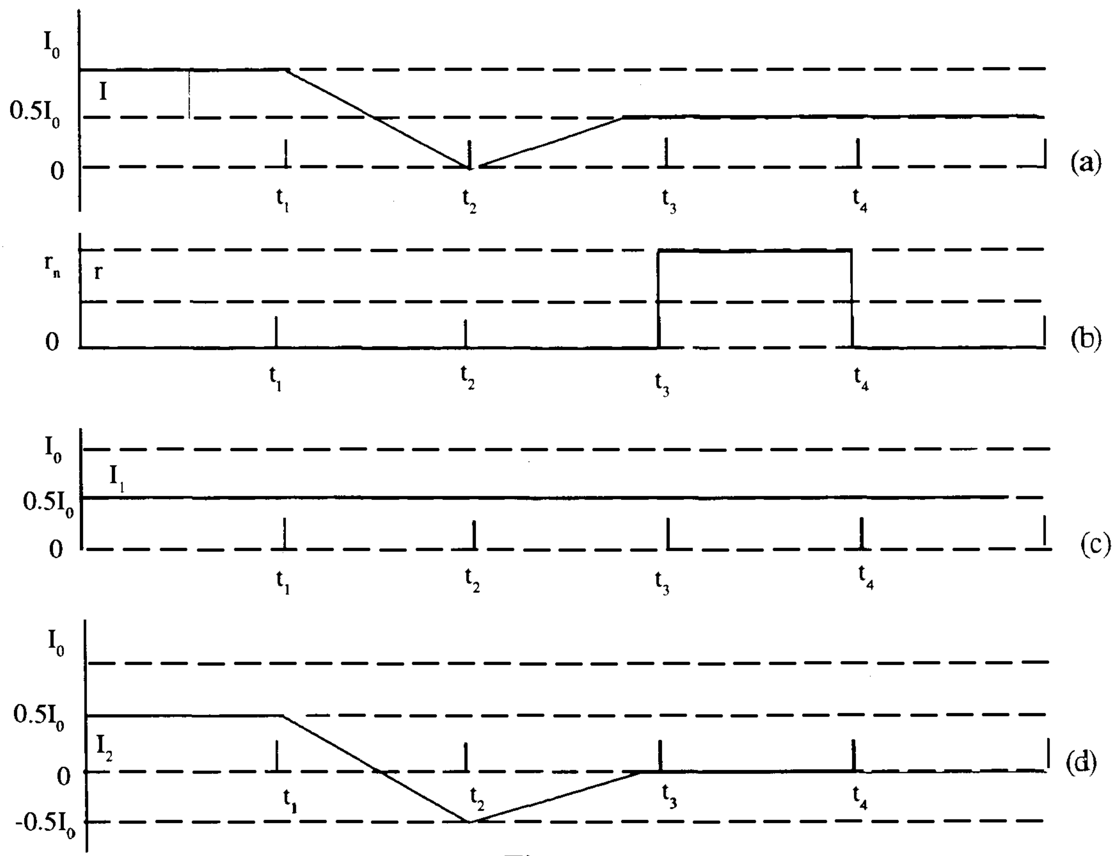



**Fig.** 25.11

2.2 From t = 0 to t = 1 s,

since $r = 0$, $V_M = L\frac{dI_1}{dt} = 0$

$I_1 =$ c where c is a constant

$I_1 = I_1(0) = 0$

$I_2 = I - I_1$

At t = 1 s, r increases from 0 to $r_n$, current I abruptly descreases from E/R to $E/(R+r_n)$ since $I_1$ cannot change suddenly due to inductance.

$$\frac{E}{R} = 0.5A;\ R = 7.5\ \Omega,\ rn = 5\ \Omega$$

Current I decreases to $\frac{E}{(R+r_n)} = \frac{3.75V}{122.5A} = 0.3A$

From t = 1 s to = 2 s; current I, $I_1$, and $I_2$ assume steady values i.e.

I = E/R = 0.5 A,

$I_1 = I = 0.5$ A

$I_2 = 0$ A

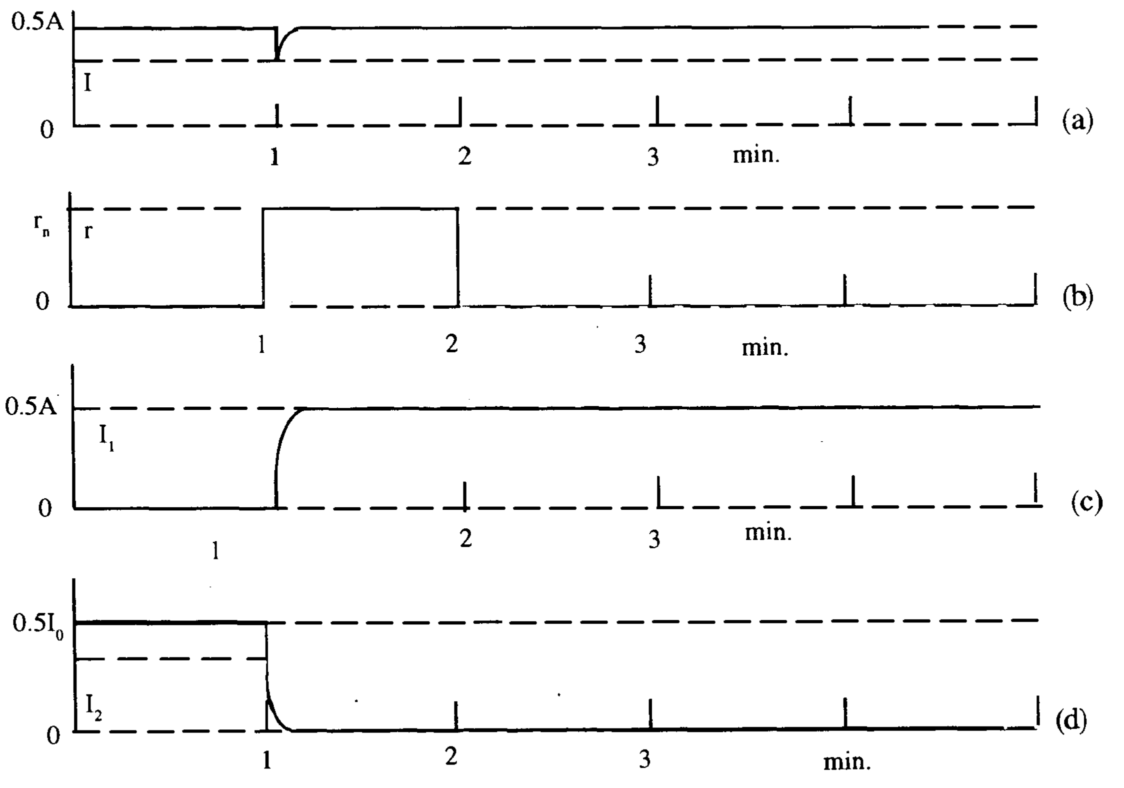



**Fig.** 25.12

2.3

**Step 1.**

Turn on switch K, and slowly increase the total current I to the same value with $I_1$ i. e. 20 A by adjusting external resistance R.

Since $$V_M = L\frac{dI_1}{dt} = 0,$$

$I_1$ therefore remains at 20 A, while $I_2$ increases from - 20 A to 0.

**Step 2**

Adjust resistance of superconducting switch ,so that its value increases from 0 to $r_n$.

**Step 3**

Slowly decrease I until it reaches 0, at the same time keep $I_2$ smaller than 0.5 A.

Since $$I_2 = \frac{V_M}{r_n}$$

$$V_M = L\frac{dI_1}{dt} \ : \ L = 10\ H, rn = 5\ \Omega$$

The situation in which $I_2 < 0.5$ A implies that

$$10\frac{dI_1}{dt} < 0.5 \times 5$$

$$\frac{dI_1}{dt} < 0.25$$

and $I_1$ drops less tha 15 A in 1 minute, and $\frac{dI}{dt} \sim 0.1 A/s$ while $\frac{dI_1}{dt}$ is close to being 0.

**Step 4**

Make resistance of the superconducting switch zero,when VM = 0 then turn off the power switch K.

The results are depicted in Fig. 25. 13

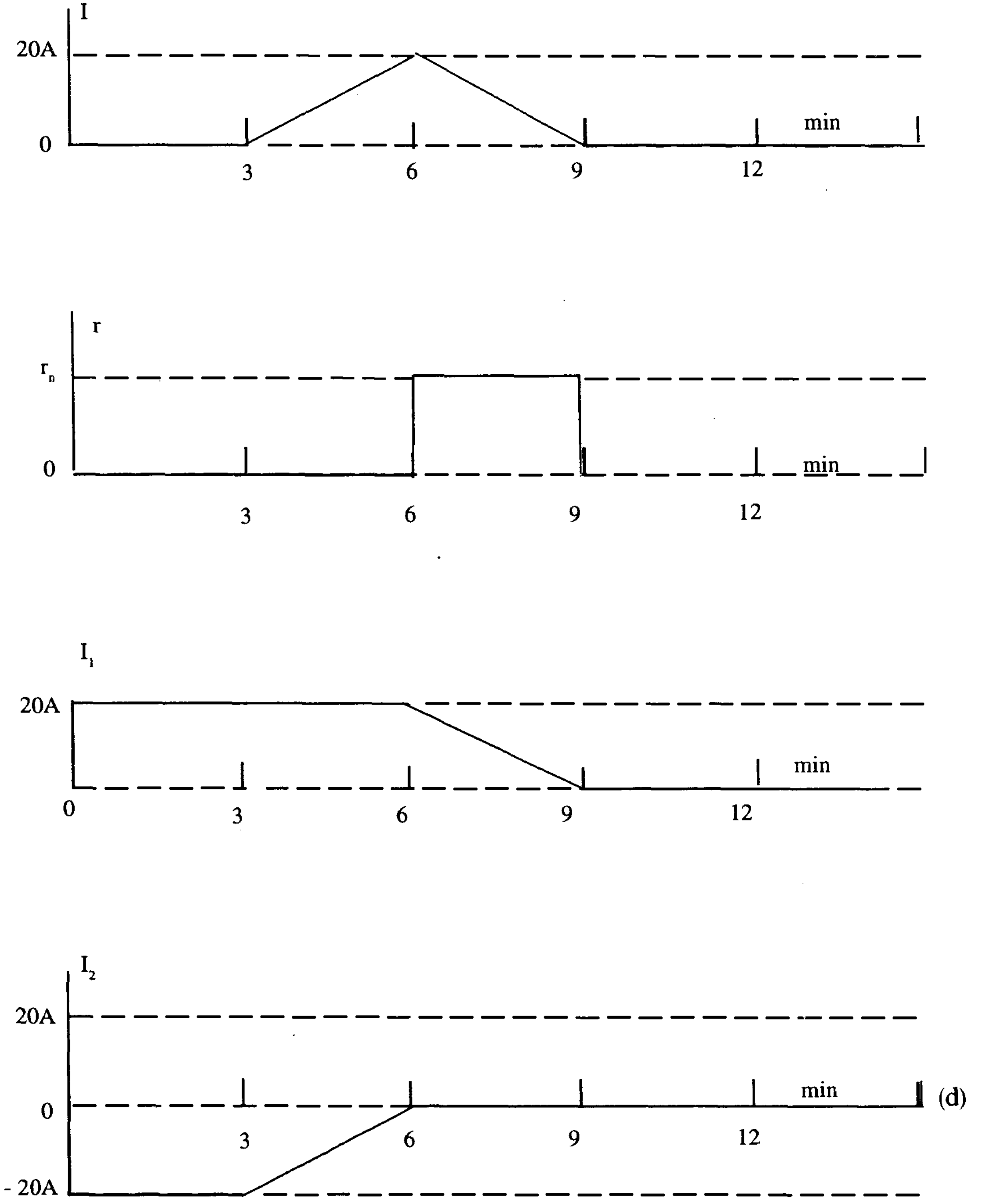

I
20A
0
3
6
9
12
min
r
$r_n$
0
3
6
9
12
min
$I_1$
20A
0
3
6
9
12
min
$I_2$
20A
0
- 20A
3
6
9
12
min
(d)

**Fig.** 25.13

2.4

**Step 1.** As in Step 1 of 2.3

**Step 2.** As in Step 2 of 2.3

**Step 3.** Increase I from from 10 A to 30 A to make $I_2 < 0.5A$

**Step 4.** Adjust switch r to make $r = 0$ when $V_M = 0$

**Step 5** Decrease I to 0 A, thus $I_1$ remains at 30 A, as $V_M = 0$, while $I_2$ which is equal to $I - I_1$ becomes - 30 A.

**Step 6** Turn off power switch K. The magnet is now in a persistent mode.

The results are demonstrated in Fig 25.14

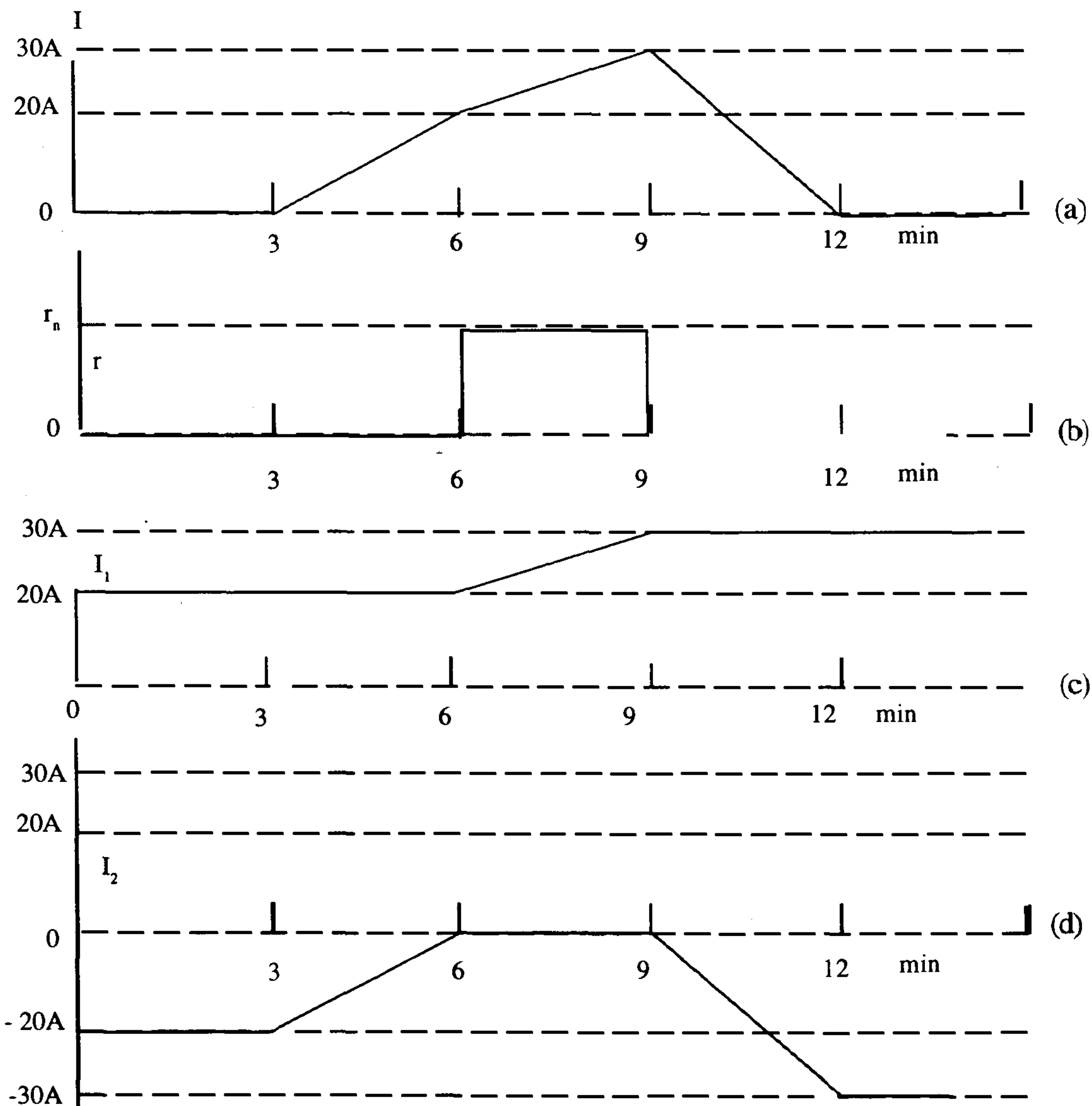



**Fig.** 25.14

# Problem 3. Collision of balls with surface friction

A uniform ball A of mass $m_A$ and radius $R_A$ moves translationally in the x-y plane in the x direction at a distance b above the x-axis with velocity V. It collides with a stationary ball B of $m_B$ and radius $R_B$ at the origin of the coordinates system. During the collision, at their point of contact, their surface friction is assume to be large enough to ensure that the tangential velocities of the two balls are the same after collision, while along the normal the collision is elastic.

3.1 Write down the condition for such a collision to occur.

3.2. For such a collision , determine the x and y components of the velocities of the two balls after the collision, i.e. $V_{Ax}'$, $V_{Ay}'$, $V_{Bx}'$ aand $V_{By}'$ in terms of $m_A$, $m_B$, $R_A$, $R_B$, V and b.

3.3 Also determine the kinetic energies of ball $E_A'$ for ball A, $E_B'$ for ball B after collision in terms of $m_A$, $m_B$, $R_A$, $R_B$, V and b.

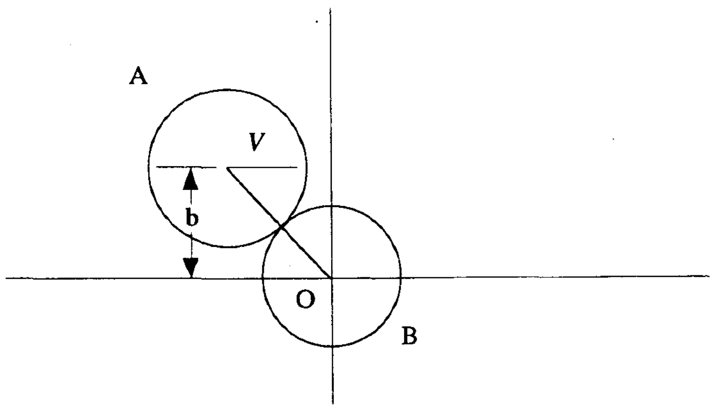



**Fig.** 25.15

# Solution

3.1. The condition for collision to take palce is

$$b \leq R_A + R_B$$

3.2 The problem is relatively easier to solve in the centre-of mass system in which the centre of mass of the system is at rest.

Let $\mathbf{V}_A$ and $\mathbf{V}_B$ are velocities of $m_A$ and $m_B$ in the centre- of -mass system.

Thus in the centre of mass system

$$m_A\mathbf{v}_A + m_B\mathbf{v}_B = 0$$
$$\mathbf{v}_A - \mathbf{v}_B = \mathbf{V}$$

where $\mathbf{V}$ is the velocity of $m_A$ relative to $m_B$.

It follows
$$\mathbf{v}_A = \frac{m_B\mathbf{V}}{m_A + m_B}$$
$$\mathbf{v}_B = -\frac{m_A\mathbf{V}}{m_A + m_B}$$

With the unit vector $\mathbf{1}_x$ removed
$$v_A = \frac{m_B V}{m_A + m_B}$$
$$v_B = -\frac{m_A V}{m_A + m_B}$$

The negative sign means $m_B$ moving towards the left hand side.

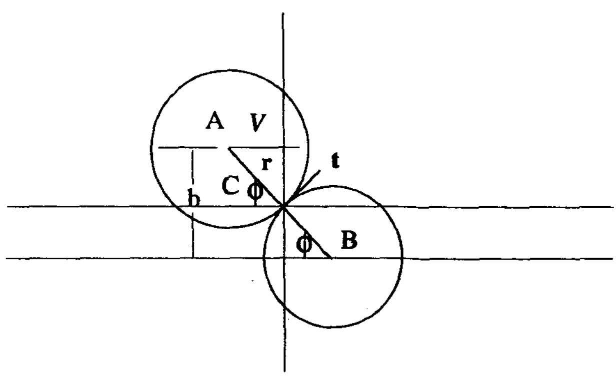



**Fig.** 25.16

Let

$v'_{Ar}$, $v'_{Br}$ be magnitude of velocities of $m_A$ and $m_B$ along the radial after the collision and $v'_{At}$, $v'_{Bt}$ magnitude of velocities of $m_A$ and $m_B$ along the tangent after the collision.

Along the radial the collision is elastic,

$$v'_{Ar} = -V_A\cos\phi = -\frac{m_B V}{m_A + m_B}\cos\phi$$

$$v'_{Br} = -V_B\cos\phi = -\frac{m_B V}{m_A + m_B}\cos\phi$$

where $\phi$ is the angle between the line joining the centres of $m_A$ and $m_B$ makes with the horizontal line.

Consider motion along the tangent afterthe collision.
Let $\mathbf{f}_A$ be friction which slows down motion of $m_A$ along the tangent and at the same time provides torque for spinning motion of $m_A$.
Likewise $\mathbf{f}_B$, friction on $m_B$.

$$\mathbf{f}_A = -\mathbf{f}_B$$

and $$|\mathbf{f}_A| = |\mathbf{f}_B| = f$$

Impulse along $\mathbf{1}_t$ on $m_A$ $\quad f\Delta t = m_A V_A \sin\phi - m_A V'_{At}$

along $-\mathbf{1}_t$ on $m_B$ $\quad f\Delta t = m_B V_B \sin\phi - m_B V'_{Bt}$

Equation of rotational motion of $m_A$ and $m_B$ about their respective centres

for $m_A$ $$fR_A = I_A \frac{(\omega_A - 0)}{\Delta t}$$

for $m_B$ $$fR_B = I_B \frac{(\omega_B - 0)}{\Delta t}$$

Where $I_A$ $I_B$ are moment of inertia of $M_A$ and $m_B$ about their centres of mass
and $\omega_A$ $\omega_B$ are angular velocities after the collision.

From (3) $$f\Delta t = \frac{2}{5} m_A R_A \omega_A$$

$$f\Delta t = \frac{2}{5} m_B R_B \omega_B$$

Soon after the collision, the point of contact of mA and mB must have the same velocity.

Take note of the direction of the velocities:

$$V'_{At} - \omega_A R_A = \omega_B R_B - V'_{Bt}$$

Substitute $V'_{At}, V'_{Bt}, \omega_A$ and $\omega_B$ from equations (3), (4),(5) and (6) in (7) to arrive at

$$-\frac{f\Delta t}{m_A} + V_A \sin\phi - \frac{5}{2}\frac{f\Delta t}{m_A} = \frac{5}{2}\frac{f\Delta t}{m_B} + \frac{f\Delta t}{m_B} - V_B \sin\phi$$

$$f\Delta t\left(-\frac{1}{m_A} - \frac{5}{2m_A} - \frac{5}{2m_B} - \frac{1}{m_B}\right) = (-V_A - V_B)\sin\phi$$

Substitute the values of $V_A$ and $V_B$

$$f\Delta t\left(-\frac{7}{2m_A} - \frac{7}{2m_B}\right) = \left(\frac{-m_A - m_B}{m_A + m_B}\right)\sin\phi.\mathrm{V}$$

$$f\Delta t = \frac{2}{7}\left(\frac{m_A m_B}{m_A + m_B}\right) V \sin\phi$$

From (5) $$\omega_A = \frac{5}{2}\frac{f\Delta t}{m_A R_A} = \frac{5}{2m_A R_A}\times\frac{2}{7}\frac{m_A m_B V\sin\phi}{(m_A+m_B)}$$

$$= \frac{5}{7}\frac{m_B}{(m_A+m_B)}\frac{V\sin\phi}{R_A}$$

$$= \frac{5}{7}\frac{m_B}{(m_A+m_B)}\frac{bV}{R_A(R_A+R_B)}$$

Likewise from (6) $$\omega_B = \frac{5}{7}\frac{m_A}{(m_A+m_B)}\frac{bV}{R_B(R_A+R_B)}$$

From (1) $$V'_{At} = \frac{1}{m_A}\left[-f\Delta t + \frac{m_A m_B V\sin\phi}{(m_A+m_B)}\right]$$

$$= \frac{1}{m_A}\left[-\frac{2}{7}\frac{m_A m_B V\sin\phi}{(m_A+m_B)} + \frac{m_A m_B V\sin\phi}{(m_A+m_B)}\right]$$

$$= \frac{5}{7}\frac{m_B V\sin\phi}{(m_A+m_B)}$$

Likewise from (2) $$V'_{Bt} = \frac{1}{m_B}\left[-\frac{2}{7}\frac{m_A m_B V\sin\phi}{(m_A+m_B)} + \frac{m_A m_B V\sin\phi}{(m_A+m_B)}\right]$$

$$= \frac{5}{7}\frac{m_A V\sin\phi}{(m_A+m_B)}$$

While $\omega_A$ , $\omega_B$, and the y-components of $m_A$ and $m_B$ are the same for centre of mass and laboratory frames , the x-components of $m_A$ and $m_B$ in the cntre of mass frame must be added to the velocity $(m_A V)/(m_A+m_B)$ to obtain x-components in the laboratory frame.

x-component of $m_A$ $$= V'_{Ar}\cos\phi + V'_{At}\sin\phi + \frac{m_A V}{(m_A+m_B)}$$

$$= -\frac{m_B V\cos^2\phi}{(m_A+m_B)} + \frac{5m_B V\sin^2\phi}{7(m_A+m_B)} + \frac{m_A V}{(m_A+m_B)}$$

$$= \frac{m_B V}{7(m_A+m_B)}\left[-7m_B + 7m_B\sin^2\phi + 5m_B\sin^2\phi + \frac{7m_A}{m_B}\right]$$

$$= \frac{m_B V}{(m_A+m_B)}\left[\frac{12\sin^2\phi}{7} - 1 + \frac{m_A}{m_B}\right]$$

$$= \frac{m_B V}{(m_A+m_B)}\left[\frac{12b^2}{7(R_A+R_B)} - 1 + \frac{m_A}{m_B}\right]$$

x component of $m_B$ $= V'_{Br}\cos\phi - V'_{Bt}\sin\phi + \dfrac{m_A V}{(m_A+m_B)}$

$$= \frac{m_A V\cos^2\phi}{(m_A+m_B)} - \frac{5m_A V\sin^2\phi}{7(m_A+m_B)} + \frac{m_A V}{(m_A+m_B)}$$

$$= \frac{m_A V}{(m_A+m_B)}\left[\frac{7-7\sin^2\phi-5\sin^2\phi+7}{7}\right]$$

$$= \frac{m_A V}{(m_A+m_B)}\left[\frac{14-12\sin^2\phi}{7}\right]$$

$$= \frac{m_A V}{(m_A+m_B)}\left[2-\frac{12}{7}\frac{b^2}{(R_A+R_B)^2}\right]$$

y-component of $m_A$ $= V'_{Ar}\sin\phi + V'_{At}\cos\phi$

$$= \frac{m_B V\cos\phi\sin\phi}{(m_A+m_B)} + \frac{5m_B V\sin\phi\cos\phi}{7(m_A+m_B)}$$

$$= \frac{12m_B V\sin\phi\cos\phi}{7(m_A+m_B)}$$

$$= \frac{12m_B V}{7(m_A+m_B)}\frac{b\sqrt{(R_A+R_B)^2-b^2}}{(R_A+R_B)^2}$$

y-component of $m_B$ $= -V'_{Br}\sin\phi - V'_{At}\cos\phi$

$$= -\frac{m_A V\cos\phi\sin\phi}{(m_A+m_B)} - \frac{5m_A V\sin\phi\cos\phi}{7(m_A+m_B)}$$

$$= -\frac{12m_B V\sin\phi\cos\phi}{7(m_A+m_B)}$$

$$= -\frac{12m_B V}{7(m_A+m_B)}\frac{b\sqrt{(R_A+R_B)^2-b^2}}{(R_A+R_B)^2}$$

3.3 After the collision, kinetic energy of ball $m_A$ is

$$E'_A = \frac{1}{2}m_A\left[(V'_{LAx})^2 + (V'_{LAy})^2\right] + \frac{1}{2}I_A\omega_A^2$$

$$=\frac{1}{2}\frac{m_A m_B}{(m_A+m_B)}\left[\frac{m_B V^2}{(m_A+m_B)}\right]\left[\left(\frac{12}{7}\frac{b^2}{(R_A+R_B)}+\frac{m_A}{m_B}-1\right)^2+\left(\frac{12}{7}\frac{b\sqrt{(R_A+R_B)^2-b^2}}{(R_A+R_B)^2}\right)^2\right]$$

$$+\frac{1}{2}\times\frac{2}{5}m_A R_A^2\left(\frac{5}{7}\frac{m_B V b}{(m_A+m_B)(R_A+R_B)R_A}\right)^2$$

$$=\frac{1}{2}\frac{m_A m_B}{(m_A+m_B)}\left[\frac{m_B V^2}{(m_A+m_B)}\right]\left[\left(\frac{12}{7}\frac{b^2}{(R_A+R_B)}+\frac{m_A}{m_B}-1\right)^2+\left(\frac{12}{7}\frac{b\sqrt{(R_A+R_B)^2-b^2}}{(R_A+R_B)^2}\right)^2\right.$$

$$\left.+\left(\frac{12}{7}\frac{b\sqrt{(R_A+R_B)^2-b^2}}{(R_A+R_B)^2}\right)^2+\frac{2}{5}\left(\frac{5}{7}\frac{b}{(m_A+m_B)(R_A+R_B)}\right)^2\right]$$

$$E_B^{\prime}=\frac{1}{2}m_B\left[\left(V_{LBx}^{\prime}\right)^2+\left(V_{LBy}^{\prime}\right)^2\right]+\frac{1}{2}I_B\omega_B^2$$

$$=\frac{1}{2}\left(\frac{m_A m_B}{(m_A+m_B)}\right)\left[\frac{m_A V^2}{(m_A+m_B)}\right]\left[\left(2-\frac{12}{7}\frac{b^2}{(R_A+R_B)^2}\right)^2+\left(\frac{12}{7}\frac{b\sqrt{(R_A+R_B)^2-b^2}}{(R_A+R_B)^2}\right)^2\right]$$

Straightforward algebra gives

$$E_A^{\prime}=\frac{(12m_A-m_B)m_A m_B V^2 b}{7(m_A+m_B)^2(R_A+R_B)^2}+\frac{m_A(m_A-m_B)^2V^2}{(m_A+m_B)^2}$$

$$E_B^{\prime}=\frac{m_A^2 m_B^2 V^2}{(m_A+m_B)^2}\left[1-\frac{b^2}{7(R_A+R_B)^2}\right]$$

The algebra becomes very simple for the case of $m_A = m_B$.

# Experiment

## Problem 1. Determination of light relectivity of a transparent dielectric surface

### Experimental Apparatus

1. He-Ne Laser (~ 1.5 mW)

2. Two polarizers with degree scale disks ( Fig 25.17 ), one is mounted in front of the laser output window as a polariser, and another can be fixed in a proper place of the drawing board by push pins when it is necessary.

3. Two light intensity detectors each of which consists of a photocell and a microammeter. ( Fig 25.18 )

4. Glass beam splitter

5. Transparent dielectric plate, whose reflectivity and refractive index are to be determined.

6. Sample table mounted on a semicircular degree scale plate with a coaxial swivel arm.

7. Several push-pins for fixing the sample table on the drawing board and serving as its rotation axis.

8. Slit aperture and viewing screen for adjusting the laser beam in the horizontal direction and for alignment of optical elements.

9. Lute for fixing optical elements in a fixed place.

10. Wooden drawing board.

11. Plotting papers.

### Experimental Requirements

1. Determine the reflectivity of the p- component as a function of the incident angle ( electric field component, which is parallel to the plane of incidence is called the p-component):

1.1. Specify the transmission axis of the polarizer (A) by position of the marked line on the degree scale disk in the p-component meaasurement ( the transmission axis is the direction of vibration of the electric field vector of the transmitted light).

1.2 Choose any one of the light intensity detector and set its microammeter at the range"x5". Verify the linear relationship between the light intensity and the micro-ammeter reading.

1.3 Draw an optical schematic disagram for determining the reflectivity of the p-component as a function of the incident angle.

1.4 Show lyour measured data and calculated reflectivity ( including the calcualtion formula) in the form of a table. Plot the reflectivity as a function of the incident angle.

2. Calculate the refracttive index of the sample using the measured results.

3. Determine the reflectivity of the s-component as a function of the incident angle (The electric field component, which is perpendicular to the plane of incidence, is called s-component): Specify the transmission axis of the polarizer (A) by the position of marked line on the degree scale disk in your s- component experiment.

Draw the optical schematic diagaram.
Show your measured data and the callculated reflectivity including the calculation formula in the form of a table.
Plot the refectivity as a function of the incident angle.

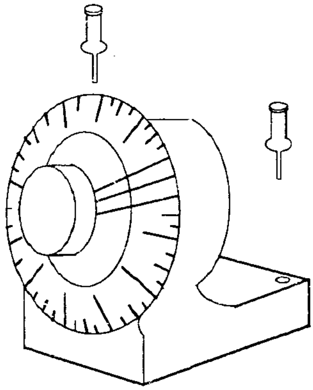
## Explanation and Suggestion

1. Avoid direct eye exposure to laser radiation.

2. Do your experiment step by step as recomended for a better chance of success.

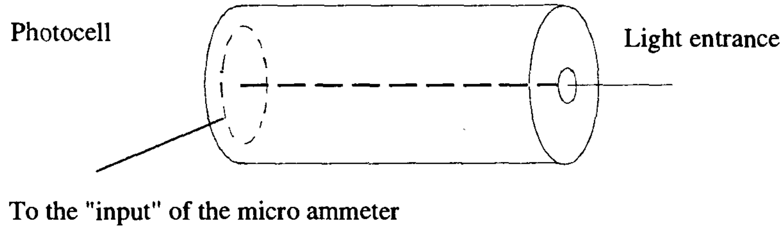
3.The output power of the laser beam may fluctuate from time to time, and needs to be monitored and requires appropriate correction in the calculation of the experiemntal results.

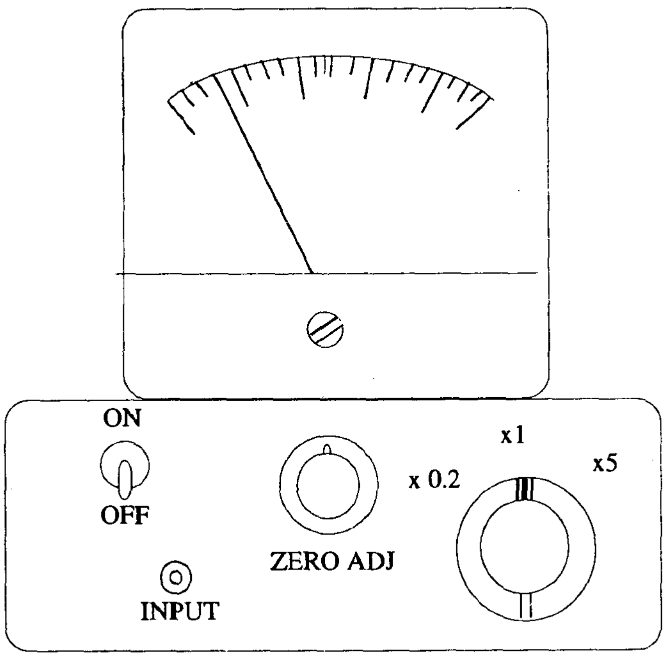
4. The laser should be lighting at all time, even when you have finished your experimentand left the examination room.



Polarizer with degree scale disk

**Fig.** 25.17




Light Intensity Detector

**Fig.** 25.18

1. Insert the plug of photocell into the "INPUT" socket of microammeter.

2. Switching on microammeter.

3. Block off the light entrance hole in front of the photocell and adjust the scale reading of microammeter to"0".

4. Set the "Multiple" knob to a proper range.

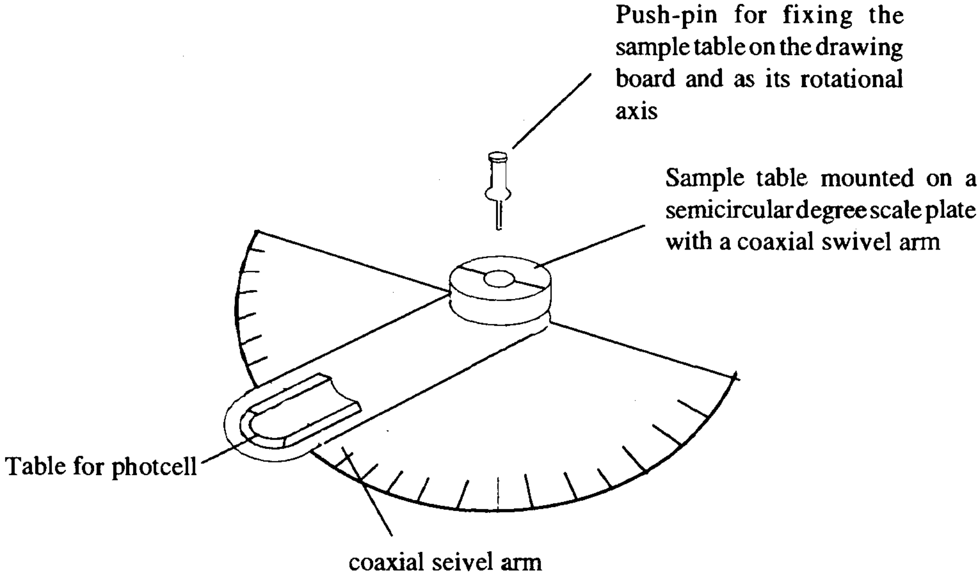



**Fig.** 25. 19 Sample table mounted on a semicircular degree scale

# Solution

## 1.1 Determination of the p-component as a function of incident angle

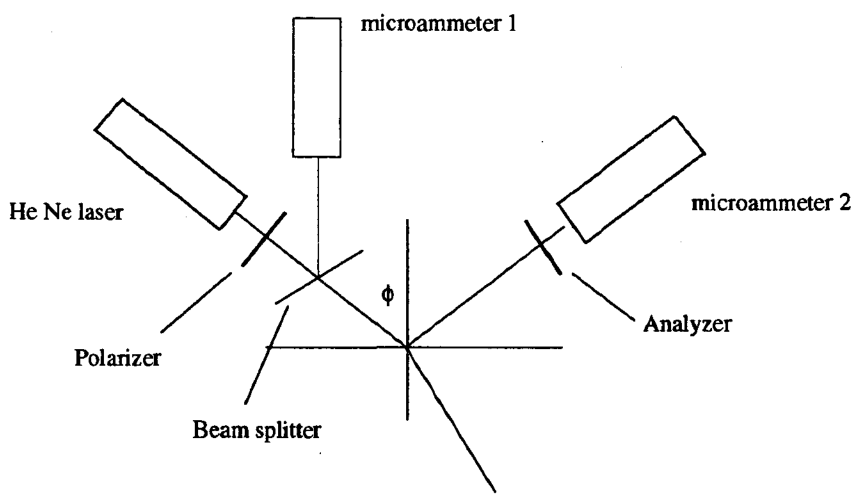



**Fig.**25.20

Set up experiment as shown in Fig. 25. 13 Light beam from He Ne laser is allowed to pass through the polarizer the transmision axis of which is not known at this stage, and through the beam splitter before it is incident on the dielectic slab. The beam reflected of the beam splitter, monitored by micrammeter 1 will be refered to as the reference beam. The beam through the beam splitter is incident on the dielectric slab before being reflected and picked up by micrammter 2.

In the beginning part of the experiment the analyzer is not yet placed infront of micro-ammeter 2.

For the sake of simplicity, we will assume that light intensity before going through the beam splitter is $2I_0$ and the beam isplitted into two beams of equal intensity $I_0$.

Choose a convenient value of angle of incidence. Rotate the polarizer until maximum value of $I_2$ or rather $I_2/I_1$ is maximum. In this position the axis of the tranmission of the polarizer is normal to the plane of incidence. Mark its position on the scale.

1.1. 2 Using the same experimental setup, place the polarizer and the analyzer in the position that their transmission axes are perpendicular to the plane of incidence.

Rotate the analyzer so that its axis makes angle θ with its original position.
The intensity of the signal monitored by microammeter 2 changes by a factor of $\cos^2\theta$.
For each value of angle of incidence, measure current $I_1$ and $I_2$ from microammeter 1 and 2 repectively.

Record experimental results in the table below.

| θ | $\cos^2\theta$ | $I_1$ | $I_2$ | $I_2/I_1$ |
|---|---|---|---|---|
| | | | | |

Plot a graph of $\cos^2\theta$ against $I_2/I_1$ . The curve should be linear.

1.1.3 Use the same experimental setup. Place the transmission axes of the polarizer and the analyzerparalle to the plane of incidence.
Vary the angle of incidence. For each value of angle of incidence $\phi$, measure current $I_1$ and $I_2$ from microammeter 1 and 2 repectively.

Tabulate the results in the table.

| $\phi$ | $I_1$ | $I_2$ | $I_2/I_1$ |
|---|---|---|---|
| | | | |

1.1.3 Plot the value of $I_2/I_1$ as a function of angle of incidence $\phi$.
The curve should give a minimum value of angle of incidence at $\phi_B = 57°$.This angle is known as Brewster's angle.

1.1.4 If the axis of the plane polarized incident beam makes some angle with the plane of incidence, at Brewster's angle the reflected beam will be polarized in the direction normal to the plane of incidence, while the refracted beam will be polarized in the direction parallel to the plane of incidence.

Referring to the diagram in Fig. 25.

2.At Brewster's angle, the angle wbetween the refracted and reflected beam is $90°$.

Hence the refractive index of the dielectric slabe is given by

$$n = \frac{\sin \theta_B}{\sin(90 - \theta_B)} = \tan \theta_B$$

3. Use the same experimental setup , but place the transmission axes of both the polarizxer and the analyzer in the direction normal to the plane of incidence.

Vary the angle of incidence $\theta$ . For every value of $\theta$ , record current readings from ammetesr 1 and 2.

Tabulate reflectivity and angle of incidence.

Plot a graph of reflectivity of s-component ($I_1/I_2$) as a function of $\theta$.

# Problem 2.

## A Black Box

Given a black box with two similar terminals. There are no more than three passive elements inside the black box. Find the values of elements in the equivalent circuit between the terminals. this box is not allowed to be opened.

## Experimental Apparatus

1. Double channel oscilloscope with a panel illustration, showing the name and function of each knob.

2. Audio frequency signal generator with a panel illustration, showing the name and function of each knob.

3. Resistance box with a fixed value of 100 Ω ( ≤ ± 10.5% )

4. Several connecting wires.

5. For the coaxial cables, the wire in black colour at terminal is grounded.

6. Log-log paper, semi-log paper, and millimeter paper are provided for use if necessary.

**Note** :The knobs, which were not shown on the panel illustration of the "generator" and "oscilloscope", have been set to the correct positions. It should not be touched by the student.

## Experimental Requirements

1. Draw the circuit diagram in your experiment.

2. Show your measured data and the calculated results in the form of tables. Plot the experimental curves with the obtained results on the coordinate charts provided (indicate the title of the diagram and the titles and scale units of the coordinate axes).

3. Given the equivalent circuit of the black box and the names of the elements with their values in the equivalent circuit ( write down the calculation formulas).

## Instruction

1. Perform your experiment in the frequency range between 100 Hz and 50 kHz.

2. The output volatage of signal generator should be less than 1.0 V (peak-to-peak). Fix the "output attenuation" switch at "20" db during the experiment.

There are two kinds of passive elements , induction coil and capacitor. For three elements to be identified as three separate units with respective charactistic values, they must be connected as one element in series with the other two connected in parallel.

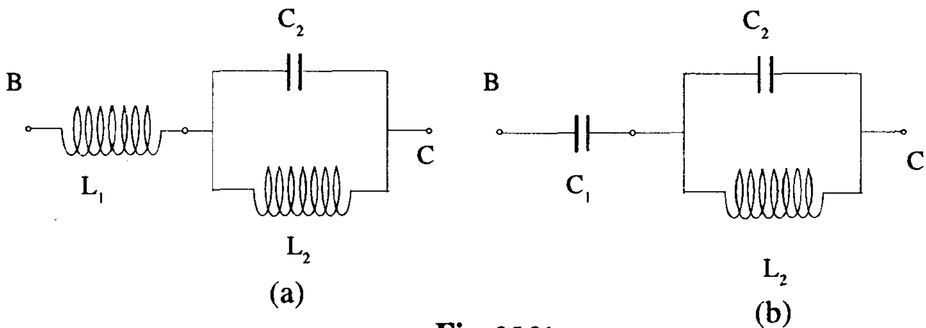



**Fig.** 25.21

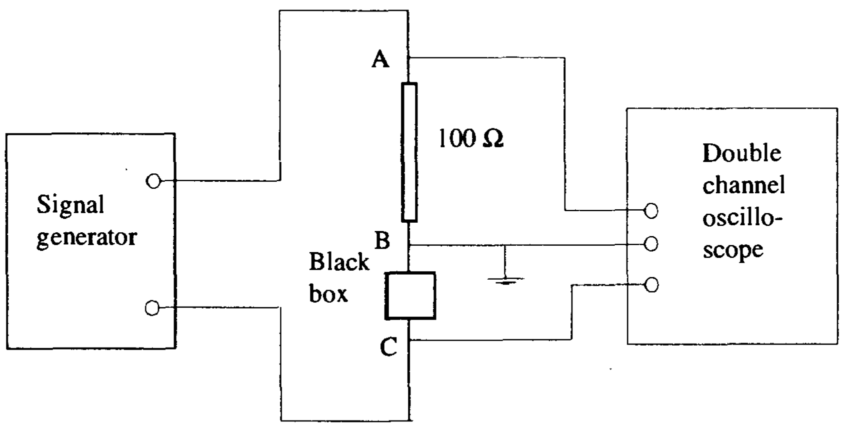



**Fig.** 25.22

Our first task is to determine whether the elements are conneced as in (a) or in (b).

**Case(a)**

At low frequencies, the impedance (or rather reactance) across BC is $\frac{1}{\omega C_2}$.

Current through 100 Ω resistance and $C_2$ is $\frac{V_{AB}}{100}$ in the unit of $\frac{V'}{\Omega}$.

$V_{BC}$ voltage across BC = current through $C_2 \times \frac{1}{\omega C_2}$

Current through $C_2$ = $\omega C_2 V_{BC}$ in the unit of $\frac{V'}{\Omega}$.

$$\frac{V_{AB}}{100} = \omega C_2 V_{BC}$$

$$\frac{V_{AB}}{V_{BC}} = 100\omega C_2$$

A graph of $\frac{V_{AB}}{V_{BC}}$ against $\omega$ should be linear, and the slope , say, m = $100C_2$.

At high frequencies, the impedance across BC is $\omega(L_1 + L_2)$. In this case,

$$\frac{V_{AB}}{V_{BC}} = \frac{100}{\omega(L_1 + L_2)}$$

A graph of $\frac{V_{AB}}{V_{BC}}$ against $\frac{1}{\omega}$ should be linear, and the slope , say, n $= \frac{1}{(L_1 + L_2)}$

On e may also identify case (a) by looking at phase difference between $V_{AB}$ and $V_{BC}$ at low and high frequencies.
Bearing in mind because of the way we connect $V_{AB}$ and $V_{BC}$ to the input of the oscilloscope, the signal of $V_{BC}$ suffer an additional phase change of 180°.
At low frequencies, when the impedance across BC is that of $C_2$, the phase of $V_{AB}$ will lag behind $V_{AB}$ by another 90°.
At high frequencies, when the impedance across BC is mainly inductance $L_1 + L_2$, the phase of $V_{AB}$ will lead $V_{AB}$ by 90°.

**Case (b)**

At low frequencies, effective impedance across BC is $\frac{1}{\omega C_1} + \frac{1}{\omega C_2}$.

Peak current through AB (and also BC) $\frac{V_{AB}}{100}$, where $V_{AB}$ is peak voltage across AB measured in the unit of V′.

Peak Voltage across BC $V_{BC} = \frac{V_{AB}}{100} \Big/ \left( \frac{1}{\omega C_1} + \frac{1}{\omega C_2} \right)$

$$\frac{V_{BC}}{V_{AB}} = \frac{\omega C_1 C_2}{100\left(C_2 + C_2\right)}$$

A graph of $\frac{V_{BC}}{V_{AB}}$ against $\omega$ should be linear, with the slope $m = \frac{C_1 C_2}{100\left(C_2 + C_2\right)}$

At high frequencies, the effective impedance across BC is $\omega L_2$

$$V_{BC} = \frac{V_{AB}}{100\omega L_2}$$

A graph of $\frac{V_{BC}}{V_{AB}}$ against $1/\omega$ should be linear, with the slope $n = \frac{1}{100L_2}$

Also at resonance $\omega = \frac{1}{\sqrt{L_2 C_2}}$

From which $C_1$, $L_2$ and $C_2$ can be calculated.
As in case (a), case(b) can also be identified through phase changes at low and high frequencies.

# International Physics Olympiad XXVI
# 1995
# Canberra, ACT Australia

## Theory

### Problem 1. Gravitational Red Shift and the Meaurement of Stellar Mass

1.1 A photon of frequency f possesses an effective inertial mass m determined by its energy. We may assume that it has a gravitational mass equal to this inertial mass. Accordingly, a photon emitted at the surface of a star will lose energy when it escapes from the star's gravitational field. Show that the frequency shift $\Delta f$ of a photon when it escapes from the surface of the star to infinity is given by

$$\frac{\Delta f}{f} = -\frac{GM}{Rc^2} \quad \text{for } \Delta f <<< f$$

where G = gravitational constant
R = radius of the star
c = velocity of light
M = mass of the star

Thus the red shift of a known spectral line measured a long way from the star can be used to measured the ratio M/R. Knowledge of R will allow the mass of the star to be determined.

1.2 An unmanned spacecraft is launched in an experiment to measure both the mass M and radius R of a star in our galaxy. As the spacecraft approaches its objective radially, photons emitted from $He^+$ ions on the surface of the star are monitored via resonance excitation of a beam of $He^+$ ions in a test chamber inside the spacecraft. Resonance absorp tion occurs only if the $He^+$ ions are given a velocity towards the star to allow exactly for the red shifts. The velocity ( $v = \beta c$) of the $He^+$ ions in the spacecraft relative to the star at absorption resonance is measured as a function of the distance d from the (nearest) surface of the star.
The experimental data are displayed in the accompanying table. Fully utilize the data to determine graphically the mass M and radius R of the star. There is no need to estimate the uncertainties in your answer.

**Data for Resonamce Condition**

| | | | | | | |
|---|---|---|---|---|---|---|
| Velocity of $He^+$ | $\beta = v/c$ | 3.352 | 3.279 | 3.195 | 3.077 | 2.995 |
| Distance from surfaceof star | d= $10^{-8}$ m | 38.90 | 19.98 | 13.32 | 8.99 | 6.67 |

1.3 In order to determine R and M in such an experiment, it is usual to conduct the frequency correction due to the recoil of the emitting atom, (Thermal motion causes emission lines to be broadened without displacing distribution maxima, and we may therefore assume that all thermal effects have been taken into account)

Let E be energy difference between two atomic energy levels, with the atom at rest in each case. Assume that the atom decays at rest, producing a photon and a recoiling atom. Obtain the relativistic expression of the energy hf of a photon emitted in terms of E and the inertial rest mass $m_0$ of the atom.
Hence make a numerical estimate of the relativistic frequency shift $(\Delta f/f)_{recoil}$ for the case of $He^+$ ions.
Your answer should turn out to be much smaller than the gravitational red shift obtained in Part 1.2.

**Data**

| | | |
|---|---|---|
| Velocity of light | $c =$ | m/s |
| Rest energy of He | $m_0c^2 =$ | MeV |
| Bohr energy | $E_n =$ | eV |
| Gravitational constant | $G =$ | $Nm^2kg^{-2}$ |

# Solution

1.1 Effective inertial mass of photon is given by

$$m = \frac{hf}{c^2}$$

where f is photon's normal frequency.
Gravitational and inertial masses are assumed to be equal..
The photon after travelling from its initial position $r_i$ to its final position $r_f$ loses its energy in the amount of potential energy gained.

i.e.

$$hf_i - hf_f = -\frac{GMm}{r_f} - \left[-\frac{GMm}{r_i}\right]$$

$$h(f_i - f_f) = GMm\left[\frac{1}{r_i} - \frac{1}{r_f}\right]$$

Substitute m from (1) in (2) to arrive at,

$$-\Delta f = \frac{GMf}{c^2}\left[\frac{1}{r_i} - \frac{1}{r_f}\right]$$

where $\Delta f = f_f - f_i$

When the photon escapes from the star's gravitational pull, $r_f$ corresponds to $\infty$. Hence frequency shift towards the red end of the spectrum is given by,

$$\frac{\Delta f}{f} = -\frac{GM}{c^2 R}$$

where R is the radius of the star( from the centre to its surface).
The negaive sign indicates that frequency becomes lower, as it loses energy.

1.2 In the experiment described in the problem
R is the radius of the star
d distance or position of the probe measured from the surface of the star.

Hence frequency shift measured at distance d from the surface of the star

$$\frac{\Delta f}{f} = -\frac{GM}{c^2}\left[\frac{1}{R} - \frac{1}{R+d}\right]$$

This frequency shift is to be rectified by Doppler's shift in order to have resonant absorption by $He^+$

We may use classical Doppler's shift without loss in accuracy.
Doppler's frequency for observer moving towards a light source is given by

$$f = \frac{f_0}{1-\beta} \qquad \text{where } \beta = v/c$$

$$f - f\beta = f_0$$

$$f - f_0 = f\beta$$

$$\frac{\Delta f}{f} = \beta$$

The two types of shift are cancelled out, hence

$$\left[\frac{\Delta f}{f}\right]_{Gravitation} + \left[\frac{\Delta f}{f}\right]_{Doppler} = 0$$

$$-\frac{GM}{c^2}\left[\frac{1}{R} - \frac{1}{R+d}\right] + \beta = 0$$

$$\beta = \frac{GM}{c^2}\left[\frac{1}{R} - \frac{1}{R+d}\right]$$

$$\beta = \frac{GM}{c^2}\left[\frac{d}{R(R+d)}\right]$$

$$\frac{1}{\beta}=\left[\frac{R^2c^2}{GM}\right]\left[\frac{1}{d}+\frac{1}{R}\right]$$

Tabulate the values of $\frac{1}{\beta}$ against $\frac{1}{d}$

| | | | | | |
|---|---|---|---|---|---|
| $\frac{1}{\beta}(10^{-8}\ m^{-1})$ | .026 | .050 | .075 | .111 | .150 |
| $\frac{1}{d}(10^{+5})$ | .298 | .305 | .313 | .325 | .338 |

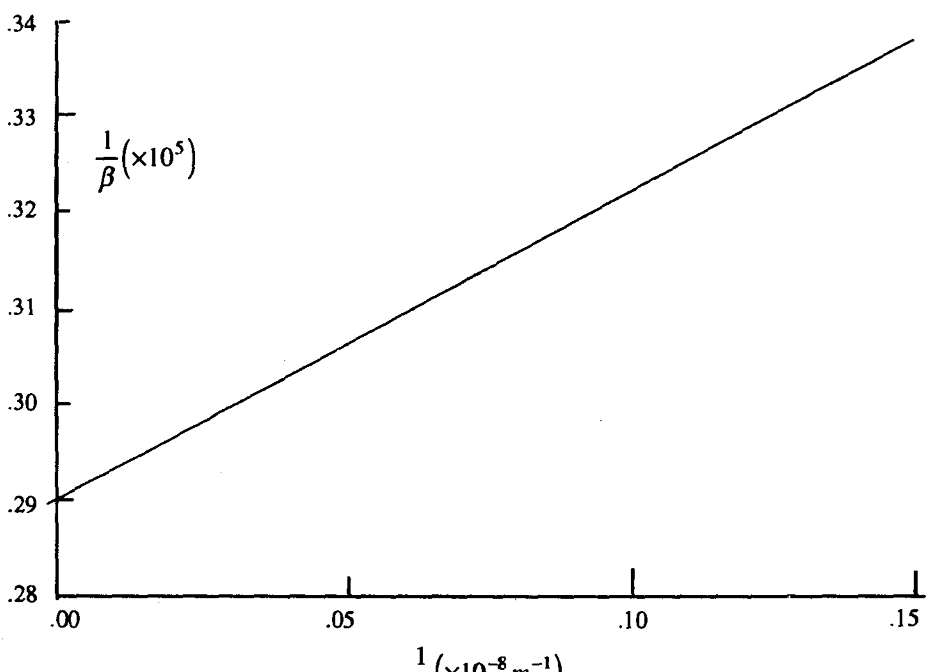



**Fig. 26.1**

From the graph above, measure the slope m, and intercept on the vertical axis.

$$\text{slope} \qquad \left[\frac{R^2c^2}{GM}\right] = 3.2\times10^{12}\,m \qquad (1)$$

$$\text{Intercept} \qquad \left[\frac{Rc^2}{GM}\right] = 0.29\times10^{5} \qquad (2)$$

From the graph $R = \dfrac{3.2\times10^{12}\,m}{0.29\times10^{5}} = 1.10\times10^{8}\,m\mathrm{R}$

Substitute the values of R, G and c in (2) to obtain

$$\frac{1.10\times10^{8}\times9.0\times10^{6}}{6.7\times10^{-11}} = 0.29\times10^{5}$$

$$M = 5.11\times10^{30}\,kg$$

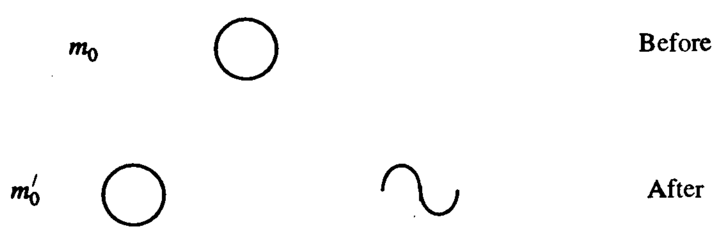



**Fig. 26.2**

Total energies before and after emission are the same.

Total energy before emission of photon $= m_0c^2$

Total energy after emission of photon

$$E = \sqrt{p^2c^2 + (m_0')^2c^4} + hf$$

And from data given $\Delta E = (m_0 - m_0')c^2$

Conservation of momentum gives

$$m_0c^2 = \sqrt{p^2c^2 + (m_0')^2c^4} + hf$$

$$p = \frac{hf}{c}$$

$$m_0c^2 - hf = \sqrt{(hf)^2 + (m_0')^2c^4}$$

$$m_0^2c^4 + (hf)^2 - 2m_0c^2hf = (hf)^2 + (m_0')^2c^4$$

$$\left[m_0^2c^4 - \left(m_0^{\prime}\right)^2 c^4\right] = 2m_0c^2hf$$

$$2m_0c^2hf = \left[m_0c^2 - m_0^{\prime}c^2\right]\left[m_0c^2 + m_0^{\prime}c^2\right]$$

$$= \Delta E\left[m_0c^2 - m_0^{\prime}c^2 + 2m_0^{\prime}c^2\right]$$

$$= \Delta E\left[2m_0c^2 - \left(m_0c^2 - m_0^{\prime}c^2\right)\right]$$

$$hf = \Delta E\left[1 - \frac{\Delta E}{2m_0c^2}\right]$$

Since $\Delta E <<< 2m_0c^2$ frequency shift due to recoil is negligible when compared to gravitational shift.

## Problem 2. Sound Propagation

### Introduction

The speed of propagation of sound in the ocean varies with depth, temperature and salinity. Fig. 26.3 a below shows the variation of sound speed c with depth z for a case where a minimum speed value $c_0$ occurs midway between the ocean surface and the sea bed. Note that for convenience z = 0 at the depth of this sound speed minimum; $z = z_S$ at the surface and $z = - z_B$ at the sea bed.

Above z = 0, c is given by;

$$c = c_0 + b\,z$$

Below z = 0, c is given by;

$$c = c_0 - b\,z$$

In each case, b = |dc/dz |, whereb is the magnitude of the sound speed gradient with depth; b is assumed constant.

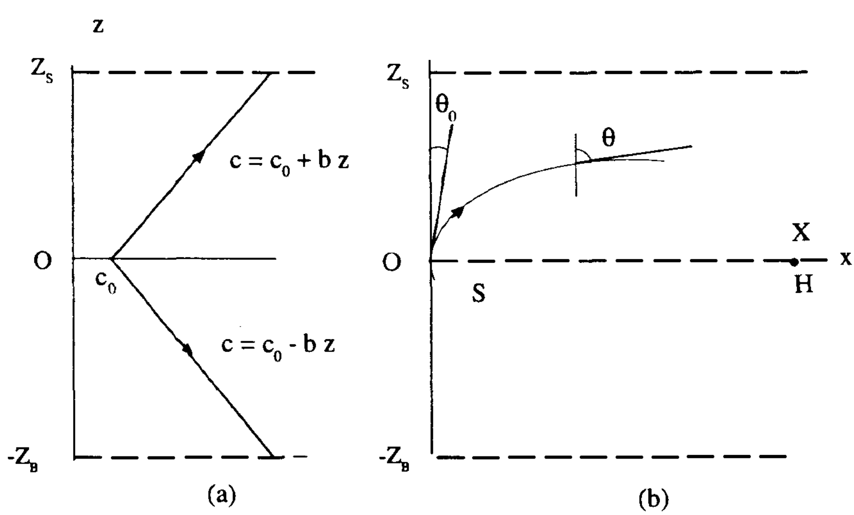



**Fig.** 26.3

FIG. 26.1 b above shows a section of the z-x plane through the ocean, where x is a horizontal direction. At all points along the z-x section the sound speed profile c(z) is as shown in Fig 26.3 a .

At the position z = 0, x = 0, a sound source S is located. Part of the output from this source is described by a sound ray emerging from S with initial angle $\theta_0$ as shown. Because of the variation of sound speed with z, the ray will be refracted, leading to varying values along the trajectory of the ray.

2.1 Show that the initial trajectory of the ray leaving the source S and constrained to the z-x plane is an arc of a circle with radius R where :

$$R = \frac{c_0}{b \sin \theta_0} \quad \text{for } 0 \leq \theta_0 \leq \frac{\pi}{2}$$

2.2 Derive an expression involving $z_S$ ,$c_0$ and b to give the smallest value of the angle $\theta_0$ for upwardly directed rays which can be transmitted without the sound wave reflecting from the sea surface.

2.3 Fig. 26.1 b shows the position of a sound reciever H which is located at the position z = 0, x = X. Derive an expression involving b, X and $c_0$ to give the series of values of angle $\theta_0$ required for the sound ray emerging from S to reach the reciever H. Assume that $z_S$ and $z_b$ are sufficiently large to remove the possibility of reflection from sea surface or sea bed.

2.4 Calculate the smallest four valaues of $\theta_0$ for refracted rays from S to reach the reciever H, when;

X = 10,000 m
$c_0$ = 1,500 m
b = 0.02000 $s^{-1}$

2. 5 Derive an expression to give the time taken for sound wave to travel from S to H following the ray path associated with smallest value of angle $\theta_0$ , as determined in Part 1.3 . Calculated the value of this transit time for the conditions given in Part 1.4. The following result may be of assistance;

$$\int \frac{dx}{\sin x} = \ln \tan\left(\frac{x}{2}\right)$$

Calculate the time taken for the direct ray to travel for S to H along z = 0. Which of the two rays will arrive first, the ray for which $\theta_0 = \frac{\pi}{2}$ or the ray with the smallest value of $\theta_0$ as calculated for Part 2.4.

# Solution

2.1

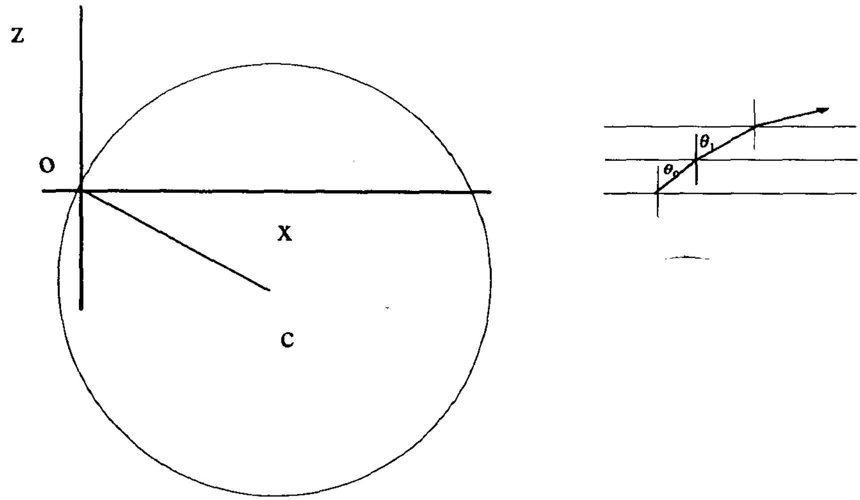



FIG. 26.4

Let the ocean be divided into layers of small thickness so that within each layer the refractive index can be considered constant.
From the principle of refraction:

$$n_0 \sin\theta_0 = n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 = \ldots..$$

where $n_0, n_1, n_2, n_3 \ldots.$ *and* $\theta_0, \theta_1, \theta_2, \theta_3$ are refractive indices and incident angles associated with o,1,2,3 ... layers respectively.

$$\text{Or}\quad \frac{\sin\theta_0}{c_0} = \frac{\sin\theta_1}{c_1} = \frac{\sin\theta_2}{c_2} = \frac{\sin\theta_3}{c_3} \ldots$$

where $c_0, c_1, c_2, c_3 \ldots..$ are velocities of sound wave in o,1,2,3 ... layers respectively.

Consider refraction which takes place at the boundary separating z and $z + \Delta z$ layers. Law of refaction gives

$$\frac{\sin\theta}{c_z} = \frac{\sin(\theta + \Delta\theta)}{c_{z+\Delta z}}$$

$$\frac{\sin\theta}{c_z} = \frac{\sin\theta\cos\Delta\theta + \cos\theta\sin\Delta\theta}{c_z + \frac{dc}{dz}\Delta z}$$

$$\cos\Delta\theta \cong 1 \text{ and } \sin\Delta\theta \cong \Delta\theta$$

$$\frac{\sin\theta}{c_z} = \frac{\sin\theta + \cos\theta.\Delta\theta}{c_z + b\Delta z}$$

$$c_z \sin\theta + b\Delta z \sin\theta = c_z \sin\theta + c_z \cos\theta.\Delta\theta$$

$$b\Delta z \sin\theta = c_z \cos\theta.\Delta\theta$$

$$\frac{b\Delta z \sin\theta}{c_z} = \cos\theta.\Delta\theta$$

Note that

$$\Delta z = \frac{dz}{dx}\Delta x = \tan\left(90^o - \theta\right)\Delta x = \cot\theta\Delta x \tag{1}$$

$$\frac{\sin\theta_0}{c_0} = \frac{\sin\theta}{c_z}$$

$$\frac{b\cot\theta\Delta x\sin\theta_0}{c_0} = \cos\theta.\Delta\theta$$

$$\frac{b\sin\theta_0}{c_0}\Delta x = \sin\theta.\Delta\theta$$

$$\frac{\Delta x}{R} = \sin\theta.\Delta\theta \tag{2}$$

$$\frac{x}{R} = -\cos\theta + k$$

where $R = \dfrac{c_0}{b\sin\theta_0}$ and k an arbitrary constant.

Since x = 0 when $\theta = \theta_0$, $k = \cos\theta_0$

$$x = R\left(\cos\theta_0 - \cos\theta\right) \tag{3}$$

The above equation is recognized as an equation of a circle.

Or substitute $\Delta x$ from (2) in equation (1) to obtain,

$$\Delta z = R\cos\theta.\Delta\theta$$

$$z = R\sin\theta + k$$

Since z= 0 when $\theta = \theta_0$, $k = -R\sin\theta_0$

$$z = R(\sin\theta - \sin\theta_0) \tag{4}$$

From (3) $\left(x - R\cos\theta_0\right)^2 = R^2\cos^2\theta$ (5)

From (4) $\left(z + R\sin\theta_0\right)^2 = R^2\sin^2\theta$ (6)

$(5)^2 + (6)^2$ gives

$$\left(z + R\sin\theta_0\right)^2 + \left(x - R\cos\theta_0\right)^2 = R^2$$

which is in the more familiar form the equation of a circule of radius R having its center at

$$z = -R\sin\theta_0 \quad \text{and} \quad x = R\cos\theta_0$$

2.2 The condition for minimum angle $\theta_0$ for upwardly directed ray not to be reflected at the sea surface( at $z_S$ )
This condition implies that

$$\frac{\sin\theta_0}{c_0} = \frac{\sin 90^o}{c_0 + bz_S}$$

$$\sin\theta_0 = \frac{c_0}{c_0 + bz_S}$$

$$(\theta_0)_{MIN} = \sin^{-1}\left(\frac{c_0}{c_0 + bz_S}\right)$$

2.3

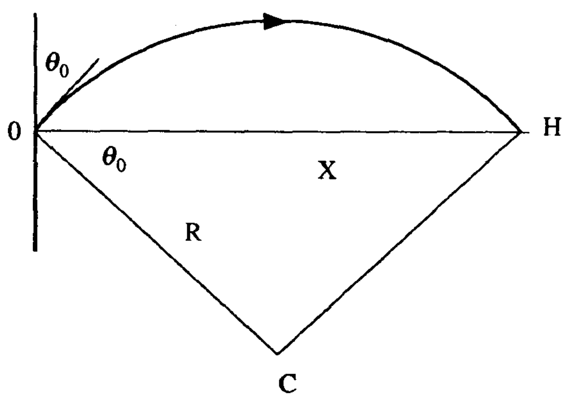



**Fig. 26.5**

For the ray to reach H by the trajectory shown above,

$$\frac{X}{2} = R\cos\theta_0$$

Substitute $R = \dfrac{c_0}{b\sin\theta_0}$

$$X = \frac{2c_0}{b\sin\theta_0}\cos\theta_0$$

$$X = \frac{2c_0}{b}\cot\theta_0$$

Or the beam may undergoes two reflections, one in each region above and under the line z = 0 as shown in the Fig 26.6

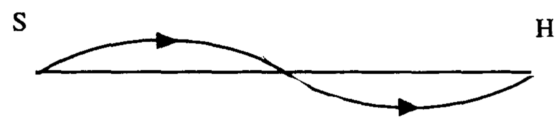



FIG. 26.6

In this case, $$\frac{X}{2} = R\cos\theta_0$$

$$X = \frac{2c_0}{b}\cot\theta_0$$

In general, the ray may make multiple numbers of reflection.

$$X = \frac{nc_0}{b}\cot\theta_0$$

$$\cot\theta_0 = \frac{bX}{nc_0}$$

$$\theta_0 = \cot^{-1}\left(\frac{bX}{nc_0}\right) = \tan^{-1}\left(\frac{nc_0}{bX}\right)$$

where n = 1,2,3,4........

For X = 10,000 m

$c_0$ = 1,500 m/s

b =.0 2000 $s^{-1}$

| n | $\theta_0$ |
|---|---|
| 1 | 86.2 |
| 2 | 88.1 |
| 3 | 88.7 |
| 4 | 89.0 |

2.5 For smallest $\theta_0$ , n =1 time taken for the sound ray to reach H

$$t = \int_{\theta_0}^{\phi} \frac{Rd\theta}{c} = 2\int_{\theta_0}^{\frac{\pi}{2}} \frac{Rd\theta}{c}$$

$$= 2\int_{\theta_0}^{\frac{\pi}{2}} \left[ \frac{c_0}{b\sin\theta_0} \Big/ \left( c_0 + \frac{bc_0}{b\sin_0}(\sin\theta - \sin\theta_0) \right) \right] d\theta$$

$$= 2\int_{\theta_0}^{\frac{\pi}{2}} \frac{d\theta}{b\sin\theta}$$

$$= \frac{2}{b}\left[ \ln\tan\frac{\theta}{2} \right]_{\theta_0}^{\frac{\pi}{2}}$$

$$= -\frac{2}{b}\ln\tan\frac{\theta_0}{2}$$

Substitute $\theta_0$=86.2 , b= 0.0200 $s^{-1}$

t = 6.655 s

For axial or direct beam $t = \dfrac{X}{c_0}$

Substitute X =10,000 m ,$c_0$ = 1,500 m

t = 6.667 s

For n =1 , the sound along the curve path will arrive earlier than that along the direct path by 0.012 s **Ans**

# Problem 3

3.1 A bouy consists of a solid cylinder, radius a, length l, made of lightweight material of uniform density d, with a uniform rigid rod protruding directly outwards from the bottom halfway along the length. The mass of the rod is equal to that of the cylinder, its length is the same as the diameter of the cylinder and the density of the rod is greater than that of the seawater. This buoy is floating in seawater ( of density $\rho$ ). In equilibrium derive an expression relating the floating angle $\alpha$, as drawn, to $d/\rho$. Neglect the volume of the rod.

3.2 If the buoy, due to some perturbation, is depressed vertically by a small amount z, it will cause it to begin oscillating up and down, about the equilibrium floating position. Determine the frequency of this vertical mode of vibration in terms of $\alpha$, g and a, where g is the acceleration due to gravity, assuming the influence of water motion on the dynamics of the buoy is such as to increase the effective mass of the bouy by a factor of one third. You may assume that $\alpha$ is not small.

3.3 In the approximation that the cylinder swings about it's horizontal central axis, determine the frequency of swing again in terms of g and a. Neglect the dynamics and viscosity of the water in this case. The angle of swing is supposed to be small.

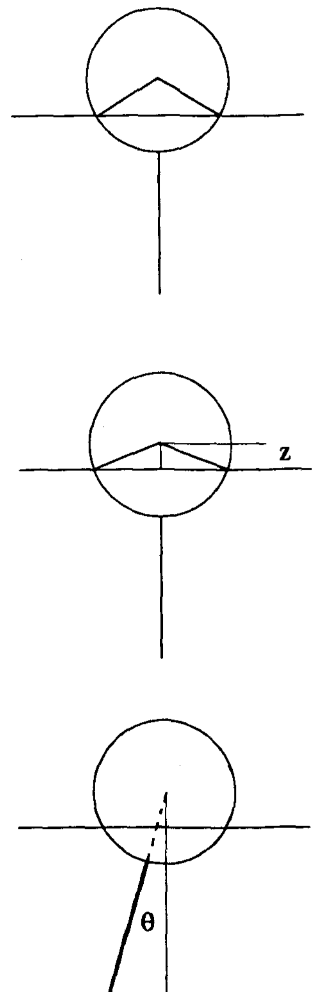



FIG. 26.7

3.4 The buoy contains sensitive accelerometers which can measure the vertical and swinging motions and then relay this information by radio to shore. In relatively calm waters it is recorded that the vertical oscillation period is about 1 second and the swinging oscillation period is about 1.5 seconds. From this information, show that the float angle is about 90° and thereby estimate the radius of the buoy and its total mass, given the cylinder length l is equal to a.

[ You may take it that $\rho \sim 1000$ kgm$^{-3}$ and $g \sim 9.8$ ms$^{-2}$.]

# Solution

3.1

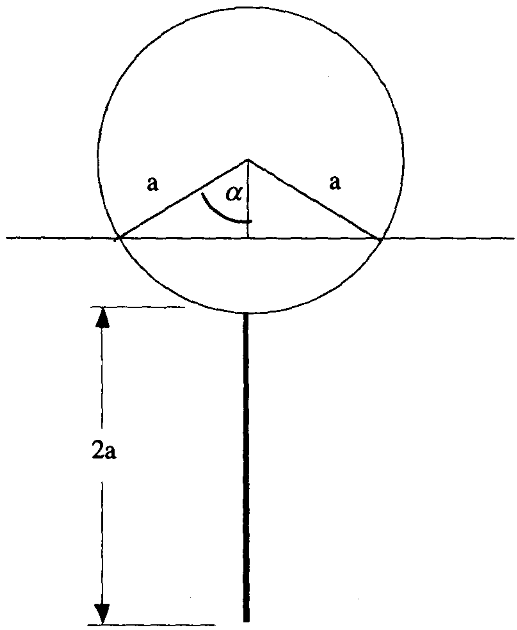



**Fig** 26.8

Mass of cylinder $= \pi a^2 ld$

Hence mass of attached rod $= \pi a^2 ld$

Mass of the whole system consisting of the cylinder and the attached rod $= 2\pi a^2 ld$

Volume of displaced sea water $= \frac{2\alpha}{2} a^2 l - 2 \times \frac{1}{2} \sin\alpha\cos\alpha . a^2 l = (\alpha - \sin\alpha\cos\alpha)a^2 l$

where $\alpha$ is the float angle ( see Fig 26. )

At equilibrium, the buoyancy force ( which is equal to the weight of the displaced sea water ) is equal to the weight of the system of the buoy i.e. $2\pi a^2 ld\,g$

Hence $(\alpha - \sin\alpha\cos\alpha)a^2 l\rho g = 2\pi a^2 ldg$

$$(\alpha - \sin\alpha\cos\alpha)\rho = 2\pi d$$

$$(\alpha - \sin\alpha\cos\alpha) = \frac{2d\pi}{\rho}$$ **Ans**

3.2 Let $\alpha + \Delta\alpha$ be the float angle when the buoy is slightly depressed.
Volume of displaced sea water associated with float angle $\Delta\alpha$ is given by

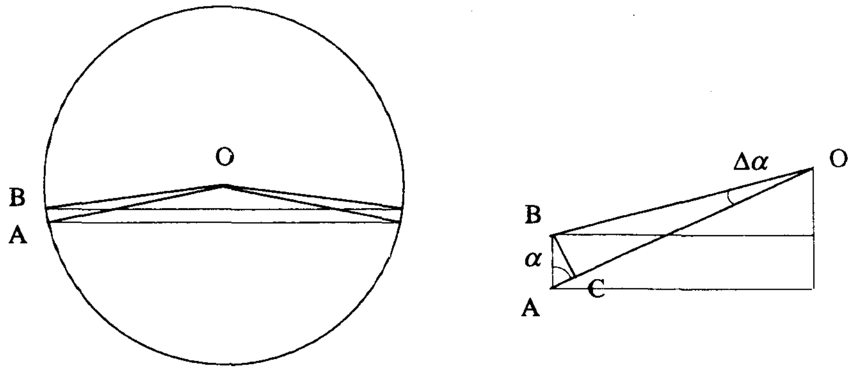



**Fig 26.9**

$$\Delta V = \left[\alpha + \Delta\alpha - \cos(\alpha + \Delta\alpha)\sin(\alpha + \Delta\alpha) - \alpha + \cos\alpha \sin\alpha\right] a^2 l$$

For small $\Delta\alpha$ and large $\alpha$

$$\Delta V = \left[\Delta\alpha - \sin\alpha\left\{\cos\alpha - \cos(\alpha + \Delta\alpha)\right\}\right] a^2 l$$

$$\Delta\alpha = \sin\Delta\alpha = \frac{BC}{a} = \frac{z\sin\alpha}{a}$$

Since z becomes more negative as $\alpha$ increases

$$\Delta\alpha = -\frac{z\sin\alpha}{a}$$

$$z = a\cos\alpha - a\cos(\alpha + \Delta\alpha)$$

$$\Delta V = -2z\sin\alpha . al$$

where z is displacement of the centre of mass measured from the equilibrium position. In this case it is the same as displacement of the centre of mass of the cylinder from the position of the equilibrium.
Equation of motion of the system of the bouy and the rod is

$$2\pi a^2 l d\ddot{z} = -z.2\sin\alpha . a.l.g.\rho$$

$$\pi a d\ddot{z} = -\sin\alpha . g.\rho z$$

$$\ddot{z} = -\left(\frac{\rho g \sin\alpha}{\pi d a}\right) z$$

We recognize this equation as the equation describing simple harmonic motion with angular frequency given by

$$\omega_z = \sqrt{\frac{\rho g \sin\alpha}{\pi d a}}$$

Substitute $\frac{\pi d}{\rho} = \frac{\alpha - \cos\alpha\sin\alpha}{2}$ to obtain

$$\omega_z = \sqrt{\frac{\rho g \sin\alpha}{\pi d a}} = \sqrt{\frac{2g\sin\alpha}{a(\alpha - \cos\alpha\sin\alpha)}} \quad \textbf{Ans}$$

3.3 It is reasonable to assume that the system swings about the centre of the cylinder and the volume of of displaced sea water remains the same throughout the motion.

Observe also that the centre of mass of the system is at the point where the rod joins the cylinder.

Moment of inertia of the cylinder about its centre of mass is given by

$$I_C = \frac{1}{2}M_C a^2$$

where mass of the cylinder $M_c = \pi a^2 ld$

Moment of inertia of the rod about its centre of mass is given by

$$I_R = \frac{1}{3}M_C a^2$$

Apply Steiner's theorem to determine moment of inertia of the rod about the centre of mass of the cylinder

$$I_{R'} = \frac{1}{3}M_C a^2 + M_C(2a)^2 = \frac{13}{3}M_C a^2$$

The net moment of inertia of the system of the cylinder and the rod about the centre of mass of the cylinder is,

$$I_T = \frac{13}{3}M_C a^2 + \frac{1}{2}M_C a^2$$
$$= \frac{29}{6}M_C a^2$$

At the position the rod makes angle $\theta$ with the vertical, torque on the system is given by

$$\tau = M_C g 2a\sin\theta$$

For small $\theta$

$$\tau \approx 2agM_C\theta$$

Equation of motion of the system is

$$2agM_C\theta = -I_T\ddot{\theta}$$

$$2agM_C\theta = -\frac{29}{6}M_C a^2\ddot{\theta}$$

$$\ddot{\theta} = -\frac{12g}{29a}\theta$$

Thre angular frequency of the rotational motion of the system of the cylinder and the rod is

$$\omega_\theta = \sqrt{\frac{12g}{29a}} \quad \textbf{Ans}$$

3.4 From data provided by the problem

$$\frac{T_\theta}{T_z} \cong \frac{7}{4}$$

Or $$\left(\frac{\omega_z}{\omega_\theta}\right)^2 \cong \frac{49}{16} \cong 3.06$$

Substitute the values of $$\omega_z = \sqrt{\frac{2g\sin\alpha}{a(\alpha - \cos\alpha\sin\alpha)}}$$

and $$\omega_\theta = \sqrt{\frac{12g}{29a}}$$

$$\left[\frac{2g\sin\alpha}{a(\alpha-\cos\alpha\sin\alpha)}\Big/\frac{12g}{29a}\right] = 3.06$$

$$\alpha - \cos\alpha\sin\alpha \cong \frac{5}{3}\sin\alpha$$

$$\alpha - \frac{\sin 2\alpha}{2} \cong 1.6\sin\alpha$$

Since $\frac{\pi}{2} \approx 1.6$, the above equation is satisfied when $\alpha = \frac{\pi}{2} = 1.6$ **Ans**

And the expression for $\omega_z$ becomes

$$\omega_z = \sqrt{\frac{2g}{a\left(\pi/2\right)}} = \sqrt{\frac{4g}{\pi a}}$$

Since the period of vertical oscilation $T_z = \frac{2\pi}{\omega_z} = 1$

Hence $$2\pi\sqrt{\frac{\pi a}{4g}} = 1$$

$$\frac{\pi^3 a}{g} = 1$$

$$a = \frac{g}{\pi^3} = \frac{9.8}{31.4} = 0.32$$

Mass of the the buoy system is given by,

$$2M_c = 2\pi a^2 l d$$

$$= 2\pi a^3 d$$

$$= 2 \times 3.14 \times .32^3 \times 500$$

$$= 50 \quad \text{kg} \qquad \textbf{Ans}$$

# Experiment

## Problem 1 Terminal velocity in a viscous fluid

An object falling in the liquid will eventually reach a constant velocity, called the terminal velocity. The aim of this experiment is to measure the terminal velocities of objects falling through glycerine.

For a sphere of radius r moving at velocity v through viscous liquid, the viscous force F is given by $F = 6\pi\eta r v$. Here $\eta$ is property of the liquid, called viscosity. In this experiment you will measure the terminal velocity of metal cylinders ( because cylinders are easier to make than spheres). The diameter of each cylinder is equal to its length, and we will assume the viscous force on such a cylinder is similar to the viscous force on a sphere of the same diameter, 2r:

$$F_{cyl} = 6\pi\kappa\eta r^m v \qquad (1)$$

where $\kappa = 1$, $m = 1$ for a sphere.

### Premliminary: Calculation of Terminal Velocity

If $\rho$ is the density of the cylinder and $\rho'$ is the density of the liquid, show that the terminal velocity $v_T$ of the cylinder is given by

$$v_T = C r^{3-m} (\rho - \rho') \qquad (2)$$

where C is a constant . Derive an expression for C.

### Experiment

Use the equipment available to determine the numerical value of the exponent m and the density of glycerine.

### Notes

- For consistency, try to ensure that the cylinders fall in the same orientation, with the axis of the cylinder horizontal.
- The tolerance on the diameter and the length of the cylinders is 0.05 mm (you need not measure them yourself).

- There is a brass sieve inside the container that you should use to retrieve the metal cylinder. **Important**: Make sure the sieve is in place before dropping objects in glycerine, otherwise you will not be able to retrieve them for repeating measurements.

- When glycerine absorbs water from the atmosphere, it becomes less viscous. Ensure that the cylinder of glycerine is covered with the plastic film provided, when not in use.

- Do not mix cylinders of differrent sizes and different materials after the experiment.

| **Material** | **Density ( kg $m^{-3}$)** |
|---|---|
| Aluminum | 2.70 103 |
| Titanium | 4.54 103 |
| Stainless steel | 7.87 103 |
| Copper | 8.96 103 |

- Provided are :

1 1000 mL measuring cylinder filled with glycerine
1 container of glycerine for topping up the measuring cylinder
1 electronic stopwatch
1 30 cm ruler
1 clothpeg
1 sieve for retrieving metal cylinders
1 pair of tweezers
16 10.00 mm diameter aluminium cylinders
6 × 8.00 mm diameter aluminium cylinders
6 × 5.00 mm diameter aluminium cylinders
6 × 4.00 mm diameter aluminium cylinders
6 × 4.00 mm diameter titanium cylinders
6 × 4.00 mm diameter stainless steel cylinders
6 × 4.00 mm diameter copper cylinders
Linear and log-log graph paper

# Solution

At terminal velocity, the net force on the cylinder as well as the acceleration is zero, i.e

$$V\rho g - 6\pi\kappa\eta r^m v_T - V\rho' g = 0$$

where the volume of the falling cylinder $V = 2\pi r^3$ ( lenght of the cylinder = 2r ).

Thus $$v_T = C r^{3-m}(\rho - \rho')$$

where $$C = \frac{g}{3\kappa\eta}$$

1. Measurement of terminal velocity

- Calibrate the scale on the measuring cylinder using the ruler provided.
- Release a 5 mm aluminum cylinder from the position in which its axis lying in the horizontal position.
- Measure time taken t for the cylinder to fall over distance s in the range of constant velocity. Repeat the experiment
- Plot a graph of s versus time t. The curve should be a linear one with the slope equal to $v_T$.

2. Determination of variation of terminal velocity with size of the cylinder r.

- For aluminum cylinders of different radii, measaure falling time t over fixed distance s.
- Plot a graph of logt versus radius log r.

The curve shoul be linear, with slope = 3 - m .

3. Determination of variation of terminal velocity with density .

- For all 4 mm cylinders in tin, copper, steel and aluminum ( same volume but different densities), measure time taken tfor the cylinders to fall over fixed distance.
- Plot a graph of $v_T$ versus density $\rho$.

The curve should be linear ,with the intercept on the horizontal axis divided by slope = $\rho'$.

# Problem 2. Diffraction and Scattering of Laser Light

**Donot look directly into the laser beam , it may damge you r vision.**

The aim of this experiment is to demonstrate and quantify to some extent the reflection, diffraction and scattering of light, using visible radiation from a Laser Diode source. A metal ruler is employed as a diffraction grating. A perspex tank, containing water and diluted milk, is used to determine reflection and scattering phenomena.

**Section 1:**

Place the 150 mm length ruler provided so that its polished section is nearly normal to the incident laser beam which illuminates several rulings on it. Observe a number of "spots" of light on the white paper screen provided, caused by the phenomenon of diffraction.

Measure the position and separation of these spots with the screen at a distance of approximately 1.5 meters from the ruler, and draw the overall geometry you have employed.

Using the relation

$$N\lambda = \pm h \sin\beta$$

where N is the order of diffraction
$\lambda$ is the radiation wavelength
h is the grating period
$\beta$ is the angle of diffraction

and the information obtained from your measurements , determine the wavelength of the laser radiation and the experimental error in this determination.

**Section 2:**

Insert the empty perspex tank provided into the space between the laser and the white paper screen. Set the tank at approximately normal incidence to the laser beam.

2.1 Observe a reduction in the emergent beam intensity, estimate the percentage value of the reduction. Some calibrated transmission discs are provided to assist with this estimation. Remember that the human eye has a logarithmic response.

This intensity reduction is caused primarily by reflection losses at the air/perspex boundaries, of which there are four in this case.
The reflection coefficient for normal incidence at each boundary R, which is the ratio of the reflected to the incident intensities, is given by

$$R = [(n_1 - n_2)/(n_1 + n_2)]^2$$

Where $n_1$ and $n_2$ are the refractive indices before and after the boundary. The corresponding transition coefficient, assuming zero absorption in the perspex, is given by

$$T = 1 - R$$

2.2 Assuming a refractive index of 1.59 for the perspex, and neglecting the effect of multiple reflections and coherence, calculating the intensity transmission coefficient of the empty perspex tank. Compare this result with the estimate you make in Part ( 2.1 ) of this section.

**Section 3**

Without moving the perspex tank, repeat the observations and calculations in Section 2 with 50 ml of water provided in a beaker now added to the tank. Assume the refractive index of water to be 1.33.

**Section 4**

4.1 Add 0.5 ml (12 drops) of milk (the scattering material) to the 50 ml of water in the perspex tank, and stir well. Measure as accurately as possible the total angle through which the laser light is scattered, and the diameter of the emerging light patch at the exit face of the tank, noting that these quantities are related. Also estimate the reduction in transmitted intensity, as in the earlier sections.

4.2 Add an additional 0.5 ml of milk to the tank, and repeat the measurements requested in Part 4.1.

4.3 Repeat the process in Part 4.2 until very little or no transmitted laser light can be observed.

4.4 Determine the relationship between scattering angle and milk concentration in the tank.

4.5 Use your results, and the relationship

$$I = I_0 e^{-\mu z} = T_{milk} I_0$$

where $I_0$ is the input intensity
$I$ is the emerging intensity
$z$ is the distance in the tank
$\mu$ is the attenuation coefficient = constant × C (concentration of the scatterer)
$T_{milk}$ is the transmission coefficient for the milk

to obtain an estimate for the value of $\mu$ for a scatterer concentration of 10%.

## Apparatus

Students may not need to use all equipment, i.e.

- Laser diode light source
- Metal ruler as diffraction grating
- Perspex tank containing water and milk for reflection and scattering phenomena.
- Tape measure
- White paper screen
- Diverging lens
- Converging lens
- Protractor
- Transmission filters
- Beaker and dropper
- Stirrer
- Linear-linear and log-linear graph paper.

# Solution

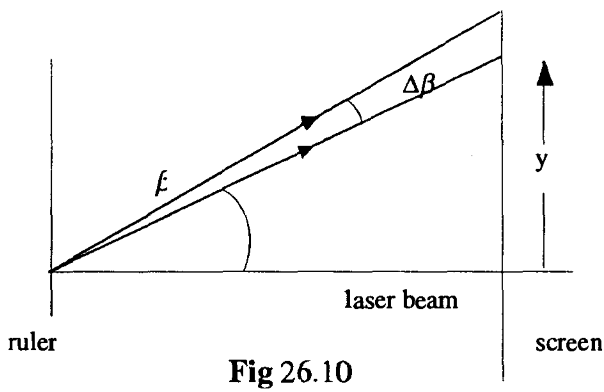



**Fig** 26.10

Distance between the ruler and the screen should be of the order of 15 to 20 cm.

At the position of the dot of Nth order

$$h\sin\beta = N\lambda \tag{1}$$

also for the dot of (N +1)th order

$$h\sin(\beta+\Delta\beta) = (N+1)\lambda \tag{2}$$

For small $\beta$ $\quad \sin\beta \approx \beta = \frac{y}{D}$

$$\sin(\beta+\Delta\beta) \approx \beta + \Delta\beta = \frac{y+\Delta y}{D}$$

where D is the distance between the ruler and the screen.

(2) - (1) $\quad \frac{h\Delta y}{D} \approx \lambda$

$\Delta y$ is the separation between two consecutive dots.
Thus measure the distance of ,say, 10 dots and calculate the separation between two nearest dots and wavelength from the formula $\lambda = \dfrac{h\Delta y}{D}$

The error in $\lambda$ is mainly due to error in $\Delta y$

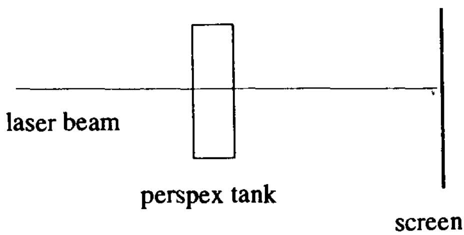



**Fig 26.11**

1.2 Insert the perspex tank between the laser source and the white screen. The tank should be normal to the incident beam as shown in diagram.

As the beam passes through the tank it loses its intensity mainly by relflection. The reflectivity or coefficient of reflection for normal incidence at each surface or boundary is given by,

$$R = \left[\frac{(n_1 - n_2)}{(n_1 + n_2)}\right]^2$$

where $n_1$ and $n_2$ are the refractive indices of the materials before and after the boundary. Also note that reflectivity for light entering and emerging from the perspex is the same.

If the absorption is assumed to be negligligble, the corresponding transmission coeeficient in the perspex is given by

$$T = 1 - R$$

For an empty perspex tank, $n_1 = 1, n_2 = 1.59$

$$T = 1 - R = \frac{4n_1n_2}{(n_1 + n_2)^2}$$

$$= \frac{4 \times 1.59}{2.59^2} = 0.948$$

For transmission at the four boundaries

$$T_{Total} = 0.948^4 = .808$$

1.3 With water in the perspex tank, transmission coefficient at the boundary of the perspex and water is

$$T_{Perspex/water} = \frac{4 \times 1.59 \times 1.33}{(1.59 + 1.33)^2} = \frac{4 \times 1.59 \times 1.33}{2.92^2}$$

$$= 0.992$$

$$T_{Total} = 0.948^2 \times 0.992^2 = 0.884$$

1.4

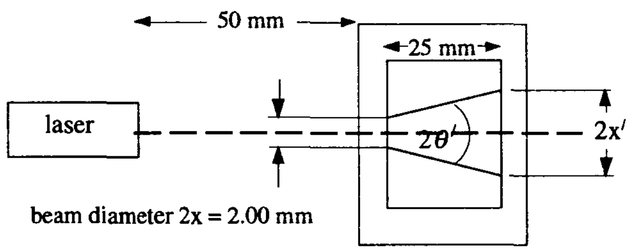



**Fig** 26.12

With 0.5 ml milk in 50 ml water concentration of milk as scatterer, C $= \frac{0.50}{50} = 0.01$

Measure the diameter $2x'$ of the divergent beam at the inner surface of the perspex tank, and calculate the scattering angle $2\theta'$.

Without the tank, use a combination of filters to produce transmitted beam of the same intensity of laser light emerging from the exit face of the perspex tank. Calculate total transmission $T_{Total}$ from the known values of transmission coeeficients of filters employed.

$$T_{Total} = T_{Milk} \cdot T_{Water}$$
$$T_{Milk} = T_{Total} / T_{Water}$$

Tabulte the results in the table such as the one below

| Milk Volume Ml | 0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
|---|---|---|---|---|---|---|---|---|---|
| Concentration(%) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $2\theta'$ ($\phi$) | | | | | | | | | |
| $T_{Milk}$ | | | | | | | | | |

Plot a graph of $\phi$ against C to demonstrate a linear relationship of the two variables in the form;

$$\phi = kC$$

From the realtionship,

$$I = I_0 e^{-\mu z} = T_{Milk} I_0$$

$$T_{Milk} = e^{-\mu z}$$

$$\log_e T_{Milk} = -\mu z = -kCz$$

where $\mu = kC$

Plot a graph of $\log_e T_{Milk}$ against C, the graph should be linear.

The slope m of the curve can be read of the graph .

$$m = kz$$

For z = 25 mm, k can be calculated, and thus $\mu$ can also be calculated for C = 10 %.

# Syllabus for International Physics Olympiads

In order to make International Physics Olympiads interesting and have impacts on the teaching and learning of Physics in participating countries, the International Physics Olympiad Committee has listed topics -theoretical as well as practical, which may be asked in the contest.

Since the First International Physics Olympiad held in 1967, there has been a continuing effort to amend and modify the list of t opics for the contest. Only during the years 1985-1986, a syllabus was drafted and submitted for approval at the meeting of International Physics Olympiad Committee held in Warsaw, Poland in 1989. This syllabus was amended and approved at the meeting of the International Physics Olympiad Committee held in Gronningen, the Netherlands in 1990. Since then there have been a few minor changes but the essence of this sytllabus still remains in effective use up to the present times.

## General

1. Knowledge of calculus(differential and integral) and complex numbers to solve differential equations in theoretical and practical works.
2. Questions may cover topics not listed in the syllabus. In such case, the question must supply necessary information so that the contestants who have no previous knowledge on the topic are not at disadvantage.
3. In the practical contest, the contestants may be required to use complicated instruments, in such case, data and instruction on the use of that instruments must be provided.
4. The contestants are expected to have knowledge on units in use in the country in which those units originate.
5. The contestants should have experience and skills in solving International Physics Olympiad problems of all preceding years.

## a.Theory

(Information in the first column lists main topics and those in the second column is description in further detail)

| | |
|---|---|
| 1.Basic principles on motion of particles. | use of vectors to describe position and velocity |
| 2.Newton's laws of motion, inertial frame | question involving change of mass may be asked |
| 3. Closed system, open system momentum, energy | |

| | |
|---|---|
| 4. Law of conservation of energy, law of conservation of momentum, impulse | |
| 5.Elasticforce,frictionalforce,universal law of gravitation | Hooke's law, coefficient of friction $F \leq \mu N$ , static friction |

## 2. Mechanics of Rigid Bodies

| | |
|---|---|
| 1. Statics, centre of mass,torque | Couple, condition for equilibrium |
| 2.Motion of rigid bodies angular velocity, angular acceleration conservation of angular momen tum | Conservation of angular momentum about a fixed axis |
| 3.External and internal forces, equation of motion of rigid body about a fixed axis, moment of inertia, kinetic energy of rigid body in rotational motion | Parallel -axes Theorem (Steiner's Theorem), addition of moment s of inertia |
| 4. Accelerated frame of reference (non-inertial frame) | Not include coriolis force |

## 3. Fluid Mechanics

There will be no question on fluid mechanics, the contestants however are expected to have basic knowledge on pressure, bouyancy, and equation of continuity.

## 4.Thermodynamics and Molecular Physics

| | |
|---|---|
| 1.Internal energy, work, and thermal energy, 1st and 2nd laws of thermodynamics | Thermal equilibrium quantities which depend on states and processes |
| 2. Model of ideal gas, pressure and molecular kinetic enrgy,Avogadro's number, equation of state for ideal gas, absolute temperature. | Calculation of molecular changes for liquid and solid, such as boiling point, melting point etc. |
| 3.Work done by expanding gas, isothermal and adiabic processes. | Knowledge on the derivation of adiabatic equation is not required |

| | |
|---|---|
| 4.Carnot's cycle, thermal efficiency, reversible and non-reversible processes | Entropy in the form that does not depend on the path of change for non-reversible and quasi-static process. |

## 5. Oscillation and Waves

| | |
|---|---|
| 1.Simple harmonic motion, simple harmonic equation. | Solutions of equation of simple harmonic motion, attenuation and resonance( nocalculation) |
| 2.Harmonic wave, motion of transver sal and longitudinal waves, linear polarization, classical Doppler's effect for sound wave. | Motion of particles in a medium during transmission of sound wave. wave diagram, measurement of velocities of light and sond waves, Doppler's effect in one dimension.Propagation of wave in homogeneous medium, reflection and refraction of waves,Fermat's principle. |
| 3. Superposition of harmonic waves, coherent waves, interference and beats | Wave intensity in direct proportion to square of amplitude. Knowledge of Fourier's analysis is not required. The contestants are however expected to know that non-harmonic wave consists of harmonic waves of various different frequencies.<br>Interference in thin film and others, superposition of wavelets (secondary waves) |

## 6. Electric Charge and Field

| | |
|---|---|
| 1. Conservation of charge | |
| 2. Electric field electric potential Guass's law | Application of Gauss's law to symmetric systems such as spheres planes, etc. dipole and dipole moment |
| 3.Capacitor, capacitance, dielectric constant, energy density of electric field | |

## 7. Current and Magnetic Field

| | |
|---|---|
| 1. Electric current, resistance, internal resistance of current sources. Ohm's law, Kirchhoff's law, power of AC current, Joule's law | Knowledge on circuits conisting of non-ohmic electronic items, having their own characteristic V-I relationships. |

| | |
|---|---|
| 2.Magnetic field due to electric current, Lorentz's force | Charged particle in magnetic field, application of the principle to cyclotron and magnetic dipole. |
| 3. Ampere's law | Manetic field of systems with symmetry such as an infinitely long straight wire, a ring, and a solenoid carrying DC current. |
| 4.Electromagnetic induction, mag netic flux, inductance, permeability, energy density of magnetic field. | |
| 5. AC current, resistors, inductors and capacitors in AC circuit, resonance. | Simple AC circuits, knowledge on formulas pertaining to resonant circuits consisting of various types of parameters is not required. |

**8. Electromagnetic Waves**

| | |
|---|---|
| 1. Oscillating circuit, oscillation frequency feedback and resonance. | |
| 2. Light waves, diffraction at single, and doubled slits. Grating, resolv ing power of grating. Bragg's scattering. | |
| 3. Dispersion spectrum and diffrac tion, line spectrum of gas. | |
| 4. Electromagnetic waves, properties of transversal waves, polarization due to reflection. | |
| 5. Resolving power of image-giving optical instruments. | |
| 6. Black body, Stefan -Boltzman 's law. | Not finclude Planck's formula. |

**9.Quantum Physics**

| | |
|---|---|
| 1.Photoelectric effect, photon's impulse. | Include Einstein's formula. |
| 2. De Broglie'swavelength,Heisenberg's Principle of Uncertainty | |

1. The principle of theory of Relativ ity, addition of velocity vectors using principles of theory of relativity. Phenomena explained by theory of relativity.

2. Equation of motion momentum, energy relationship between mass and energy. Law of conservation of energy and mass based on theory of relativity.

**11.Material Science**

1. Simple application of Bragg's formula

2. Energy levels in aoms and mol cules. (Exact calculation is not required) Photon emission and absorption. Hydrogen-like spectrum.

3. Nuclear energy levels( Exact calcu lation is not needed), alpha, beta and gamma decay.Absoption half- life , exponentialdecay,nuclus constituents, mass defect, nuclear reac tion.

## b. Experiment

It is to be understood that theoretical knowledge as described in the syllabus also forms the basis for practical work, and every practical problem will aways involve measurements.

Contestants should have knowledge and skills in the following:

1. Aware that measuring devices influence the values of what are to be measured or measured.
2. Knowlege on the principles and techniques of various measurements described in the syllabus.
3. Knowledge and skills in the use of instruments commonly employed in regular physics laboratories.
4. Knowlege and skills in performing calculation and estimating errors or error of measured or calculated values.

5. Knowledge and skills in the use of standard or sophisticated instruments when provided with manuals on their uses: such as dual -channeled ocilloscope, counter, ratemeter, signal and function generator, analog-to- digital converter, interphase for computer, amplifier, integrater, differentiater, power supply, analog and digital voltmeter, ohmmeter, ammeter etc..

6. Calculation and estimation of absolute and relative errors, accuracy of measuring devices, errors of individual measurements, errors of a series of measurements, error of derivative quantity as a function of quantities which constitute that derivative quantity

7. Transformation of function into a linear type by choosing appropriate variables and drawing the best possible line through points obtained via the experiment

8. Drawing graphs using different scales and suitable scales ( such as on a polar or logarithm graph paper

10. Knowledge of the standards of safety in the laboratory ( in case of an experiment with risks to injuries or accident, there will be instruction for the contestant to observe and to take proper precaution against such mishaps)

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